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LF

WITH SOME COMMENTS.

Institutt for fysikk

Eksamensoppgave i TFY4280 Signalanalyse

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Tlf.: 41466510

Eksamensdato: 20. mai

Eksamenstid (fra-til): 15.00-19.00

Hjelpemiddelkode/Tillatte hjelpemidler: C – formula tables are included as a resource

Annen informasjon:

Merk! Studenter finner sensur i Studentweb. Har du spørsmål om din sensur må du kontakte instituttet ditt. Eksamenskontoret vil ikke kunne svare på slike spørsmål.

1 Answer Problem 1

Read the course presentation below, especially about the Contents, Learning Outcome and Learning activities.

https://www.ntnu.edu/studies/courses/TFY4280#tab=omEmnet

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TFY4280 - Signal Processing

About Timetable Examination

Examination arrangement

Examination arrangement: Written examination
Grade:

Evaluation form	Weighting	Duration	Examination aids
Written examination	100/100	4 hours	C

Course content

The course focuses on basic tools in analysis of analogue and digital signals and systems. Time and frequency domain description of signals. Use of Laplace, Fourier, and Z-transforms. Basic analogue and digital filter design, frequency response, data sampling. Excitation-response analysis of linear systems. Description and analysis of stochastic signals and measured signals with noise, correlations and energy spectrum analysis. Analysis of signals and systems using mathematical methods involving differential and integral calculus, as well as numerical methods using Matlab or python.

Learning outcome

The student is expected to: 1. Obtain, through a combined theoretical and experimental approach to the subject, a fundamental understanding of signal processing and needed theoretical and mathematical background to describe signals and systems, experimental measurement signals and time series. 2. Learn how to analyze various problems in signal processing using mathematical methods involving differential and integral calculus, as well as ICT-based/numerical methods by using Matlab or python.

Learning methods and activities

Lectures, calculation assignments, compulsory computer laboratory exercises (MATLAB or python). When lectures and lecture material are in English, the exam may be given in English only. Students are free to choose Norwegian or English for written assessments.

Compulsory assignments

Laboratorieøvinger

Please comment with some 10 - 25 sentences: What do you think was the most positive parts of the course. What moments of the course can be better? Is there any particular signal processing area you think should be added or emphasized more?

Fill in your answer here

Format

Σ

THANKS FOR MANY ADVICES

N.B. MANY EVALUATED THE EXECUTION OF THE COURSE. I ONLY WANTED COMMENTS ON "CONTENTS" (MAINLY).

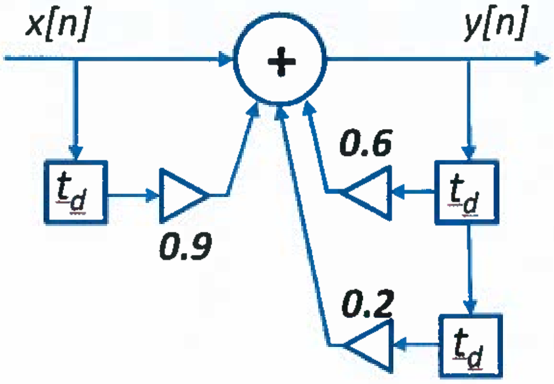
ANYWAY, ALL THAT WRITE SOMETHING GOT 20p. THIS WAS A BONUS FOR YOUR GREAT JOB WITH THE ASSIGNMENTS.

Words: 0

Maximum marks: 20

2 Answer Problem 2

Consider the following digital net.



where the boxes are delays and the triangles with their gain factors correspond to amplifiers. You only give answers:

a) Write down the difference equation associated by the net. (in the form y[n]= ......)

b) Write down the first 4 terms of the digital impulse response: {..,0,0,**1**,0,0,..}

c) Write down the first 4 terms to the input signal: x[n] = {**3**,1,2}.

Notation for b) and c): In the sequence {a,**b**,c,d} 'bold' **b** denotes index '0'.

