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**Department of Physics**

**Examination paper for TFY4280 Signal Processing (Signalanalyse)**

**Examination date: 2020-05-27**

**Examination time (from-to): 09.00 – 13.00**

**Permitted examination support material:** All support material is allowed

**Academic contact during examination:** Prof. Mikael Lindgren

   **Phone: 41466510**

**Technical support during examination:** Orakel support services

   **Phone:** 73 59 16 00

**OTHER INFORMATION**

If a question is unclear/vague – make your own assumptions and specify in your answer the premises you have made. Only contact academic contact in case of errors or insufficiencies in the question set.

**Saving:** Answers written in Inspera are automatically saved every 15 seconds. If you are working in another program remember to save your answer regularly.

**Cheating/Plagiarism:** The exam is an individual, independent work. Examination aids are permitted. All submitted answers will be subject to plagiarism control. *Note that the multiple choice questions are automatically scrambled so the question sets will be different to each student.*

**Notifications:** If there is a need to send a message to the candidates during the exam (e.g. if there is an error in the question set), this will be done by sending a notification in Inspera. A dialogue box will appear. You can re-read the notification by clicking the bell icon in the top right-hand corner of the screen. All candidates will also receive an SMS to ensure that nobody misses out on important information. Please keep your phone available during the exam.

**Weighting and points:** How the questions exactly are weighted should be shown on the at each question automatically in Inspera. *The first question is important.* The answering of each of the bullet points here: a), b), c)... together, will give 65 points, eligible for the grade 'C'. The remaining questions will be awarded 1 - 2 points each for correct answer. Typically the multiple choice/pairing question gives 1p for each correct, answer questions give 2-4 p/each. There are no negative points. The total number of points will be 100 (including the first question giving 65 points if answered).

**1** # TFY4280 V20 - AP1

**Answer Problem 1: Read the course presentation below.**

## Course content

The course focuses on basic tools in analysis of analogue and digital signals and systems. Time and frequency domain description of signals. Use of Laplace, Fourier, and Z-transforms. Basic analogue and digital filter design, frequency response, data sampling. Excitation-response analysis of linear systems. Description and analysis of stochastic signals and measured signals with noise, correlations and energy spectrum analysis. Analysis of signals and systems using mathematical methods involving differential and integral calculus, as well as numerical methods using Matlab or python.

## Learning outcome

The student is expected to: 1. Obtain, through a combined theoretical and experimental approach to the subject, a fundamental understanding of signal processing and needed theoretical and mathematical background to describe signals and systems, experimental measurement signals and time series. 2. Learn how to analyze various problems in signal processing using mathematical methods involving differential and integral calculus, as well as ICT-based/numerical methods by using Matlab or python.

**Please comment with some 10 - 25 sentences:**
a) What do you think was the most positive parts of the course.
b) What moments of the course can be better?
c) Was there a good balance between 'analogue' and computer assignments?
d) The tutorials arranged at the end of the course, were these detailed enough/useful?
e) Are there signal processing areas you think is missing/should be emphasized more?
The answer of the questions a - e will render you 65 points for grade 'C'. The answers do not need to be long/extensive. Fill in below a) ..text... b) ..text...  etc

**Fill in your answer here**

| Format ▾ | B | I | U | x₂ | x² | Iₓ | ⎘ | ⎗ | ↶ | ↷ | ↺ | ≟ | ≔ | Ω | ⊞ | ✎ | Σ | ⤢ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

