NORWEGIAN UNIVERSITY OF SCIENCE AND TECHNOLOGY
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EXAM
TFY4280 Signal Processing

Mon 06th of June 2011. 09:00

Examination support materials:

- Simple calculator (according to NTNU exam regulations)
- K. Rottmann: Matematisk formelsamling (eller tilsvarende)
- Carl Angell og Bjørn Ebbe Lian: Fysiske størrelser og enheter, navn og symboler (eller tilsvarende)

Answer must be written in English or Norwegian. Number of points given to each sub-question is given in bold font. The maximum score for the exam is **100p**. The exam consists of 7 questions: 4 questions 10 point each (section A) and 3 questions 20p each (section B). Attachment: 2 pages with transform tables and properties.

A

maximum 10 points for each of the question.

A1 (10p) Plot auto-correlation and self-convolution for each of the two signals shown in Figure 1. Both signals are defined for t in the range $[-\pi, \pi]$ and zero outside that range.

A2 (10p) Calculate response (output) for a unit step function input ($x(t) = \varepsilon(t)$) and delta impulse input $x(t) = \delta(t)$ from a given impulse response function in the time domain:

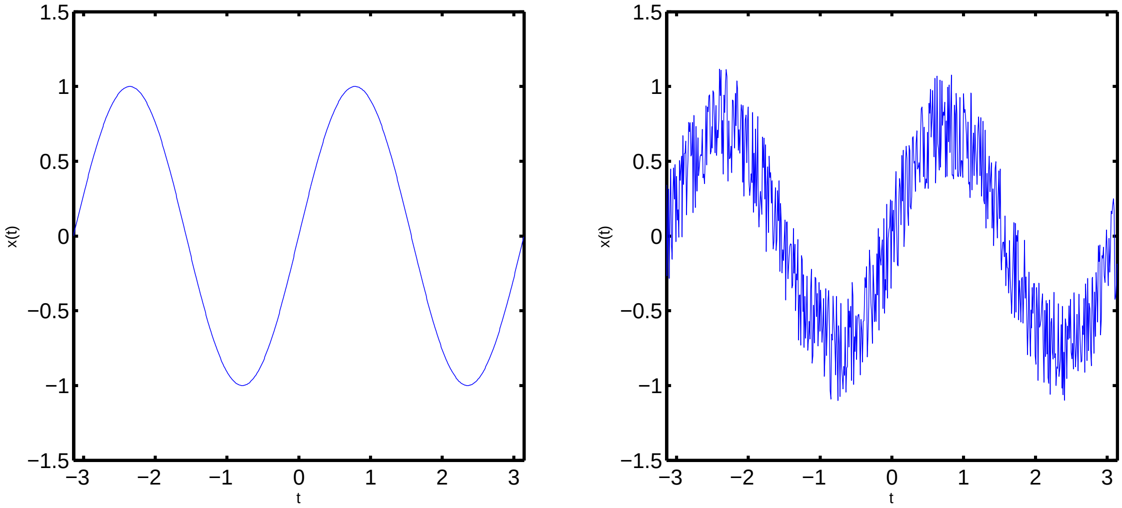
$$h(t) = 2\varepsilon(t) - 0.5e^{-3t}\varepsilon(t)$$

A3 (10p) Two uncorrelated processes $x(t)$ and $y(t)$ have the ensemble averages 2 and 0, respectively. Moreover, $E\{x^2(t)\} = 5$ and $E\{y^2(t)\} = 2$. Define the random process $z(t) = x(t) + y(t)$. Determine:$\mu_z(t)$, $E\{z^2(t)\}$ and $\sigma_z^2(t)$

A4 (10p) The signal $x(t)$ passes through a square law device giving output $y(t) = [x(t)]^2$

$$x(t) = \cos(10t) + \cos(11t)$$

Determine the Fourier transform of the output $\mathcal{F}\{y(t)\}$.



Figur 1: Question A1

B

maximum 20 points for each of the question.

B1 (20p)

1. Show that

$$\mathcal{L}\{t \cdot x(t)\} = -\frac{\mathrm{d}X(s)}{\mathrm{d}s}$$

2. Use above property to calculate output of LTI system where input $x(t) = te^{-9t}$ defined for $t > 0$ and the impulse response is given by:

$$H(s) = \frac{1}{(s+10)}$$

B2 (20p) A Hypothetical measured signal can be represented by an analytical formula:

$$x(t) = \left[1 - e^{-t} + \sin(10t)\right]\varepsilon(t)$$

Design and draw circuit diagram for a 1-st order low pass Butterworth filter which will remove 90.9090% of the power carried by the sin part of the signal. For 1-st order low pass Butterworth filter:

