Q1 DTFT and DFT (25p)

1. Explain what is described by the term "discrete frequency".
2. Find DTFT of the following discrete time signal:

$$x_1[n] = a^n u[n]$$

   where $u[n]$ is the discrete unit step function. Use $t_s = 1\text{ms}$.
3. Sketch the amplitude response for $a = 0.8$ using discrete frequency on the x-axis.
4. Sketch the amplitude response for $a = 0.8$ using frequency on the x-axis.
5. What would you needed to be able to calculate DFT of the same signal? What would be the main difference between DFT and DTFT? Please explain.

Q2 System output and frequency response. (25p)

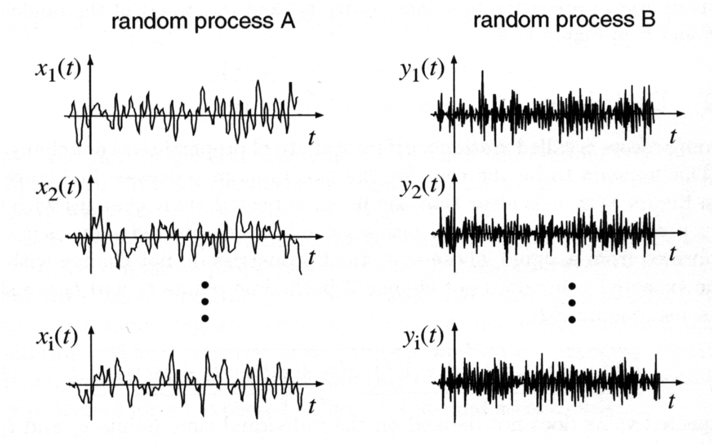
1. A LTI system is characterized by the impulse response function given below. Calculate and plot the response of that system to a unit step input $x_1(t) = \varepsilon(t)$ and to a delta impulse input $x_2(t) = \delta(t)$.

$$h(t) = \varepsilon(t)\mathrm{e}^{-t} - 2\mathrm{e}^{-3t}\varepsilon(t)$$

   Also here, $\varepsilon(t)$ is the unit step function.
2. Calculate frequency response of this system for $\omega = 0$ and $\omega = 2\pi\text{s}^{-1}$. Explain how you would calculate the frequency response for any given frequency.

Q3 Stochastic Signals (25p)

1. What is an "ergodic random process"? What is a "stationary random process"?
2. Define auto-correlation function (ACF). Sketch ACF for two signals, for which few sample functions are shown below, assuming that they are drawn on the same time scale.



Q4 Filters. (25p)

1. For time-discrete systems, filters are often characterized as IIR or FIR. Explain what is described by these terms.
2. For which of these filter types, can we use discrete convolution to calculate output for an input signal similar to $x[n] = \{\underline{1}, 1, 1, 1, 1, 1\}$? Do not calculate but explain.
3. A low pass filters is described by the difference equation given below. Use zeros/poles diagram on an appropriate frequency plane to illustrate that this system is indeed a low pas filter.

$$y[n] = \alpha x[n] + (1 - \alpha)y[n-1]$$

   where α is a constants and $0 < \alpha < 1$
4. Is this a IIR or FIR filter? Please explain.

Appendix B.1 Bilateral Laplace Transform Pairs

$x(t)$	$X(s)=\mathcal{L}\{x(t)\}$	ROC
$\delta(t)$	1	$s \in \mathbb{C}$
$\varepsilon(t)$	$\frac{1}{s}$	$\text{Re}\{s\}>0$
$e^{-at}\varepsilon(t)$	$\frac{1}{s+a}$	$\text{Re}\{s\}>\text{Re}\{-a\}$
$-e^{-at}\varepsilon(-t)$	$\frac{1}{s+a}$	$\text{Re}\{s\}<\text{Re}\{-a\}$
$t\varepsilon(t)$	$\frac{1}{s^2}$	$\text{Re}\{s\}>0$
$t^n\varepsilon(t)$	$\frac{n!}{s^{n+1}}$	$\text{Re}\{s\}>0$
$te^{-at}\varepsilon(t)$	$\frac{1}{(s+a)^2}$	$\text{Re}\{s\}>\text{Re}\{-a\}$
$t^ne^{-at}\varepsilon(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\text{Re}\{s\}>\text{Re}\{-a\}$
$\sin(\omega_0 t)\varepsilon(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\text{Re}\{s\}>0$
$\cos(\omega_0 t)\varepsilon(t)$	$\frac{s}{s^2+\omega_0^2}$	$\text{Re}\{s\}>0$
$e^{-at}\cos(\omega_0 t)\varepsilon(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\text{Re}\{s\}>\text{Re}\{-a\}$
$e^{-at}\sin(\omega_0 t)\varepsilon(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\text{Re}\{s\}>\text{Re}\{-a\}$
$t\cos(\omega_0 t)\varepsilon(t)$	$\frac{s^2-\omega_0^2}{(s^2+\omega_0^2)^2}$	$\text{Re}\{s\}>0$
$t\sin(\omega_0 t)\varepsilon(t)$	$\frac{2\omega_0 s}{(s^2+\omega_0^2)^2}$	$\text{Re}\{s\}>0$