Words: 0

Maximum marks: 65

**2**

# TFY4280 V20 - AP2

**Answer Problem 2.** A signal x(t) is displayed in the plot below where *t* is the time. Write a mathematical expression for the signal using the unit step function *u(t)* and the variable *t*.
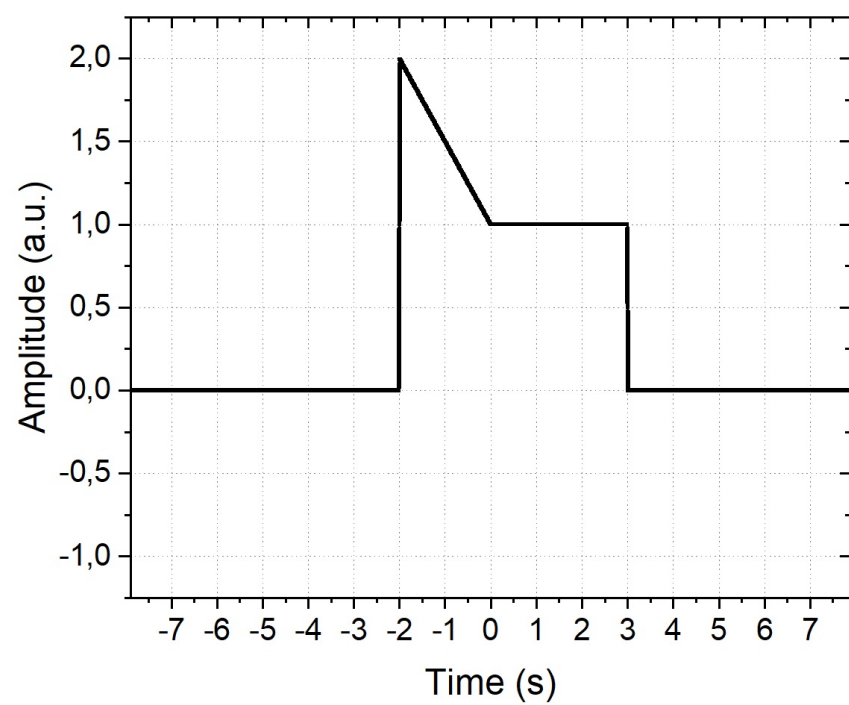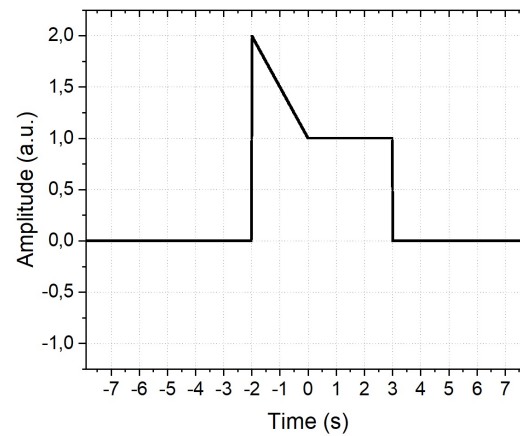


Figure x(t):

