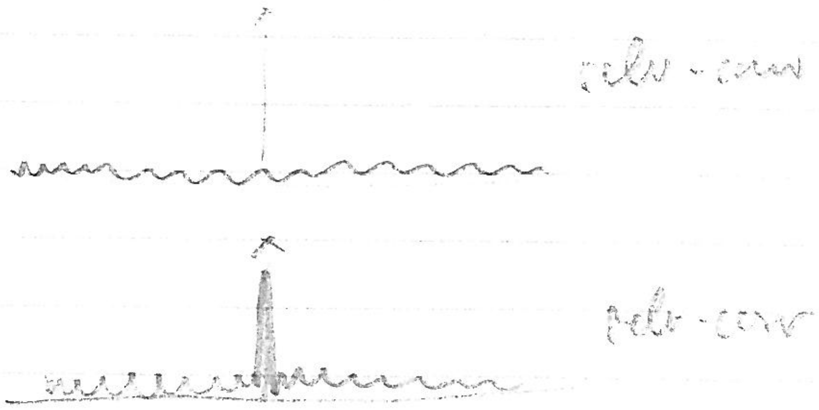
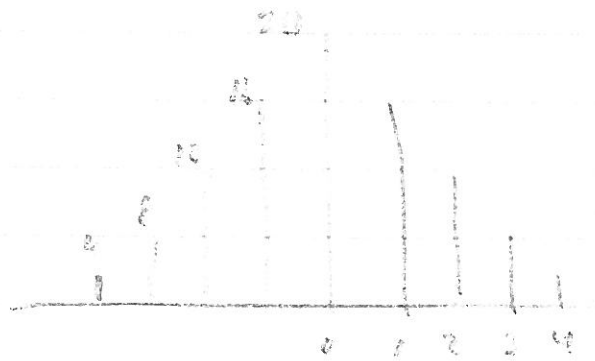
Del A - only answers

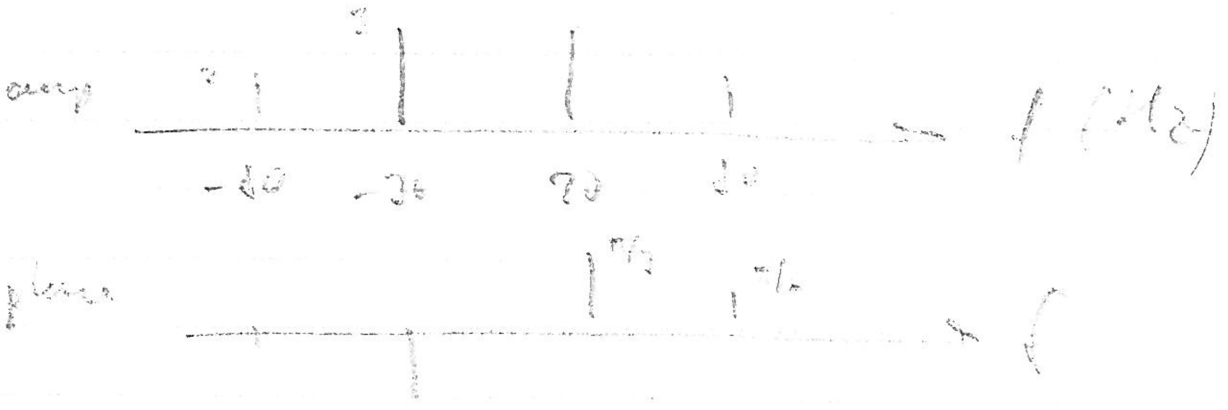
X1:



X2:



A3:



A4 $m(t) = 0.8\, e^{-t} - 0.2\, e^{-6t}$

Solution B1:

a) after z-transformation $Y(z) = H(z) \cdot X(z)$

$H(z) = 0.5 + 1.5z^{-1} + z^{-2}$

$X(z) = \frac{z}{z-1}$ from table (unit step respon)

hence $Y(z) = \frac{(0.5 + 1.5z^{-1} + z^{-2})\ z}{(z-1)} = \frac{0.5z + 1.5 + z^{-1}}{(z-1)}$

long division gives

$Y(z) = ?$

$$
\begin{array}{l}
(0.5z + 1.5 + z^{-1}) : (z-1) = 0.5 + 2z^{-1} + 3z^{-2} + 3z^{-3} + 3z^{-4} \\
\underline{0.5z - 0.5} \\
\quad 2 + z^{-1} \\
\quad \underline{2 - 2z^{-1}} \\
\qquad 3z^{-1} \\
\qquad \underline{3z^{-1} - 3z^{-2}} \\
\qquad\quad 3z^{-2} \\
\qquad\quad \underline{3z^{-2} - 3z^{-3}} \\
\qquad\quad 0 \quad 3z^{-3}
\end{array}
$$

