NTNU – Trondheim
Norwegian University of
Science and Technology

NTNU, Department of Physics

Exam TFY4280 Signal Processing spring 2024

Lecturer: Professor Jens O. Andersen
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Monday June 3 2024 15:00-19:00

Permitted examination aids according to code H:
No printed or hand-written support material is allowed
All calculators allowed

Good luck! Bonne chance! Viel Glück! Veel succes! Lykke til!

Problem 1

Fig. 1 shows a continuous time LTI system with input signal $x(t)$ and output signal $y(t)$, with $t \geq 0$. The unit impulse response function of the system is denoted by $h(t)$.

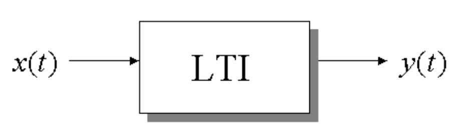

Figure 1: Continuous time LTI system.

a) Assume that the unit impulse response function is $h(t) = u(t)$. Show that the system is an integrator and that an integrator is BIBO metastable.

b) Find the impulse response function $h_i(t)$ of the inverse LTI system $y(t) \to x(t)$ *without* using the Laplace transform, i.e. directly in the time-domain. Show that the inverse system is a differentiator. A differentiator is a system such that $x(t) \to y(t) = x'(t)$.

c) Fig. 2 shows an RC circuit with an input voltage $V_{\text{in}}(t)$ and output voltage $V_{\text{out}}(t) = V_C(t)$. There is no charge on the capacitor at $t = 0^-$, i.e. $V_{\text{out}}(0^-) = 0$. The current also vanishes at $t = 0^-$, $i(0^-) = 0$.

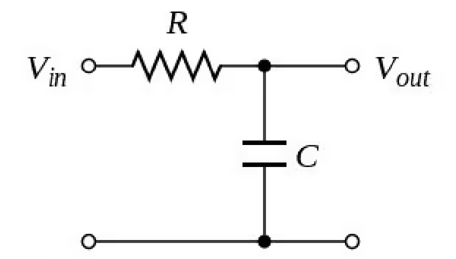

Figure 2: RC series circuit with output $V_{\text{out}} = V_C(t)$.

Write down the integral equation that governs the RC circuit. What is the engineering dimension of RC?

d) Calculate the transfer function $H_C(i\omega)$. What type of filter is this circuit?

e) Let $V_{\text{in}}(s)$ and $V_C(s)$ denote the Laplace transform of $V_{\text{in}}(t)$ and $V_C(t)$, respectively. Show that $V_C(s) \approx \frac{V_{\text{in}}(s)}{s}$ for $s \gg \frac{1}{RC}$. Use this to show that the system is an integrator for $s \gg \frac{1}{RC}$.

f) Consider now the case where the output $V_{\text{out}}(t)$ is taken to be the voltage $V_R(t)$ across the resistor R, see Fig. 3. The initial conditions are the same as before. Calculate the transfer function $H_R(i\omega)$ in this case and determine the type of filter.

g) Let $V_R(s)$ denote the Laplace transform of $V_R(t)$. Show that $V_R(s) \approx RCsV_{\text{in}}(s)$ for $s \ll \frac{1}{RC}$. Use this to show that the system is a differentiator for $s \ll \frac{1}{RC}$.

h) Show that the systems in Figs. 2 and 3 are BIBO stable. Explain why this is not in disagreement with the result in **a)** that an integrator is BIBO metastable.

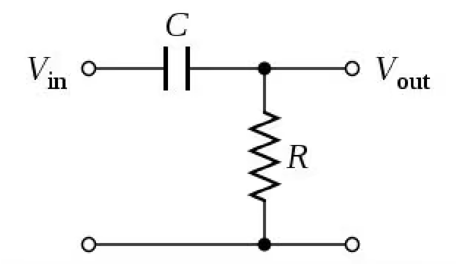

Figure 3: RC series circuit with output $V_{\text{out}} = V_R(t)$.

Problem 2

Consider the function

$$f(t) \quad = \quad t\ ,\ \text{for}\ 0 \leq t \leq T_0, \tag{1}$$

where T_0 is a constant. The function $x(t)$ is the periodic extension of $f(t)$. Calculate the Fourier coefficients of $f(t)$ and sketch its frequency spectrum. For which values of t does the Fourier series converge to the actual function $x(t)$?

Problem 3

A discrete-time LTI system has a discrete impulse response function $h[n]$ given by

$$h[n] \quad = \quad a^n u[n]\ , \tag{2}$$

where $a \in \mathbb{C}$ and $u[n]$ is the discrete-time unit step function.

a) Find the z-transform $H[z]$ of $h[n]$ and the difference equation of the system.

b) For which values of a is the system causal?

c) For which of values of a is the system BIBO stable?

d) Find the difference equation for the inverse system and the associated discrete impulse response function $h_i[n]$.