**Fill in your answer here**



Words: 0

Maximum marks: 2

## 3  TFY4280 V20 - MC3



**Multiple Choice Question 3.** Consider the following signal:

**Pair the time translations with the corresponding plot:**
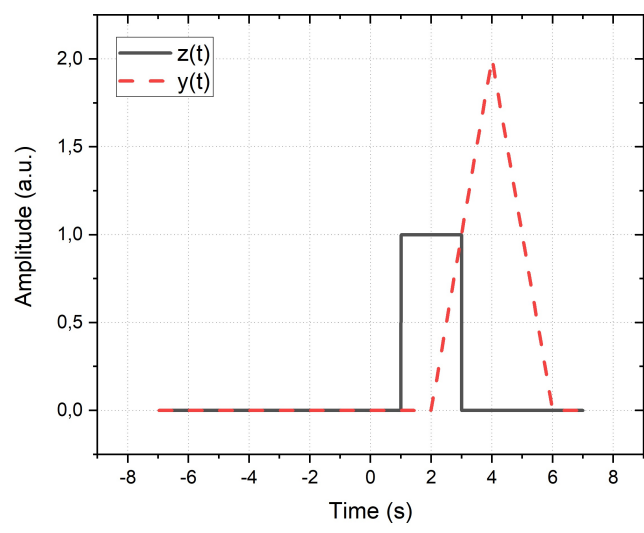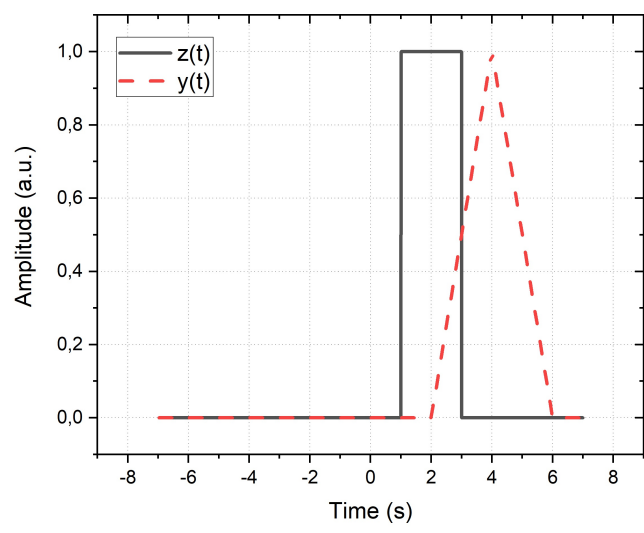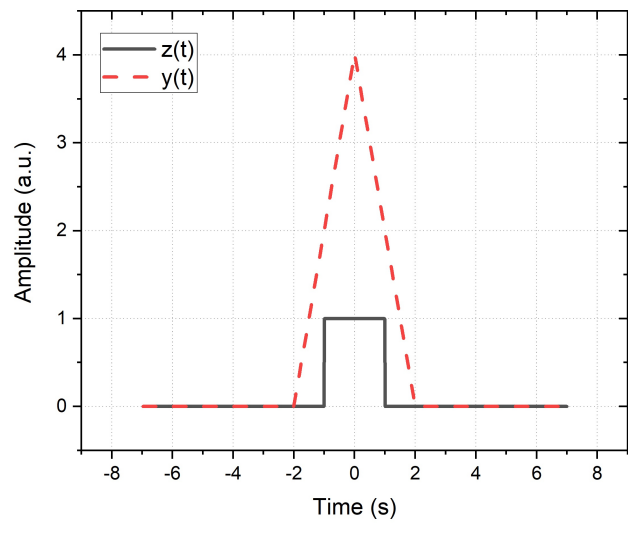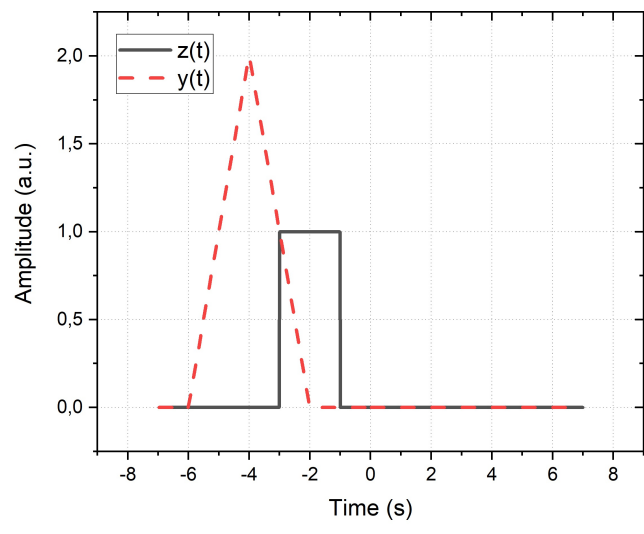


Maximum marks: 4

## 4  TFY4280 V20 - MC4

**Multiple Choice Question 4.** Consider the two cascaded LTI-systems as shown in the figure.
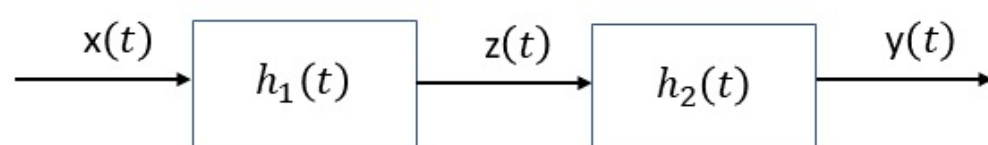


The impulse responses of the two systems are identical with $h_1(t) = h_2(t) = u(t-1) - u(t-3)$ where $u(t)$ is the unit step response. If x(t) is the impulse response, what are the signals $z(t)$ and $y(t)$? ($t$ is the time, given in seconds.) Make your choice from the plots below.

**Select one alternative:**









Maximum marks: 2

## 5  TFY4280 V20 - MC5

**Multiple Choice Question 5.** Along the column to the left are some time-functions listed ($u(t)$ is as usual the unit step function). Note also that the function $k(t)$ is a function of $f(t)$, and $h(t)$ is a function of $g(t)$. Match these 4 functions with their Fourier transforms found along the row on the top.