Fill in your answer here

Maximum marks: 10

a)

By inspection

$y[n] = x[n] + 0.9 \cdot x[n-1] + 0.6 \cdot y[n-1] + 0.2 \cdot y[n-2$

-2/3 FOR missing erroneous terms

b) $y[n] = (\underset{\uparrow}{1}, 1.5, 1.1, 0.96, \ldots)$

-3 FOR the wrong

c) $y[n] = (\underset{\uparrow}{3}, 5.5, 6.8, 6.98, \ldots)$

-3 FOR the wrong

AP-2

b) Make a table:

n	x[n]	x[n-1]	y[n-1]	y[n-2]	y[n]
0	1	0	0	0	1
1	0	1	1	0	$0.9 + 0.6 \cdot 1 = 1.5$
2	0	0	1.5	1	$0.6 \cdot 1.5 + 0.2 \cdot 1 = 1.1$
3	0	0	1.1	1.5	$0.6 \cdot 1.1 + 0.2 \cdot 1.5 = 0.96$
4	0	0	0.96	1.1	$0.6 \cdot 0.96 + 0.2 \cdot 1.1 = 0.796$

Answer: $\Rightarrow y[n] = (\underset{\uparrow}{1}, 1.5, 1.1, 0.96, 0.796, \ldots)$

c) Make z-transform + long division

Diff Eqn.

$$y[n] - 0.6\, y[n-1] - 0.2\, y[n-2] = x[n] + 0.9\, x[n-1]$$

$$z \downarrow \quad Y(z)\left(1 - 0.6 z^{-1} - 0.2 z^{-2}\right) = X(z)\left(1 + 0.9 z^{-1}\right)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{(1 + 0.9 z^{-1})}{(1 - 0.6 z^{-1} - 0.2 z^{-2})}$$

Now output is $H(z) \cdot X(z)$

with $X(z) = \left(3 + z^{-1} + 2 z^{-2}\right)$

AP 2

c) cont.

$$Y(z) = \frac{(1+0.9z^{-1})(3+z^{-1}+2z^{-2})}{(1-0.6z^{-1}-0.2z^{-2})} =$$

$$= \frac{(z+0.9)(3z^2+z+2)}{(z^3-0.6z^2-0.2z)}$$

$$= \frac{3z^3+z^2+2z+0.9\cdot 3z^2+0.9z+1.8}{(\qquad\qquad)}$$

$$= \frac{(3z^3+3.7z^2+2.9z+1.8)}{(z^3-0.6z^2-0.2z)}$$

Long Division:

$$\begin{array}{r|l}
 & 3+5.5z^{-1}+6.8z^{-2}+6.98z^{-3}+\ldots \\
z^3-0.6z^2-0.2z & 3z^3+3.7z^2+2.9z+1.8 \\
 & 3z^3-1.8z^2-0.6z \\
 & 0 \quad 5.5z^2+3.5z+1.8 \\
 & 5.5z^2-3.3z-1.1 \\
 & 0 \quad 6.8z+2.9 \\
 & 6.8z-4.08-1.36z^{-1} \\
 & 0 \quad 6.98+1.36z^{-1}
\end{array}$$

AP2 CHECK b) IMPULSE RESPONSE WITH LONG DIVISION

$$Y(z) = H(z) \cdot 1 = \frac{1 + 0.9z^{-1}}{1 - 0.6z^{-1} - 0.2z^{-2}}$$

$$= \frac{z^2 + 0.9z}{(z^2 - 0.6z - 0.2)}$$

LONG - DIVISION

$$1 + 1.5z^{-1} + 1.1z^{-2} + 0.96z^{-3} + \dots$$

$$\Rightarrow \quad z^2 - 0.6z - 0.2 \,\big)\, z^2 + 0.9z$$

$$z^2 - 0.6z - 0.2$$

$$0 \quad 1.5z + 0.2$$

$$1.5z - 0.9 - 0.3z^{-1}$$

$$0 \quad 1.1 + 0.3z^{-1}$$

$$1.1 - 0.66z^{-1} - 0.22z^{-2}$$

$$0 \quad + 0.96z^{-1} + 0.22z^{-2}$$

$$\Rightarrow \quad y[n] = (\underset{\uparrow}{1}, 1.5, 1.1, 0.96, \dots)$$

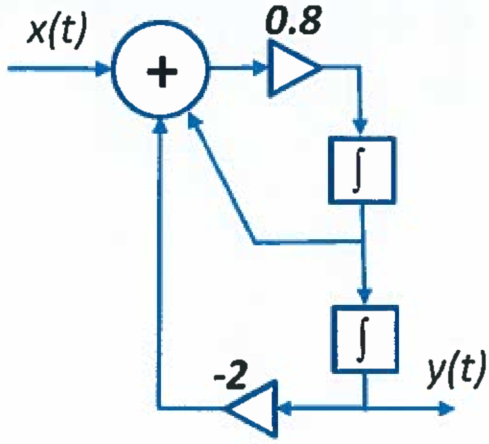
OR WITH TABLE IN b

MORE WAYS TO DOUBLE-CHECK

⋮

3 Answer Problem 3

Consider the following block diagram in direct form II:



MISSING TERMS/

Give a linear differential equation (in terms of x(t), y(t), dx/dt, etc...) that corresponds to the same system.