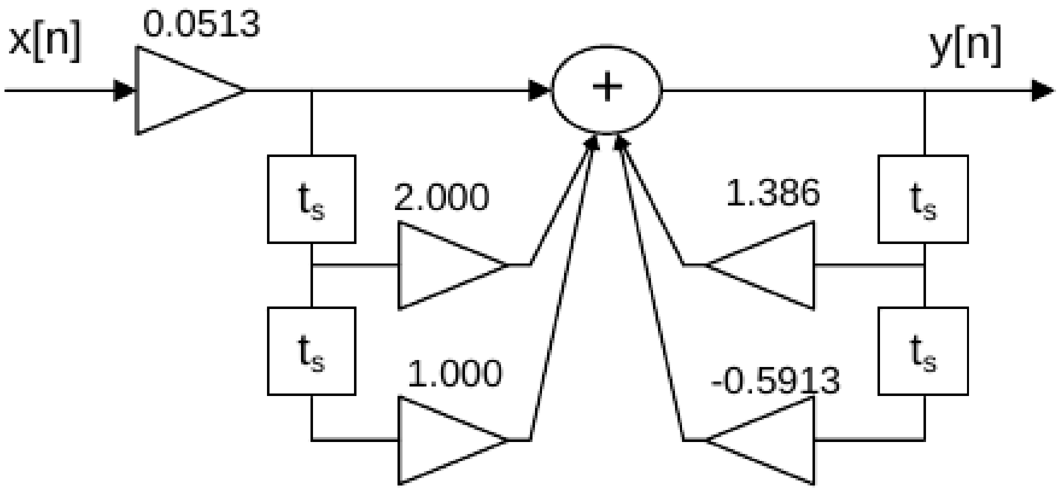
$$|H(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_c^2}}} \qquad \omega_c = \frac{1}{RC}$$

use R = 1000Ω. Calculate how this filter will effect on the non-sine exponential part of the signal, that is, calculate the output, if the input signal is given by

$$x_0(t) = \left[1 - e^{-t}\right]\varepsilon(t)$$

B3 (20p) The figure below (Figure 2) shows a digital filter in which the delays are 0.5 ms.

Write down the difference equation and from this derive Z-transform of the transfer function. Analyse the system and calculate the three first output terms ($0 \leq n < 3$) for the unit step excitation.



Figur 2: Question B3

Appendix B.1 Bilateral Laplace Transform Pairs

$x(t)$	$X(s)=\mathcal{L}\{x(t)\}$	ROC
$\delta(t)$	1	$s \in \mathbb{C}$
$\varepsilon(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\}>0$
$e^{-at}\varepsilon(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\}>\mathrm{Re}\{-a\}$
$-e^{-at}\varepsilon(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\}<\mathrm{Re}\{-a\}$
$t\varepsilon(t)$	$\frac{1}{s^2}$	$\mathrm{Re}\{s\}>0$
$t^n\varepsilon(t)$	$\frac{n!}{s^{n+1}}$	$\mathrm{Re}\{s\}>0$
$te^{-at}\varepsilon(t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}\{s\}>\mathrm{Re}\{-a\}$
$t^ne^{-at}\varepsilon(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\mathrm{Re}\{s\}>\mathrm{Re}\{-a\}$
$\sin(\omega_0t)\varepsilon(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\}>0$
$\cos(\omega_0t)\varepsilon(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\}>0$
$e^{-at}\cos(\omega_0t)\varepsilon(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\{s\}>\mathrm{Re}\{-a\}$
$e^{-at}\sin(\omega_0t)\varepsilon(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\{s\}>\mathrm{Re}\{-a\}$
$t\cos(\omega_0t)\varepsilon(t)$	$\frac{s^2-\omega_0^2}{(s^2+\omega_0^2)^2}$	$\mathrm{Re}\{s\}>0$
$t\sin(\omega_0t)\varepsilon(t)$	$\frac{2\omega_0 s}{(s^2+\omega_0^2)^2}$	$\mathrm{Re}\{s\}>0$

Appendix B.2 Properties of the Bilateral Laplace Transform

	$x(t)$	$X(s)=\mathcal{L}\{x(t)\}$	ROC
Linearity	$Ax_1(t)+Bx_2(t)$	$AX_1(s)+BX_2(s)$	ROC $\supseteq$ ROC$\{X_1\}$ $\cap$ROC$\{X_2\}$
Delay	$x(t-\tau)$	$e^{-s\tau}X(s)$	not affected
Modulation	$e^{at}x(t)$	$X(s-a)$	Re$\{a\}$ shifted by Re$\{a\}$ to the right
'Multiplication by t', Differentiation in the frequency domain	$tx(t)$	$-\frac{d}{ds}X(s)$	not affected
Differentiation in the time domain	$\frac{d}{dt}x(t)$	$sX(s)$	ROC $\supseteq$ ROC$\{X\}$
Integration	$\int_{-\infty}^{t}x(\tau)d\tau$	$\frac{1}{s}X(s)$	ROC $\supseteq$ ROC$\{X\}$ $\cap\{s:\mathrm{Re}\{s\}>0\}$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	ROC scaled by a factor of a