**Please match each of the functions with their Fourier transform.**

| | $\dfrac{3}{3+12i+i\omega} - \dfrac{2}{2+12i+i\omega}$ | $\dfrac{i\omega}{6+5i\omega-\omega^2}$ | $\dfrac{1}{3+i\omega} - \dfrac{1}{4+i\omega}$ | $\dfrac{i\omega}{12+7i\omega-\omega^2}$ |
|---|---|---|---|---|
| $k(t) = \displaystyle\int_{-\infty}^{t} f(\tau)d\tau$ | ○ | ○ | ○ ✔ | ○ |
| $f(t) = (4e^{-4t} - 3e^{-3t})u(t)$ | ○ | ○ | ○ | ○ ✔ |
| $h(t) = e^{-i12t}g(t)$ | ○ ✔ | ○ | ○ | ○ |
| $g(t) = (3e^{-3t} - 2e^{-2t})u(t)$ | ○ | ○ ✔ | ○ | ○ |

Maximum marks: 4

## 6  TFY4280 V20 - MC6

**Multiple Choice Question 6.** Along the column to the left are some time-functions listed ($u(t)$ is as usual the unit step function). Match these 4 functions with their corresponding unilateral Laplace transforms found along the row on the top.

**Please match the time functions with their Laplace transforms.**

| | $F(s) = \dfrac{3(s+3)}{s^2+25}$ | $F(s) = \dfrac{3}{s(s^2+25)}$ | $F(s) = \dfrac{64}{(s+1)^2(s^2+6s+25)}$ | $F(s) = \dfrac{3(s+2)}{s^2+8s+15}$ |
|---|---|---|---|---|
| $f(t) = (3\cos 5t + 1.8\sin 5t)u(t)$ | ○ ✔ | ○ | ○ | ○ |
| $f(t) = \dfrac{3}{25}(1 - \cos 5t)u(t)$ | ○ | ○ ✔ | ○ | ○ |
| $f(t) = [3.2t - 0.64]e^{-t}u(t) +$ $+[0.64\cos 4t - 0.48\sin 4t]e^{-3t}u(t)$ | ○ | ○ | ○ ✔ | ○ |
| $f(t) = \dfrac{3}{2}(3e^{-5t} - e^{-3t})u(t)$ | ○ | ○ | ○ | ○ ✔ |

Maximum marks: 4

# TFY4280 V20 - MC7

**Multiple Choice Question 7.** The plot below shows the out-put signals *y(t)* of two different LTI systems defined by linear differential equations (LDE). The input signal *x(t)* is either the impulse response or the unit step response function, as indicated at the LDE. Pair the LDE/input signal with the output signals designated A, B, C and D in the plot.
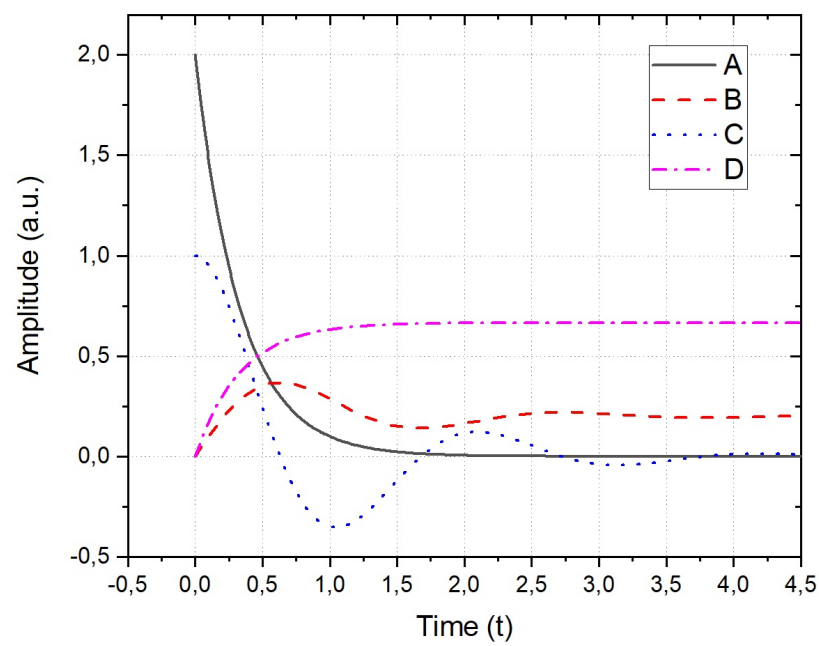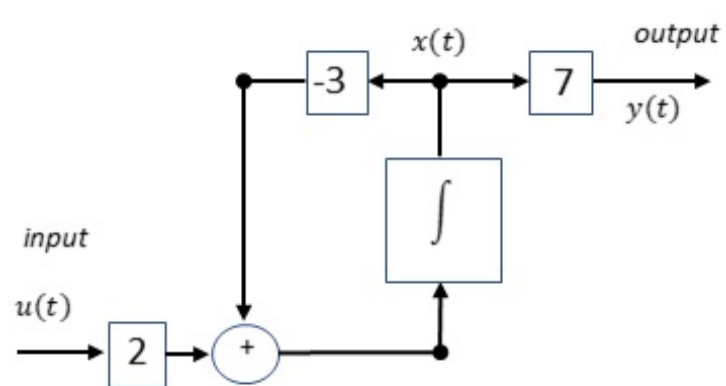


