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Department of Physics
Examination paper for TFY4280 Signal Processing (Signalanalyse)
Examination date: 2022-06-08
Examination time (from-to): 09.00 – 13.00
Permitted examination support material: All support material is allowed
Academic contact during examination: Prof. Mikael Lindgren
Phone: 41466510
Technical support during examination: Orakel support services
Phone: 73 59 16 00

If you experience technical problems during the exam, contact Orakel support services as soon as possible before the examination time expires. If you don't get through immediately, hold the line until your call is answered.

OTHER INFORMATION

Do not open Inspera in multiple tabs, or log in on multiple devices, simultaneously. This may lead to errors in saving/submitting your answer.

Get an overview of the question set before you start answering the questions.

Read the questions carefully, make your own assumptions and specify them in your answer. If a question is unclear/vague – make your own assumptions and specify in your answer the premises you have made. **If you suspect that there are errors or insufficiencies in any of the questions, pls state and describe the details with text in the answer to question 1 (the long answer question).**

Cheating/Plagiarism: The exam is an individual, independent work. Examination aids are permitted, but make sure you follow any instructions regarding citations. During the exam it is not permitted to communicate with others about the exam questions or distribute drafts for solutions. Such communication is regarded as cheating. All submitted answers will be subject to plagiarism control. *Read more about cheating and plagiarism here.*

Specific for Optics exam: Note that most of the multiple choice questionsand answers are automatically scrambled so the question sets will be different to each student. Also the majority of the questions are selected from duplicates with different numerical values giving different answers (although difficulty level is the same).

Notifications: If there is a need to send a message to the candidates during the exam (e.g. if there is an error in the question set), this will be done by sending a notification in Inspera. A dialogue box will appear. You can re-read the notification by clicking the bell icon in the top right-hand corner of the screen. All candidates will also receive an SMS to ensure that nobody misses out on important information. Please keep your phone available during the exam.

Weighting and points: How the questions exactly are weighted should be shown on the at each question automatically in Inspera. Typically, the multiple choice/pairing questions gives 2 - 7p for each correct answer. There are no negative points for one question (but if multiple answers are needed, a wrong answer may induce a negative value. So only tick as many answers as asked for). The total number of points will be normalized to 100 (%) and graded with the scale for A, B, C, etc as outlined by NTNU recommendations.

ABOUT SUBMISSION

Answering in Inspera: If the question set contains questions that are not upload assignment, you must answer them directly in Inspera. In Inspera, your answers are saved automatically every 15 seconds.

Automatic submission: Your answer will be submitted automatically when the examination time expires and the test closes, if you have answered at least one question. This will happen even if you do not click "Submit and return to dashboard" on the last page of the question set. You can reopen and edit your answer as long as the test is open. If no questions are answered by the time the examination time expires, your answer will not be submitted. This is considered as "did not attend the exam".

Withdrawing from the exam: If you become ill, or wish to submit a blank test/withdraw from the exam for another reason, go to the menu in the top right-hand corner and click "Submit blank". This cannot be undone, even if the test is still open.

Accessing your answer post-submission: You will find your answer in Archive when the examination time has expired.

1

Comment with some sentences and get 40 points:

a) What do you think was the most positive parts of the course.

b) What moments of the course can be better?

c) Was there a good balance between 'analogue' and computer assignments?

d) Are there signal processing areas you think is missing/should be emphasized more?

The answer of the questions a - d will render you 40 points. The answers do not need to be long/extensive. Fill in below a) ..text... b) ..text... etc

If you suspect some question is weird, you may write a note here.