$y[n] = \{\underset{\uparrow}{0.5};\ 2;\ 3,\ 3,\ 3,\ 3 \ldots\}$

b) $X(z) = 2z + 1 + 1.5z^{-1} + 0.5z^{-2}$

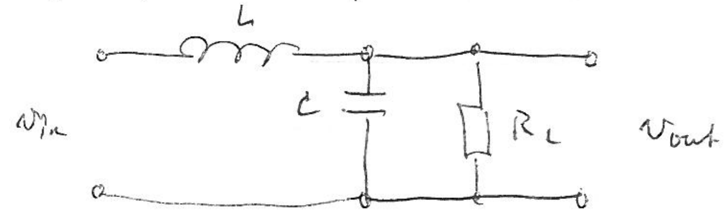
$(0.5 + 1.5z^{-1} + z^{-2})$

$$
\begin{array}{rl}
H(z)\,X(z) = & z + 0.5 + 0.75z^{-1} + 0.25z^{-2} \\
& \quad 3 + 1.5z^{-1} + 2.25z^{-2} + 0.75z^{-3} \\
& \qquad 2z^{-1} + 1.0z^{-2} + 1.5z^{-3} + 0.5z^{-4} \\
\hline
& z + 3.5 + 4.25z^{-1} + 3.5z^{-2} + 2.25z^{-3} + 0.5z^{-4}
\end{array}
$$

=) $y[n] = \{1, \underset{\uparrow}{3.5}, 4.25, 2.5, 2.25, 0.5\}$

Solution B2

a) To pass the $\omega = 0$ component of v_{in} the only possibility is)



b) derive transfer function

$$Z_{tot} = Z_L + Z_C \,||\, R_L = Z_L + \overbrace{\frac{1}{\frac{1}{R_L} + \frac{1}{(1/i\omega C)}}}^{\frac{R}{(1+i\omega CR)}}$$

$$V_{out}(\omega) = \frac{V_{in}(\omega)}{\left(i\omega L + \frac{R}{(1+i\omega CR)}\right)} \cdot \frac{R}{(1+i\omega CR)}$$

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{(i\omega L - \omega^2 CRL + R)} = \frac{1}{\left(1 - \omega^2 CL + i\frac{\omega L}{R}\right)}$$

$$|H(\omega)| = \frac{1}{\sqrt{(1-\omega^2 CL)^2 + \frac{\omega^2 L^2}{R^2}}} = \frac{1}{\sqrt{1 - 2\omega^2 CL + \omega^4 C^2 L^2 + \frac{\omega^2 L^2}{R^2}}}$$

for Butterworth form

$$\omega^4 C^2 L^2 = \frac{\omega^4}{\omega_c^4} \Rightarrow \underline{\omega_c^2 = \frac{1}{CL}}$$

$$\frac{\omega^2 L^2}{R^2} = 2\omega^2 CL \Rightarrow \underline{L = 2CR^2}$$ ok.

Solution B2 cont.

c) $R = 1000\,\Omega \Rightarrow L = 2 \cdot C \cdot 10^6$

$$L = C \cdot 2 \cdot 10^6$$

$$\Rightarrow (100)^2 = \frac{1}{C \cdot 2 \cdot 10^6 \cdot C} \Rightarrow C = \frac{1}{\sqrt{2} \cdot 10^5} = 7.07\ \mu F$$

$$\Rightarrow L = 2C \cdot 10^6 = 2 \cdot 7.07 \cdot 10^6 \cdot 10^{-6} = 14.1\ H$$

d) $|H(0)| = 1$

$$|H(377)| = \frac{1}{\sqrt{1 + \left(\frac{377}{100}\right)^4}} = 0.070$$

$$|H(100)| = \frac{1}{\sqrt{1+1}} = 0.707$$

Solution B3

a) $$\int_0^\infty f(at-b)\,u(at-b)\,e^{-st}\,dt$$

$$at-b=\tau \quad \Rightarrow \quad d\tau = a\,dt \quad ; \quad t=\frac{\tau+b}{a}$$

