NTNU – Trondheim
Norwegian University of
Science and Technology

NTNU, Department of Physics

Exam TFY4280 Signal Processing spring 2023

Lecturer: Professor Jens O. Andersen
Department of Physics, NTNU

Wednesday June 7 2023 15:00-19:00

Permitted examination aids according to code H:
No printed or hand-written support material is allowed
All calculators allowed

Problem 1

Consider the RLC series circuit shown in Fig. 1.

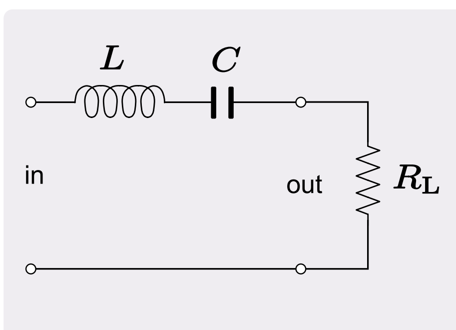

Figure 1: RLC series circuit.

a) Write down the integro-differential equation for the current $i(t)$. Assume that the charge Q on the capacitor vanishes for $t \leq 0$.

b) The output is the voltage across R_L. Calculate the frequency response function $H(s)$ and express your result in terms of the two quantities α and ω_0 defined as

$$\alpha = \frac{R}{2L} , \quad (1)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} . \quad (2)$$

What is the engineering dimension of α and ω_0?

c) Find $|H(i\omega)|$. Find the frequency ω_c that maximizes $|H(i\omega)|$. What type of filter is this circuit?

Problem 2

Fig. 2 shows a continuous time LTI system with input signal $x(t)$ and output signal $y(t)$. The unit impulse response function of the system is denoted by $h(t)$.

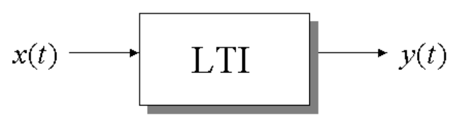

Figure 2: Continuous time LTI system.

a) Show that $h(t) = 0$ for $t < 0$ if the LTI system is causal.

b) Show that the system is BIBO stable if $h(t)$ is absolutely integrable.

c) Write down the expression for $y(t)$ in terms of $x(t)$ and the unit impulse response function $h(t)$.

d) Given $h(t) = e^{-t}u(t)$ and $x(t) = u(t)$, where $u(t)$ is the unit step function. Calculate the resulting signal $y(t)$.

Problem 3

Consider a discrete-time LTI system described by the difference equation

$$y[n] - y[n-1] + ay[n-2] \quad = \quad x[n] \ , \tag{3}$$

where a is real parameter.

a) Find the transfer function $H(z)$.

b) For what values of a is the system BIBO stable?

c) Set $a = 0$. Find the natural response $y_c[n]$. Calculate the forced response $y_p[n]$ for the discrete unit step function. Find the complete solution given the initial value $y[0] = 1$.

Problem 4

This problem consists of two questions that are independent of each other.

a) Consider the function $f(x) = 1 - |x|$ for $x \in [-1, 1]$ and its periodic extension. Its Fourier series is

$$f(x) \quad = \quad A_0 + \sum_{k=1}^{\infty} [A_k \cos(k\omega_0 x) + B_k \sin(k\omega_0 x)] \ , \tag{4}$$

where $\omega_0 = \pi$, A_k, and B_k are the Fourier coefficients. Explain why the coefficients B_k vanish for all k, $B_k = 0$. Does the Fourier series converge to $f(x)$ for all x?

b) Let $x[n]$ and $y[n]$ be two discrete time signals. The convolution of $x[n]$ and $y[n]$ is denoted by $w[n] = x[n] * y[n]$. Find the z-transform of $w[n]$.