**Please pair each LDE/input with the plots as shown above:**

|  | C | B | D | A |
|---|:---:|:---:|:---:|:---:|
| $$\frac{dy(t)}{dt} + 3y(t) = 2x(t)$$ $$x(t) = u(t)$$ | ○ | ○ | ✔ | ○ |
| $$\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 10y(t) = \frac{dx(t)}{dt} + 2x(t)$$ $$x(t) = u(t)$$ | ○ | ✔ | ○ | ○ |
| $$\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 10y(t) = \frac{dx(t)}{dt} + 2x(t)$$ $$x(t) = \delta(t)$$ | ✔ | ○ | ○ | ○ |
| $$\frac{dy(t)}{dt} + 3y(t) = 2x(t)$$ $$x(t) = \delta(t)$$ | ○ | ○ | ○ | ✔ |

Maximum marks: 4

# 8 TFY4280 V20 - AP8

**Answer Problem 8.** Consider the system below:



$y(t)$ is the output, $u(t)$ is the input, and the variable $x(t)$ is defining the initial state.

a) Find the system output $y(t)$ for $u(t) = 0$ and the initial state given by $x(0) = 3$.

b) Find the system response to the unit step signal if the initial state $x(0) = 0$.

Only provide the answers, a) $y_a(t) = ....$, etc.

**Fill in your answer here**

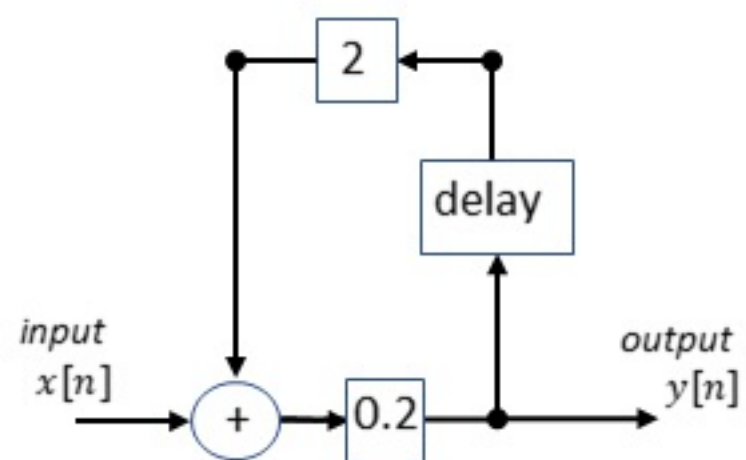| Format | B *I* U x₂ x² | | | | | | |
|---|---|---|---|---|---|---|---|

Words: 0

Maximum marks: 4

**9** # TFY4280 V20 - AP9

**Answer Problem 9.** Consider the simulation diagram below of a discrete system. Find the first 4 values of the output *y[n]* if the discrete unit step signal is applied as input. (Answer with a sequence of numbers/amplitudes.)
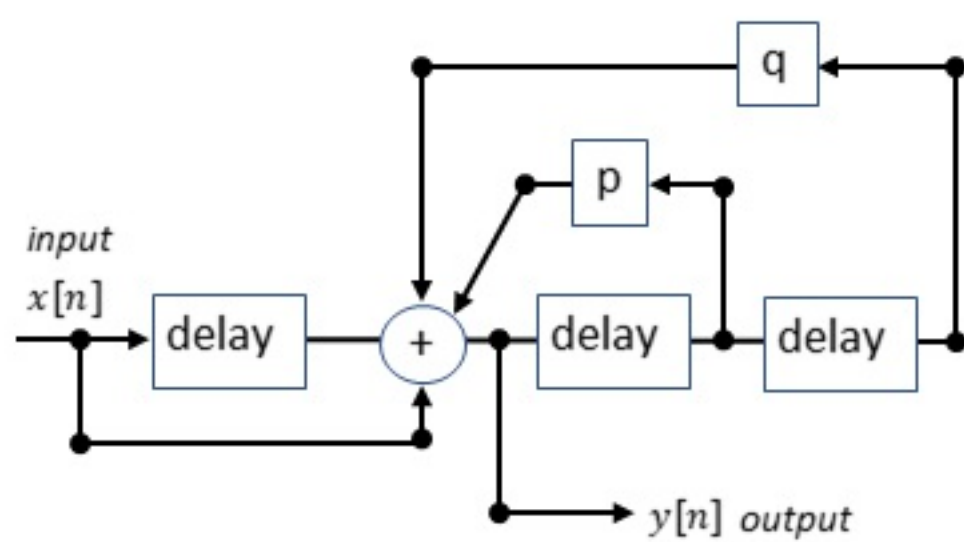