Appendix B.3 Fourier Transform Pairs

$x(t)$	$X(j\omega)=\mathcal{F}\{x(t)\}$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$\dot{\delta}(t)$	$j\omega$
$\frac{1}{T}\text{Ш}\left(\frac{t}{T}\right)$	$\text{Ш}\left(\frac{\omega T}{2\pi}\right)$
$\varepsilon(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$\mathrm{rect}(at)$	$\frac{1}{\lvert a\rvert}\mathrm{si}\left(\frac{\omega}{2a}\right)$
$\mathrm{si}(at)$	$\frac{\pi}{\lvert a\rvert}\mathrm{rect}\left(\frac{\omega}{2a}\right)$
$\frac{1}{t}$	$-j\pi\mathrm{sign}(\omega)$
$\mathrm{sign}(t)$	$\frac{2}{j\omega}$
$e^{j\omega_0t}$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$
$\sin(\omega_0t)$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$
$e^{-\alpha\lvert t\rvert},\ \alpha>0$	$\frac{2\alpha}{\alpha^2+\omega^2}$
$e^{-a^2t^2}$	$\frac{\sqrt{\pi}}{a}e^{-\frac{\omega^2}{4a^2}}$

Appendix B.4 Properties of the Fourier Transform

	$x(t)$	$X(j\omega)=\mathcal{F}\{x(t)\}$
Linearity	$Ax_1(t)+Bx_2(t)$	$AX_1(j\omega)+BX_2(j\omega)$
Delay	$x(t-\tau)$	$e^{-j\omega\tau}X(j\omega)$
Modulation	$e^{j\omega_0t}x(t)$	$X(j(\omega-\omega_0))$
'Multiplication by t' Differentiation in the frequency domain	$tx(t)$	$-\frac{dX(j\omega)}{d(j\omega)}$
Differentiation in the time domain	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t}x(\tau)d\tau$	$X(j\omega)\left[\pi\delta(\omega)+\frac{1}{j\omega}\right] = \frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right),\quad a\in\mathbb{R}\backslash\{0\}$
Convolution	$x_1(t)*x_2(t)$	$X_1(j\omega)\cdot X_2(j\omega)$
Multiplication	$x_1(t)\cdot x_2(t)$	$\frac{1}{2\pi}X_1(j\omega)*X_2(j\omega)$
Duality	$x_1(t)$ $x_2(jt)$	$x_2(j\omega)$ $2\pi x_1(-\omega)$
Symmetry relations	$x(-t)$ $x^*(t)$ $x^*(-t)$	$X(-j\omega)$ $X^*(-j\omega)$ $X^*(j\omega)$
Parseval theorem	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(j\omega)\rvert^2d\omega$

Appendix B.6 Properties of the z-Transform

Property	$x[k]$	$X(z)$	ROC
Linearity	$ax_1[k]+bx_2[k]$	$aX_1(z)+bX_2(z)$	ROC $\supseteq$ ROC$\{X_1\}\cap$ROC$\{X_2\}$
Delay	$x[k-\kappa]$	$z^{-\kappa}X(z)$	ROC$\{x\}$; separate consideration of $z=0$ and $z\to\infty$
Modulation	$a^k x[k]$	$X\left(\frac{z}{a}\right)$	ROC$=\left\{z \,\middle\vert\, \frac{z}{a}\in\text{ROC}\{x\}\right\}$
Multiplication by k	$kx[k]$	$-z\frac{dX(z)}{dz}$	ROC$\{x\}$; separate consideration of $z=0$
Time inversion	$x[-k]$	$X(z^{-1})$	ROC$=\{z \mid z^{-1}\in\text{ROC}\{x\}\}$
Convolution	$x_1[k]*x_2[k]$	$X_1(z)\cdot X_2(z)$	ROC $\supseteq$ ROC$\{x_1\}\cap$ROC$\{x_2\}$
Multiplication	$x_1[k]\cdot x_2[k]$	$\frac{1}{2\pi j}\oint X_1(\zeta)X_2\left(\frac{z}{\zeta}\right)\frac{1}{\zeta}d\zeta$	multiply the limits of the ROC

Appendix B.5 Two-sided z-Transform Pairs

$x[k]$	$X(z)=\mathcal{Z}\{x[k]\}$	ROC
$\delta[k]$	1	$z\in\mathbb{C}$
$\varepsilon[k]$	$\frac{z}{z-1}$	$\|z\|>1$
$a^k\varepsilon[k]$	$\frac{z}{z-a}$	$\|z\|>\|a\|$
$-a^k\varepsilon[-k-1]$	$\frac{z}{z-a}$	$\|z\|<\|a\|$
$k\varepsilon[k]$	$\frac{z}{(z-1)^2}$	$\|z\|>1$
$ka^k\varepsilon[k]$	$\frac{az}{(z-a)^2}$	$\|z\|>\|a\|$
$\sin(\Omega_0 k)\varepsilon[k]$	$\frac{z\sin\Omega_0}{z^2-2z\cos\Omega_0+1}$	$\|z\|>1$
$\cos(\Omega_0 k)\varepsilon[k]$	$\frac{z(z-\cos\Omega_0)}{z^2-2z\cos\Omega_0+1}$	$\|z\|>1$