TFY 4275 Classical Transport Theory

Final Exam May 16, 2015

Solution Set.

Problem 1

a) $$|\dot{v}| = \frac{v^2}{R}$$

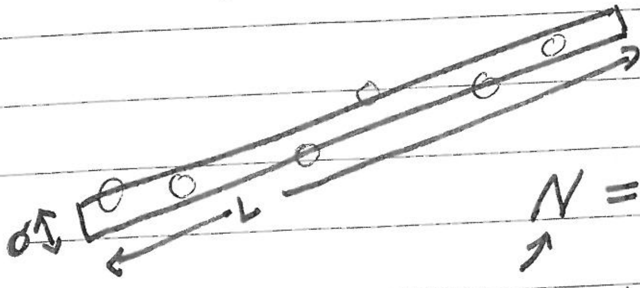
$$m\frac{v^2}{R} = evB \Rightarrow R = \frac{mv}{eB}$$

$$\left.\begin{aligned} v &= \frac{2\pi R}{T} \\ \omega &= \frac{2\pi}{T} \end{aligned}\right\} \Rightarrow \omega = \frac{eB}{m}$$

b) $$\sigma = \int_{-\pi}^{\pi} d\varphi \frac{d\sigma}{d\varphi} = \frac{a}{2}\int_{-\pi}^{\pi} d\varphi \sin\left|\frac{\varphi}{2}\right|$$

$$= a\int_{0}^{\pi} d\varphi \sin\frac{\varphi}{2} = 2a\int_{0}^{\pi/2} d\varphi \sin\varphi = 2a$$

c)



$N = nL\sigma$

\# of disks in rectangle.

$$\Lambda = \frac{L}{N} = \frac{1}{n\sigma}$$

$$\tau = \frac{\Lambda}{v} = \frac{1}{nv\sigma}$$

d) Probability density for hitting a scatterer per length is:

$$N = n L \sigma \, ; \qquad p_e = \frac{N}{L} = \frac{1}{\lambda} \quad (\text{see c)})$$

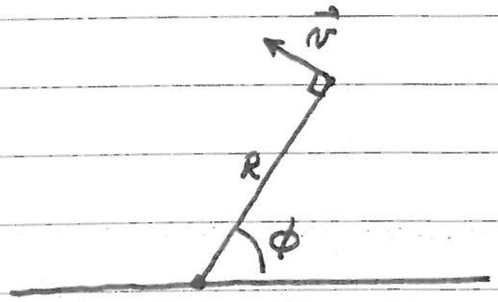
Probability of not hitting a scatterer over a distance dl is

$$1 - \frac{dl}{\lambda}$$

The full circle: $\underline{\underline{P_0}} = \lim_{dl \to 0} \left(1 - \frac{dl}{\lambda}\right)^{\frac{2\pi R}{dl}}$

$$\underline{\underline{= e^{-2\pi R/\lambda}}}$$

e)



$$\vec{v} \cdot \frac{\partial}{\partial \vec{r}} = \underbrace{(\omega R)}_{v_\phi} \frac{\partial}{\partial R\phi} = \omega \frac{\partial}{\partial \phi}$$

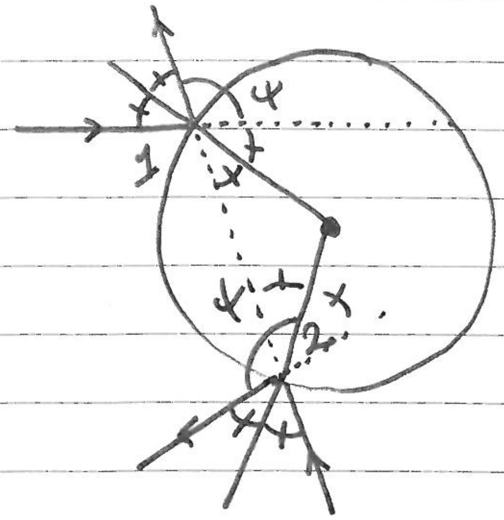
f) $\frac{1}{\tau} \int_{-\pi}^{\pi} d\psi \, g(\psi) \, f(\phi - \psi, t)$

is scattering <u>into</u> the ϕ-direction and

$$-\frac{1}{\tau} \int_{-\pi}^{\pi} d\psi \, g(\psi) \, f(\phi, t) = -\frac{1}{\tau} f(\phi, t)$$

is scattering <u>out</u> of the ϕ-direction.

g)



h) $$\frac{1}{\tau} P_0^k \int_{-\pi}^{+\pi} d\psi\, g(\psi) \left[f(\phi-(k+1)\psi, t-kT) - f(\phi - k\psi, t-kT) \right]$$

Probability that k circlings have been performed.