−1/2/3/4 SMALL ERROR IN COEFF

Fill in your answer here

$$\frac{5}{4}\frac{d^2y}{dt^2} - \frac{dy}{dt} + 2\cdot y(t) = x(t)$$

−5 AT LEAST CORRECT ORDER

VARIANT: $y(t) = \frac{1}{2}x(t) + \frac{1}{2}\frac{dy}{dt} - \frac{5}{8}\frac{d^2y}{dt^2}$

$y'' - 0.8\,y' + 1.6\,y = x$

Maximum marks: 10

4 Answer Problem 4

What is the time signal f(t) giving rise to the following unilateral Laplace transform?

$F(s) = (10s+10) / [(s^2+2s+1)(s+3)^2]$

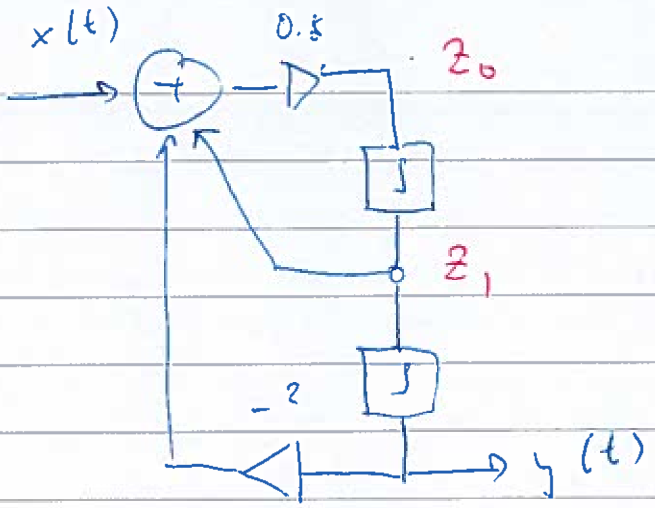
Fill in your answer here

$$f(t) = 5\cdot\left[\frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t} - t\cdot e^{-3t}\right]$$

−3/4/5 MISSING OR ERRONEOUS TERMS

Maximum marks: 10

AP 1)



BY INSPECTION, THESE EQUATIONS HOLD:

$$z_0 = \frac{dz_1}{dt} = \dot{z}_1$$

$$z_0 = (z_1 - 2y + x)\,0.8 \Rightarrow \dot{z}_1 = (z_1 - 2y + x)\,0.8$$

$$z_1 = \frac{dy}{dt} = \dot{y}$$

$$\dot{z}_1 = \ddot{y} = \frac{d^2y}{dz^2}$$

$$\Rightarrow \frac{d^2y}{dt^2} = \left(\frac{dy}{dt} - 2y + x\right)0.8$$

$$1.25\frac{d^2y}{dt^2} = \frac{dy}{dt} - 2y + x$$

$$1.25\frac{d^3y}{dt} - \frac{dy}{dt} + 2y(t) = x(t)$$

OR $$0.625\frac{d^2y}{dt^2} - 0.5\frac{dy}{dt} + y(t) = 0.5\,x(t)$$

ETC... MANY EQUIVALENT VARIANTS POSSIBLE

AP4)

$$\therefore \quad F(s) = \frac{5}{2(s+1)} - \frac{5}{(s+3)^2} - \frac{5}{2(s+3)}$$

USE TABLE

ANSWER :

$$\Rightarrow \quad f(t) = 5\left[\frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t} - t\cdot e^{-3t}\right]u(t)$$

CHECK PARTIAL FRACTION

$$F(s) = 5\cdot\left(\frac{1}{2(s+1)} - \frac{1}{2(s+3)} - \frac{2\cdot 1}{2(s+3)^2}\right)$$

$$= 5\cdot\left(\frac{\frac{1}{2}(s+3)^2 - \frac{1}{2}(s+1)(s+3) - (s+1)}{(s+1)(s+3)^2}\right)$$

$$= 5\left[\frac{\frac{1}{2}(s^2+6s+9) - \frac{1}{2}(s^2+4s+3) - (s+1)}{(s+1)(s+3)^2}\right]$$

$$= \frac{5\cdot\left[\cancel{\frac{1}{2}s^2} + \cancel{3s} + \frac{9}{2} - \cancel{\frac{1}{2}s^2} - \cancel{2s} - \frac{3}{2} - \cancel{s} - 1\right]}{(s+1)(s+3)^2}$$

$$= \frac{10}{(s+1)(s+3)^2}$$

ok!!