Fill in your answer here

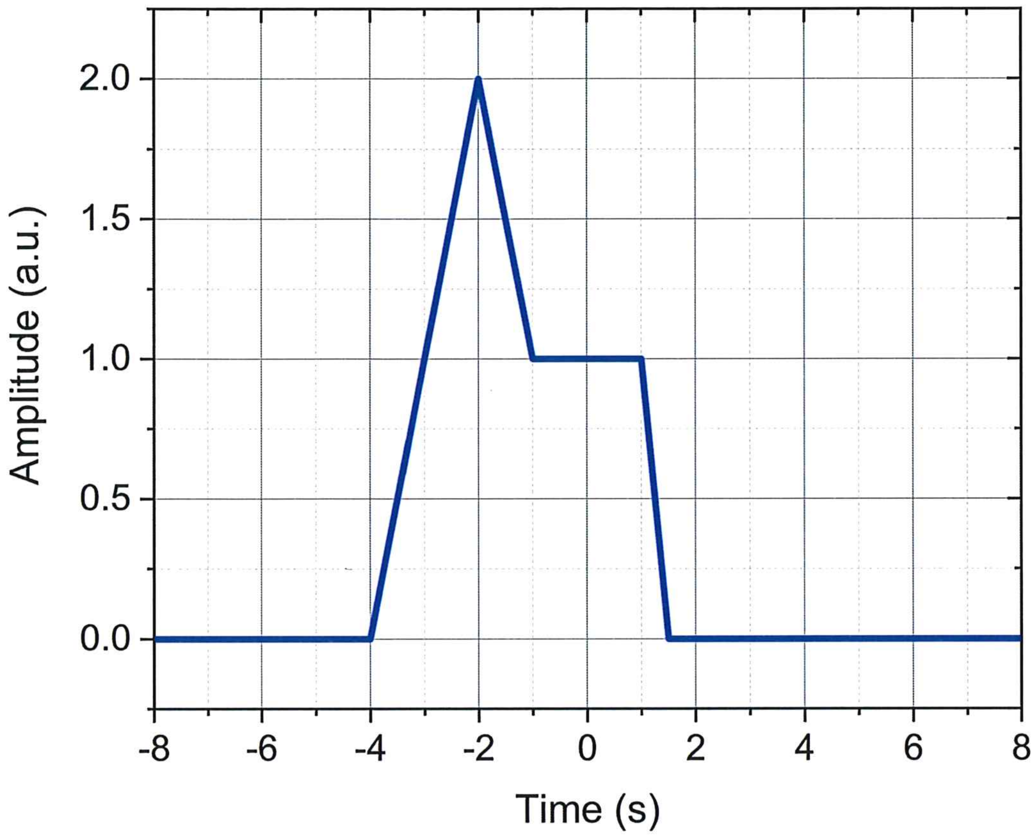
Format

Words: 0

Maximum marks: 40

2 A signal x(t) is displayed in the plot below where *t* is the time. Write a mathematical expression for the signal using the unit step function *u(t)* and the variable *t*.

Figure x(t):



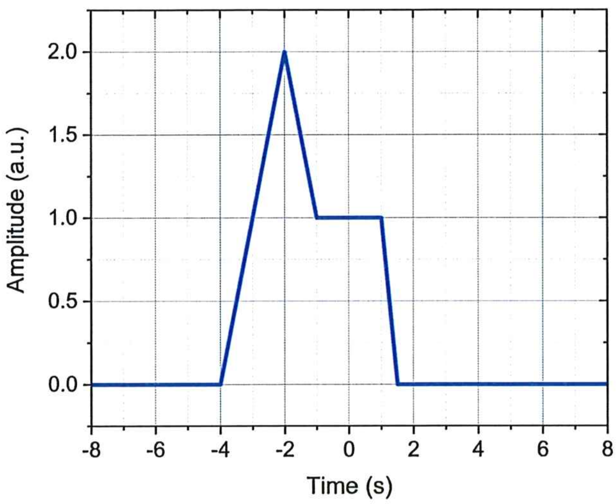
Fill in your answer here

Format

BY INSPECTION:

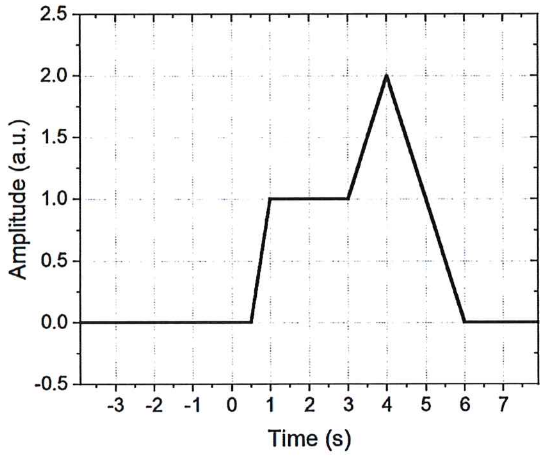
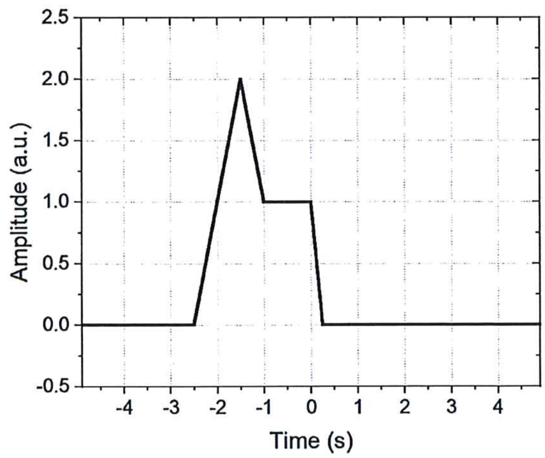
$$x(t) = (t+4)\,u(t+4) - 2(t+2)\,u(t+2) + (t+1)\,u(t+1) - 2(t-1)\,u(t-1) + 2\,(t-3/2)\,u(t-3/2)$$

3



Consider the following signal x(t):