Appendix B.2 Properties of the Bilateral Laplace Transform

$x(t)$	$X(s)=\mathcal{L}\{x(t)\}$	ROC
Linearity $Ax_1(t)+Bx_2(t)$	$AX_1(s)+BX_2(s)$	ROC $\supseteq$ ROC$\{X_1\}$ $\cap$ROC$\{X_2\}$
Delay $x(t-\tau)$	$e^{-s\tau}X(s)$	not affected
Modulation $e^{at}x(t)$	$X(s-a)$	Re$\{a\}$ shifted by Re$\{a\}$ to the right
'Multiplication by t', Differentiation in the frequency domain $tx(t)$	$-\frac{d}{ds}X(s)$	not affected
Differentiation in the time domain $\frac{d}{dt}x(t)$	$sX(s)$	ROC $\supseteq$ ROC$\{X\}$
Integration $\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	ROC $\supseteq$ ROC$\{X\}$ $\cap\{s:\text{Re}\{s\}>0\}$
Scaling $x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	ROC scaled by a factor of a

Appendix B.3 Fourier Transform Pairs

$x(t)$	$X(j\omega)=\mathcal{F}\{x(t)\}$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$\dot{\delta}(t)$	$j\omega$
$\frac{1}{T}\sqcup\!\sqcup\!\sqcup\left(\frac{t}{T}\right)$	$\sqcup\!\sqcup\!\sqcup\left(\frac{\omega T}{2\pi}\right)$
$\varepsilon(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$\text{rect}(at)$	$\frac{1}{\lvert a\rvert}\text{si}\left(\frac{\omega}{2a}\right)$
$\text{si}(at)$	$\frac{\pi}{\lvert a\rvert}\text{rect}\left(\frac{\omega}{2a}\right)$
$\frac{1}{t}$	$-j\pi\,\text{sign}(\omega)$
$\text{sign}(t)$	$\frac{2}{j\omega}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$
$\sin(\omega_0 t)$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$
$e^{-\alpha\lvert t\rvert},\ \alpha>0$	$\frac{2\alpha}{\alpha^2+\omega^2}$
$e^{-a^2t^2}$	$\frac{\sqrt{\pi}}{a}e^{-\frac{\omega^2}{4a^2}}$

Appendix B.4 Properties of the Fourier Transform

	$x(t)$	$X(j\omega)=\mathcal{F}\{x(t)\}$
Linearity	$Ax_1(t)+Bx_2(t)$	$AX_1(j\omega)+BX_2(j\omega)$
Delay	$x(t-\tau)$	$e^{-j\omega\tau}X(j\omega)$
Modulation	$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$
'Multiplication by t' Differentiation in the frequency domain	$tx(t)$	$-\frac{dX(j\omega)}{d(j\omega)}$
Differentiation in the time domain	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$X(j\omega)\left[\pi\delta(\omega)+\frac{1}{j\omega}\right]$ $=\frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right),\quad a\in\mathbb{R}\backslash\{0\}$
Convolution	$x_1(t)*x_2(t)$	$X_1(j\omega)\cdot X_2(j\omega)$
Multiplication	$x_1(t)\cdot x_2(t)$	$\frac{1}{2\pi}X_1(j\omega)*X_2(j\omega)$
Duality	$x_1(t)$ $x_2(jt)$	$x_2(j\omega)$ $2\pi x_1(-\omega)$
Symmetry relations	$x(-t)$ $x^*(t)$ $x^*(-t)$	$X(-j\omega)$ $X^*(-j\omega)$ $X^*(j\omega)$
Parseval theorem	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2\,dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(j\omega)\rvert^2 d\omega$

Appendix B.6 Properties of the z-Transform

Property	$x[k]$	$X(z)$	ROC
Linearity	$ax_1[k]+bx_2[k]$	$aX_1(z)+bX_2(z)$	ROC $\supseteq$ ROC$\{X_1\}\cap$ROC$\{X_2\}$
Delay	$x[k-\kappa]$	$z^{-\kappa}X(z)$	ROC$\{x\}$; separate consideration of $z=0$ and $z\rightarrow\infty$
Modulation	$a^k x[k]$	$X\left(\frac{z}{a}\right)$	ROC$=\left\{z \,\middle\vert\, \frac{z}{a} \in \mathrm{ROC}\{x\}\right\}$
Multiplication by k	$kx[k]$	$-z\frac{dX(z)}{dz}$	ROC$\{x\}$; separate consideration of $z=0$
Time inversion	$x[-k]$	$X(z^{-1})$	ROC$=\{z \,\vert\, z^{-1}\in \mathrm{ROC}\{x\}\}$
Convolution	$x_1[k] * x_2[k]$	$X_1(z)\cdot X_2(z)$	ROC $\supseteq$ ROC$\{x_1\}\cap$ROC$\{x_2\}$
Multiplication	$x_1[k]\cdot x_2[k]$	$\frac{1}{2\pi j}\oint X_1(\zeta)X_2\left(\frac{z}{\zeta}\right)\frac{1}{\zeta}d\zeta$	multiply the limits of the ROC

Appendix B.5 Two-sided z-Transform Pairs

$x[k]$	$X(z)=\mathcal{Z}\{x[k]\}$	ROC
$\delta[k]$	1	$z\in\mathbb{C}$
$\varepsilon[k]$	$\frac{z}{z-1}$	$\lvert z\rvert>1$
$a^k\varepsilon[k]$	$\frac{z}{z-a}$	$\lvert z\rvert>\lvert a\rvert$
$-a^k\varepsilon[-k-1]$	$\frac{z}{z-a}$	$\lvert z\rvert<\lvert a\rvert$
$k\varepsilon[k]$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert>1$
$ka^k\varepsilon[k]$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert>\lvert a\rvert$
$\sin(\Omega_0 k)\varepsilon[k]$	$\frac{z\sin\Omega_0}{z^2-2z\cos\Omega_0+1}$	$\lvert z\rvert>1$
$\cos(\Omega_0 k)\varepsilon[k]$	$\frac{z(z-\cos\Omega_0)}{z^2-2z\cos\Omega_0+1}$	$\lvert z\rvert>1$