AP4)

$$F(s) = \frac{(10s+10)}{[(s^2+2s+1)(s+3)^2]}$$

$$= \frac{10\cancel{(s+1)}}{\cancel{(s+1)}(s+1)(s+3)^2} = \frac{10}{(s+1)(s+3)^2}$$

PARTIAL FRACTIONS:

$$\frac{10}{(s+1)(s+3)^2} = \frac{A}{(s+1)} + \frac{B}{(s+3)^2} + \frac{C}{(s+3)}$$

$$s=-1 \Rightarrow A = \frac{10}{(-1+3)^2} = \frac{10}{4} = \frac{5}{2}$$

$$s=-3 \Rightarrow B = \frac{10}{(-3+1)} = -\frac{10}{2} = -5$$

NOW FIND "C"

$$\frac{2\cdot 10}{2\cdot(s+1)(s+3)^2} = \frac{5(s+3)^2}{2(s+1)(s+3)^2} - \frac{5\cdot 2(s+1)}{(s+3)^2\, 2(s+1)} + \frac{C\cdot 2(s+1)(s+3)}{(s+3)\,2(s+1)(s+3)}$$

$$\Rightarrow 20 = 5s^2 + 30s + 45 - (10s+10) + C(2s^2+8s+6)$$

IDENTIFY:

$$s^2: \quad 0 = 5s^2 + C\cdot 2s^2 \Rightarrow C = -\frac{5}{2}$$

$$s: \quad 0 = 30s - 10s + 8sC \Rightarrow -20 = 8C \Rightarrow C = -\frac{20}{8} = -\frac{5}{2}$$

ok.

5 MC2

How much amplification does a system have in dB if:

$u_{out} = 10^4 u_{in}$

u is an amplitude signal (voltage, current, etc)

Select one alternative:

DEFINITION OF AMPLITUDE GAIN in dB

$20 \cdot {}^{10}\log(\text{Gain}) = 20\ {}^{10}\log 10^4 = 80\,dB$

- 4 dB
- 10 dB
- X 80 dB
- 20 dB

Maximum marks: 5

6 MC3

HERE IT WENT WRONG

(HER BLE DET GALT) IMPEDANCE FUNCTION WAS SUPPLIED

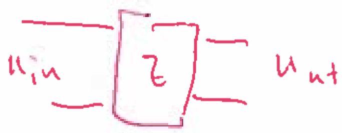
An electronic filter consiting of a resistance, a condensator and a coil is described by the following transfer function: H(s)=sL+R/(1+sRC)

Specifically, L = 1.2 H; C = 1/6 F; R = 1000 ohm. What kind of filter is this and what is the characteristic frequency.

[illegible]

Select one alternative:

- High-pass filter; cut-off frequency 0.2 rad/s
- X Low-pass filter; ~~Cut-off frequency 1.414 rad/s~~
- Band-block filter; block frequency 5 rad/s
- Band-pass filter; pass frequency 2.236 rad/s



BAND-PASS FILTER

SO THIS WAS THROWN OUT FOR THOSE WHO DID NOT SCORE.

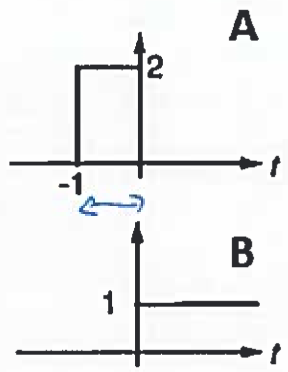
Maximum marks: 5

THIS PROBLEM EXCLUDED (ALL GOT 5)