$$\Rightarrow \int_0^\infty f(\tau)\,u(\tau)\,e^{-s\left(\frac{\tau+b}{a}\right)}\,\frac{d\tau}{a}$$

$$\Rightarrow \frac{e^{-\frac{sb}{a}}}{a}\cdot\int_{-b}^{\infty} f(\tau)\,\underline{u(\tau)}\,e^{-\frac{s}{a}\tau}\,d\tau$$

u.b. $\Rightarrow \int_0^\infty f(\tau)\,e^{-\frac{s}{a}\tau}\,d\tau$

$$\Rightarrow \frac{e^{-\frac{sb}{a}}}{a}\cdot F\left(\frac{s}{a}\right)$$

Q.E.D.

b) $f(t') = \sin 3t'$ from LT table

c) $$\Rightarrow f(t) = \sin\left[3\left(4t-\frac{\pi}{6}\right)\right]u\left(4t-\frac{\pi}{6}\right)$$

use (a), (b)

$$F(s) = \frac{e^{-\frac{s\pi}{6\cdot 4}}}{4}\cdot\frac{4\cdot 3}{\left(\frac{s^2}{16}+9\right)\cdot 4} = \underbrace{e^{-\frac{s\pi}{24}}}_{\text{time-shift (no rescale)}}\;\underbrace{\frac{12}{s^2+144}}_{\sin 12t}$$

$$\Rightarrow f(t) = \sin\left[12\left(t-\frac{\pi}{24}\right)\right]u\left(t-\frac{\pi}{24}\right)$$

B4. a) From FT tables

$$F\left\{\sum_{n=-\infty}^{\infty} g(t-nT)\right\} = \sum_{n=-\infty}^{\infty} \omega_0\, G(n\omega_0)\, \delta(\omega - n\omega_0)\ ;\ \omega_0 = \frac{2\pi}{T}$$

FT of generating function

in our case $g(t) = \mathrm{rect}(t/T')\Big|_{T'=T/2}$ (T' rect width)

$$\Rightarrow\ G(n\omega_0) = \frac{T}{2}\cdot \mathrm{sinc}\left(\frac{n\omega_0 T}{2\cdot 2}\right)$$

FT table

$$\text{then}\quad X(\omega) = \sum_{n=-\infty}^{\infty} \underbrace{\frac{2\pi}{T}}_{\omega_0}\cdot\frac{T}{2}\cdot \mathrm{sinc}\left(n\frac{2\pi}{T}\cdot\frac{T}{2\cdot 2}\right)\cdot \delta\left(\omega - n\frac{2\pi}{T}\right)$$

$$= \sum_{n=-\infty}^{\infty} \pi\cdot \mathrm{sinc}\left[n\cdot\frac{\pi}{2}\right]\cdot \delta\left(\omega - \frac{n2\pi}{T}\right)$$

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b) $T = 40\text{ ms} \Rightarrow \frac{2\pi}{T} = 50\pi$

$|H(\omega)|$ removes frequency components $> 180\pi$

$\rightarrow$ only terms $\leq n=3$ survives filter

$n=0$ $\quad Y_0(\omega) = \pi\cdot\delta(\omega)$ $\quad \neq 0$

$n=\pm1$ $\quad Y_{\pm1}(\omega) = \pi\cdot\left[\frac{\sin -\frac{\pi}{2}}{-\frac{\pi}{2}}\,\delta(\omega+50\pi) + \frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}}\,\delta(\omega-50\pi)\right]$ $\quad \neq 0$

$n=\pm2$ $\quad Y_{\pm2}(\omega) = \pi\left[\underbrace{\frac{\sin -\pi}{-\pi}}_{=0}\,\delta(\omega+100\pi) + \underbrace{\frac{\sin\pi}{\pi}}_{=0}\,\delta(\omega-100\pi)\right] = 0$

$n=\pm3$ $\quad Y_{\pm3}(\omega) = \pi\left[\frac{\sin -\frac{3\pi}{2}}{-\frac{3\pi}{2}}\,\delta(\omega+150\pi) + \frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\,\delta(\omega-150\pi)\right]$ $\quad \neq 0$

inverse transform gives $\quad y(t) = \frac{1}{2} + \frac{2}{\pi}\cos(50\pi t) - \frac{2}{3\pi}\cos(150\pi t)$