Department of Physics

Examination paper for TFY4275 Classical Transport Theory
Examination date: December 14, 2020
Examination time 09:00-13:00:
Permitted examination support material: A / All support material is allowed
Academic contact during examination: Professor Asle Sudbø Phone: 40485727
Technical support during examination: Orakel support services Phone: 73 59 16 00

OTHER INFORMATION

Make your own assumptions: If a question is unclear/vague, make your own assumptions and specify them in your answer. Contact academic contact only in case of errors or insufficiencies in the question set.
Saving: Answers written in Inspera Assessment are automatically saved every 15 seconds. If you are working in another program remember to save your answer regularly.
Cheating/Plagiarism:

NB! The exam is an individual, independent work. During the exam it is not permitted to communicate with others about the exam questions, or distribute drafts for solutions. Such communication is regarded as cheating. All submitted answers will be subject to plagiarism control.

Examination aids are permitted, but make sure you follow any instructions regarding citations.

Citations: If you use results from published scientific literature, textbooks, or lecture notes, make sure that you include a citation to those works.
Notifications: If there is a need to send a message to the candidates during the exam (e.g. if there is an error in the question set), this will be done by sending a notification in Inspera. A dialogue box will appear. You can re-read the notification by clicking the bell icon in the top right-hand corner of the screen. All candidates will also receive an SMS to ensure that nobody misses out on important information. Please keep your phone available during the exam.
Weighting: HOW THE QUESTIONS ARE WEIGHTED IS SHOWN ON EACH PROBLEM SET.

ABOUT SUBMISSION

File upload: All files must be uploaded before the examination time expires. 30 minutes are added to the examination time to manage the sketches/calculations/files. (The additional time is included in the remaining examination time shown in the top left-hand corner.) How to digitize your sketches/calculations How to create PDF documents Remove personal information from the file(s) you want to upload
NB! You are responsible for ensuring that you upload the correct file(s) for all questions. Check the file(s) you have uploaded by clicking "Download" when viewing the question. All files can be removed or replaced as long as the test is open.

An additional 30 minutes are reserved for submission. If you experience technical problems during upload/submission, you must contact technical support before the examination time expires. If you can't get through immediately, hold the line until you get an answer.

Your answer will be submitted automatically when the examination time expires and the test closes, if you have answered at least one question. This will happen even if you do not click "Submit and return to dashboard" on the last page of the question set. You can reopen and edit your answer as long as the test is open. If no questions are answered by the time the examination time expires, your answer will not be submitted.

Withdrawing from the exam: If you become ill, or wish to submit a blank test/withdraw from the exam for another reason, go to the menu in the top right-hand corner and click "Submit blank". This cannot be undone, even if the test is still open.

Accessing your answer post-submission: You will find your answer in Archive when the examination time has expired.

<u>Problem 1.</u> (Points: 5+10+10+5+10+5= 45)

The Langevin equation describes the motion of a heavy particle of mass m in the fluid background of light particles, where the heavy particle is subject to an external force F, a frictional force from the surrounding fluid, and a stochastic random force $R(t)$ from the surrounding fluid. We consider the motion of the particle to be one-dimensional, its position at time t is $x(t)$ and its velocity is $v(t)$. The frictional force is taken to be $-kv$, where $k > 0$ is a frictional coefficient. We take the external force $F = F_0 > 0$ to be independent of time and position. The stochastic force $R(t)$ is assumed to have an average over sample paths $\langle R(t)\rangle = 0$, and an auto-correlation function $\langle R(t)R(t+\tau)\rangle = \langle R^2\rangle\delta(\tau)$. At time $t = 0$, the heavy particle has a velocity v_0.

a) Set up the Langevin-equation for this problem.

b) Find $v(t)$ expressed in terms of initial conditions, elementary functions, and an appropriate time-integral involving the stochastic force $R(t)$.

c) Compute $\langle v(t)\rangle$ and $\sigma^2(t) \equiv \langle v^2\rangle - \langle v\rangle^2$.

d) From $\lim_{t\to\infty} \sigma^2(t)$, find $\langle R^2\rangle$ expressed in terms of equilibrium properties of the system.

d) Compute the autocorrelation function $\langle v(t_1)v(t_2)\rangle$ in the limit $t_1 \to \infty$, $t_2 \to \infty$, $t_2 - t_2 = \tau$.

e) Is $v(t)$ a Markov-process? Give a short reason for your answer. (Answers with no reasoning receive zero score).

Note: The problem has been solved in the lecture notes for the case $F_0 = 0$. You are allowed to use the lecture notes during the exam. A solution to the problem for $F_0 = 0$ will therefore receive zero score on the entire **<u>Problem 1</u>**.

<u>Problem 2.</u> (Points: 5+5+5+10=25)

The Boltzmann equation is a kinetic equation for the evolution of a single-particle probability distribution function $f(\mathbf{r}, \mathbf{v}, t)$, where $\mathbf{r}$ is the position and $\mathbf{v}$ is the velocity of the particle, and t is time.

a) Write down the Boltzmann equation and explain what the terms mean.

b) The microscopic equations of motion for classical particles, underlying the Boltzmann-equation, are time-reversal invariant. However, the Boltzmann-equation is not time-reversal invariant. Explain precisely which step that goes into the derivation of the Boltzmann-equation it is, that breaks time-reversal invariance.

c) Associated with the Boltzmann equation, there is a so-called H-theorem. State the theorem and explain what it means physically.

d) Consider now the Boltzmann equation on standard form and write down the simplest physically sensible non-zero approximation to the right-hand-side of the equation that you can think of. Solve the Boltzmann equation for this case, and describe the time-dependence of the answer you get.