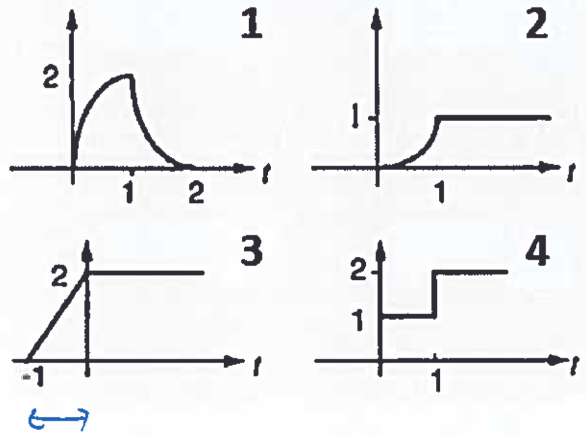
7 MC4

MC4:

Consider the following two signals:



Which would best describe their convolution product:



Select one alternative:

- Signal 4
- Signal 3
- Signal 2
- Signal 1

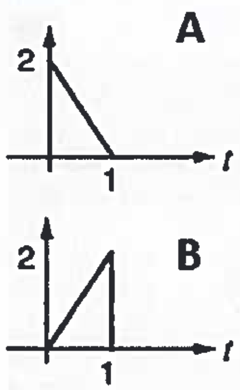
ONLY 3 CONTAINS NEG. VALUES

Maximum marks: 5

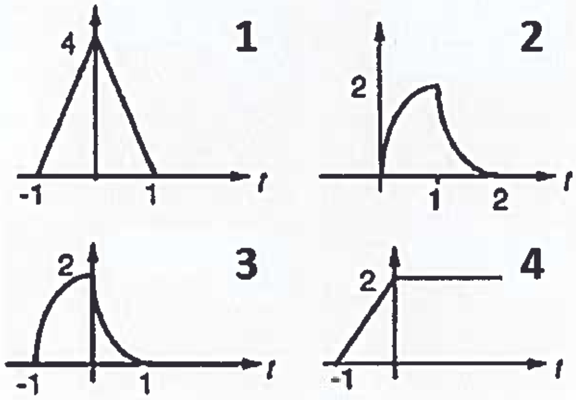
8 MC5

MC5:

Consider the following two signals:



Which would best describe their convolution product:



Select one alternative:

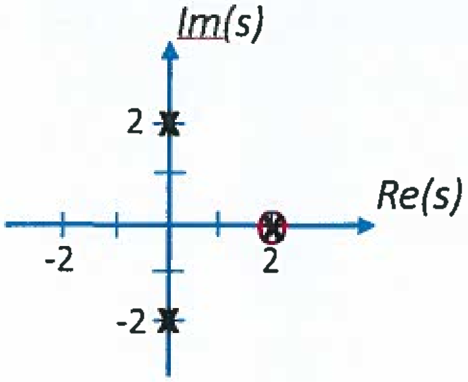
- Signal 4
- Signal 3
- X Signal 2
- Signal 1

ONLY 2 COMPLETELY POSITIVE

Maximum marks: 5

9 MC6

What transfer function has the following pole zero diagram?



$$H(s) = \frac{K(s-2)}{(s-2)(s+2i)(s-2i)} = \frac{K}{(s^2+4)}$$

K is an arbitrary real constant.

Select one alternative:

- $K(s+2)/[(s-2)(s+4)^2]$
- $K(s+2)/(s+2)^2$
- X $K/(s^2+4)$
- $K(s+2)/(s+2i)^2$

Maximum marks: 5

10 MC7

What is correct for the Fourier transform of a real, causal signal with no particular symmetry:

Select one alternative:

- Real part is odd; imaginary part is zero.
- Real part is even; odd part is even.
- X Real part is even; imaginary part is odd.
- Real part is odd; imaginary part is odd.

ALL FUNCTIONS CAN BE SEPARAT
IN ODD & EVEN PART.

$f(t)$		$F(\omega)$
REAL & EVEN	$\rightarrow$	REAL EVEN
REAL & ODD	$\rightarrow$	IMAGINARY & OD

$\Downarrow$

HERMITEAN

Maximum marks: 5

11 MC8

A second-order system is described by the following differential equation:

2 dy/dt + y = 3 d^2x/dt^2 + 2 dx/dt + x

where x and y are functions of t. x(t) is the input, and y(t) the output. What is the internal system transfer function if the initial values of y(t) = y(0) = k. (x(0) = 0; and the initial values of dy/dt and dx/dt are also = 0)

Select one alternative:

- $k^2/(2s^2+s+1)$
- k/(s+1)
- k/(s+2)
- X 2k/(2s+1)