$\phi - k\psi$ is the angle the particle had at $t-kT$, for it to be scattered an angle ψ from direction ϕ at time t.

The two terms are the "into-the-beam" and "out-of-the-beam" scattering terms – but time shifted to first collision.

i) <u>Molecular chaos</u>: There is never more than one collision with a given scatterer.

The process is <u>Markovian</u>: There is no memory in the equation.

j)

$$\left.\begin{aligned} T &= \frac{2\pi}{\omega} = \frac{2\pi m}{eB} \\ \tau &= \frac{1}{n\sigma v} \end{aligned}\right\} \quad \frac{T}{\tau} = \left(\frac{2\pi m n \sigma v}{e}\right)\frac{1}{B}$$

Hence, if $\underline{\underline{B \ll \frac{2\pi m n \sigma v}{e} \Rightarrow \frac{T}{\tau} \gg 1.}}$

k)

$$\frac{\partial}{\partial t} f(\phi, t) + \omega \frac{\partial}{\partial \phi} f(\phi, t) = \frac{1}{\tau}\int_{-\pi}^{\pi} d\psi\, g(\psi)\left(f(\phi-\psi, t) - f(\phi, t)\right)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$sF_m(s) - f_m(0) - im\omega F_m(s)$$

$$= \frac{1}{\tau}\int_{-\pi}^{\pi} d\psi\, g(\psi)\left(e^{im\psi} - 1\right) F_m(s)$$

$$\Rightarrow F_m(s) = \frac{f_m(0)}{s - im\omega + \frac{1}{\tau} - \frac{1}{\tau}\int_{-\pi}^{\pi} d\psi\, g(\psi) e^{im\psi}}$$

since $\int_{-\pi}^{\pi} d\psi\, g(\psi) = 1$

$$F_m(s) = \frac{f_m(0)}{s - im\omega + \frac{1}{\tau} - \frac{1}{\tau} g_m}$$

$$g_m = \frac{1}{1-4m^2}.$$

$$= \frac{\tau(1-4m^2)\, f_m(0)}{\tau(1-4m^2)(s - im\omega) - 4m^2}$$

b) $$F_m(s) = \int_0^\infty dt\, e^{-st} \underbrace{\int_{-\pi}^{+\pi} d\phi\, e^{im\phi} f(\phi,t)}_{= \langle e^{im\phi(t)} \rangle}$$

$$= \int_0^\infty dt\, e^{-st} \langle e^{im\phi(t)} \rangle.$$

$$F_1(s) = \int_0^\infty dt\, e^{-st} \langle e^{i\phi(t)} \rangle$$

$$F_1(0) = \int_0^\infty dt\, \langle e^{i\phi(t)} \rangle$$

$$= \int_0^\infty dt\, \langle \cos\phi(t) \rangle + i \int_0^\infty dt\, \langle \sin\phi(t) \rangle$$

$$\mathrm{Re}\, F_1(0) = \int_0^\infty dt\, \langle \cos\phi(t) \rangle. = \frac{2}{v^2 D}.$$

From equation (17):

$$f_1(0) = \int_{-\pi}^{\pi} d\phi\, e^{i\phi} f(\phi,0)$$

$$\phi(0) = 0 \Rightarrow f(\phi,0) = \delta(\phi)$$

$$\Rightarrow f_1(0) = \int_{-\pi}^{\pi} d\phi\, e^{i\phi} \delta(\phi) = \underline{1}$$

$$F_1(0) = \frac{3\tau}{4 - 3\tau i\omega}$$

$$\mathrm{Re}\, F_1(0) = \frac{1}{2}\left(F_1(0) + F_1(0)^*\right)$$

$$= \frac{12\tau}{16 + 9\tau^2\omega^2}$$

$$\Rightarrow \quad D = \frac{6\tau v^2}{16 + 9\tau^2\omega^2} = \underline{\underline{\frac{3\tau v^2}{8 + 18\pi^2(\tau/T)^2}}}$$