**Fill in your answer here**

Words: 0

Maximum marks: 3

**10**  **TFY4280 V20 - AP10**

**Answer Question 10.** Consider the block diagram of a discret system with delays and amplifiers (squares) as shown below.



Find the values *p* and *q* of the amplifiers in order to generate the output sequence $y[n] = 0.5^n$ using the digital impulse response as the input signal. (Only give the answers !)

**Fill in your answer here**

Maximum marks: 2

## 11 TFY4280 V20 - MC11

**Multiple Choice Question 11.** Recall the discrete-time Fourier transform (DTFT) of a sequence *x[n]* is given by:

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-in\Omega}$$

where *n* is an integer. Pair the following sequences (row on top) with their respective DTFT (column to left).
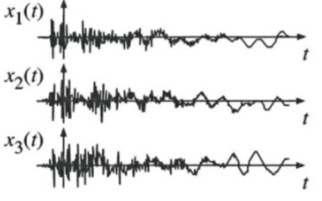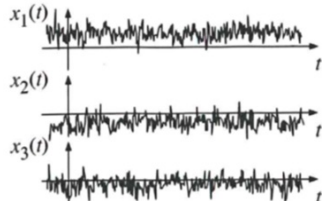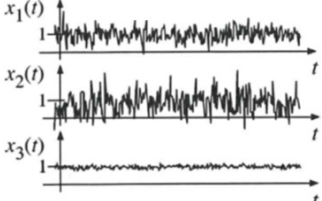
**Please match the values:**

| | $x[n] = \sum_{n=-2}^{2} (4-|n|)1^n, 0\ else$ | $x[n] = \sum_{n=0}^{\infty} (-0.5)^n$ | $x[n] = 2\sum_{n=-2}^{2} 1^n, 0\ else$ |
|---|---|---|---|
| $X(\Omega) = 2(2 + 3cos\Omega + 2cos2\Omega)$ | ○ ✔ | ○ | ○ |
| $X(\Omega) = 2(1 + 2cos\Omega + 2cos2\Omega)$ | ○ | ○ | ○ ✔ |
| $X(\Omega) = \dfrac{1}{1 + 0.5e^{-i\Omega}}$ | ○ | ○ ✔ | ○ |

Maximum marks: 3

## 12   TFY4280 V20 - MC12

**Multiple Choice Question 12.** In the column to the left are depicted three ensembles, each consisting of three time-functions. Pair what relations for the ensemble mean to the time-average of the expectation values $E\{x(t)\}$ and $E\{x^2(t)\}$ hold best with each of them.

**Please match the values:**

Column A:
$$\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}x_i(t) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x_i(t)dt$$
$$\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}x_i^2(t) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x_i^2(t)dt$$

Column B:
$$\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}x_i(t) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x_i(t)dt$$
$$\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}x_i^2(t) \neq \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x_i^2(t)dt$$

Column C:
$$\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}x_i(t) \neq \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x_i(t)dt$$
$$\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}x_i^2(t) \neq \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x_i^2(t)dt$$

| Ensemble | Column A | Column B | Column C |
|---|---|---|---|
| Ensemble 1 ($x_1(t)$, $x_2(t)$, $x_3(t)$) | ○ ✔ | ○ | ○ |
| Ensemble 2 ($x_1(t)$, $x_2(t)$, $x_3(t)$) | ○ | ○ | ○ ✔ |
| Ensemble 3 ($x_1(t)$, $x_2(t)$, $x_3(t)$) | ○ | ○ ✔ | ○ |

Maximum marks: 3