Maximum marks: 5

$$2 \cdot \frac{dy}{dt} + y = 3 \cdot \frac{d^2x}{dt^2} + 2 \cdot \frac{dx}{dt} + x$$

LAPLACE TRANSFORM:

= k

x(0) = 0 & DERIVATIVES $\frac{dx}{dt}$ SO IGNORED ON THIS SIDE

$$2 \cdot (sY(s) - y(0)) + Y(s) = 3s^2 X(s) + 2sX(s) + X(s)$$

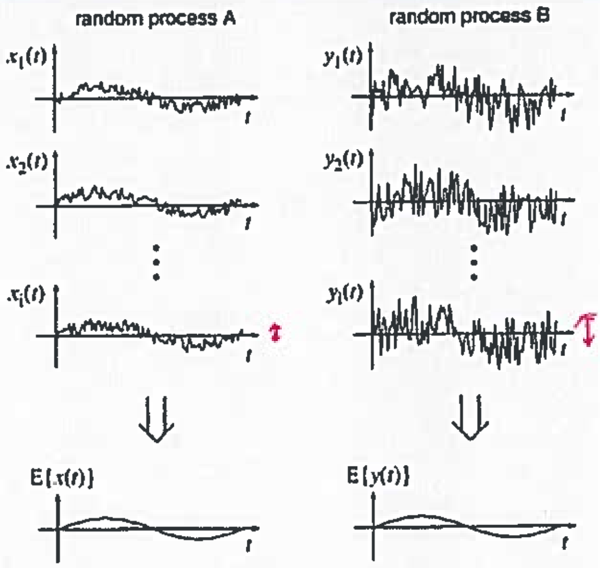
$$(2s+1)Y(s) - 2k = X(s)(3s^2+2s+1)$$

$$\Rightarrow Y(s) = \underbrace{\frac{2k}{(2s+1)}}_{\text{INTERNAL PART - DEPENDING ON START/INITIAL VALUES ETC}} + X(s)\underbrace{\frac{(3s^2+2s+1)}{(2s+1)}}_{H(s)}$$

H(s) EXTERNAL PART DEPENDING ON "EXTERNAL" INPUT

12 MC9

Consider the two random processes A and B:



Which of the following statements is true?
Select one alternative:

- None of the processes A or B can be stationary.
- The first-order expected values of processes A and B are all the same.
- X The random process A has lower variance than process B.
- It is enough to know the square average in order to calculate the standard deviation.

The book is a little unclear here on the "stationary" definition. In my opinion signal A & B ~~both~~ have a deterministic sinusoidal (time-average) part with different noise overlapped. So this alternative will also be deemed correct...

This is obvious from the definition of the variance

$$\sigma_x^2(t) = E\{(x(t)-\mu_x(t))^2\}$$

Maximum marks: 5

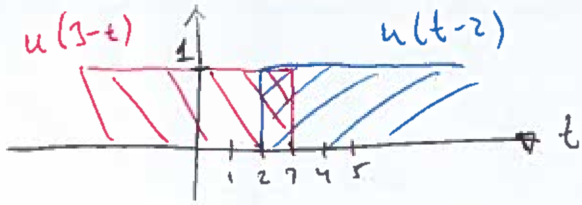
13 MC10

Find the Laplace transform of the following time function:

f(t) = -5·u(t-2)·u(3-t)

u(t) is the unit step function.

Select one alternative:



- $-5\,(e^{-2t} + e^{3t})$
- $5s\cdot\sin(2s-3s)$
- X $5(e^{-3s}+e^{-2s})/s$ ⊖
- $s\cdot(e^{3s}-e^{-2s})/5$

FROM DEFINITION

$$F(s) = -5\cdot\int_2^3 e^{-st}\,dt = \frac{5}{s}\left(e^{-3s} - e^{-2s}\right)$$

MANY DISCOVERED THIS ERRATUM.

Maximum marks: 5

14 MC11

A signal y(t) is composed of a random process x(t) and a deterministic process d(t) such that

y(t) = x(t) + d(t)

The variance of the random process is $\sigma_x^2 = 7$.

Select one alternative:

- $\sigma_y^2 = 0$
- $\sigma_y^2 = \frac{\sigma_x}{\sigma_d^2}$
- $\sigma_y^2 = 14$
- X $\sigma_y^2 = 7$

THE DETERMINISTIC PROCESS WILL NOT ADD ANY VARIANCE !!

Maximum marks: 5