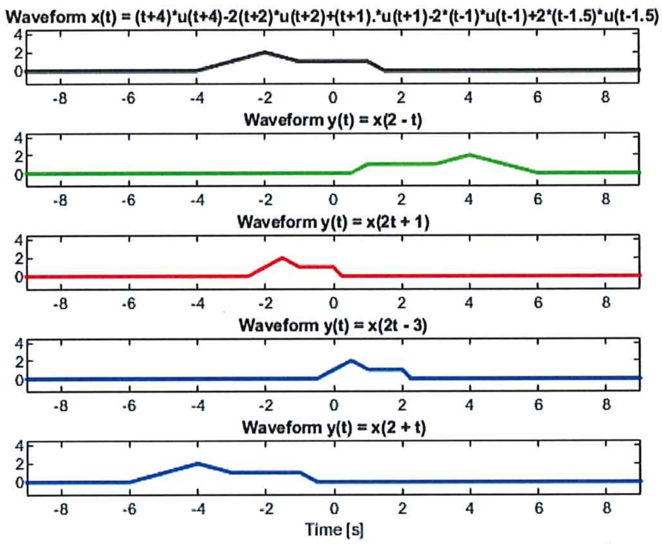
Pair the time-translation of x(t) with the corresponding plot:

	$y(t) = x(2-t)$ x(2−t)	$y(t) = x(2t+1)$ x(2t+1)	$y(t) = x(2t-3)$ x(2t−3)	$y(t) = x(2+t)$ x(2+t)
	✓	○	○	○
	○	✓	○	○

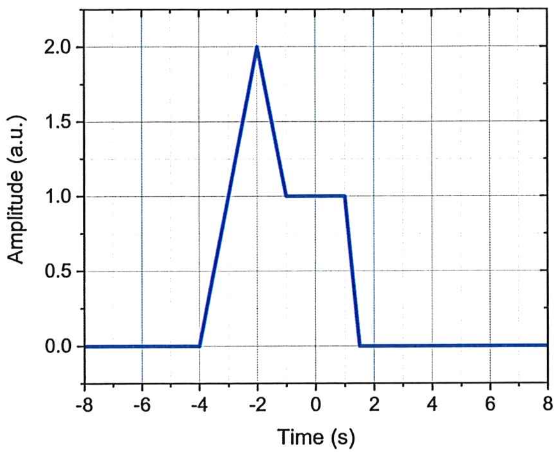
LF: SUGGESTED TO SIMULATE ALL VARIANTS →

```
clear all; close all;
% LF Oppgave 4 June 8th 2022 script for waveform + time-transforms
% Mikael Lindgren, June10th, 2022
t = -30:0.001:30;       % Time axis from -33 to 33 step of 0.01, assumed unit seconds 's'
% 'u(t)' has call name 'heaviside(t)' in Matlab.
% Waveform x(t)
xt = (t+4).*heaviside(t+4)-2*(t+2).*heaviside(t+2)+(t+1).*heaviside(t+1)-2*(t-1).*heaviside(t-1)+2*(t-1.5).*heaviside(t-1.5);
% Time-transform 1: tao = 2 - t -> tnew6 = 2 - tao
tnew1 = 2 - t;
% Time-transform 2: tao = 2t + 1 -> tnew2 = (tao - 1)/2
tnew2 = (t - 1)/2;
% Time-transform 3: tao = 2t - 3 -> tnew3 = tao/2 + 3/2
tnew3 = t/2 + 3/2;
% Time-transform 4: tao = 2 + t -> tnew4 = tao - 2
tnew4 = t - 2;

subplot(5,1,1),plot(t,xt,'k.'), axis([-9 9 -1.5 +4.5])
title('Waveform x(t) = (t+4)*u(t+4)-2(t+2)*u(t+2)+(t+1).*u(t+1)-2*(t-1)*u(t-1)+2*(t-1.5)*u(t-1.5)')
subplot(5,1,2),plot(tnew1,xt,'g.'), axis([-9 9 -1.5 +4.5])
title('Waveform y(t) = x(2 - t)')
subplot(5,1,3),plot(tnew2,xt,'r.'), axis([-9 9 -1.5 +4.5])
title('Waveform y(t) = x(2t + 1)')
subplot(5,1,4),plot(tnew3,xt,'b.'), axis([-9 9 -1.5 +4.5])
title('Waveform y(t) = x(2t - 3)')
subplot(5,1,5),plot(tnew4,xt,'b.'), axis([-9 9 -1.5 +4.5])
title('Waveform y(t) = x(2 + t)')
xlabel('Time [s]')
```

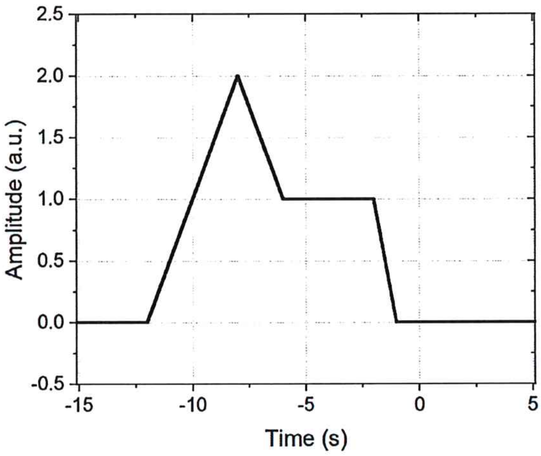
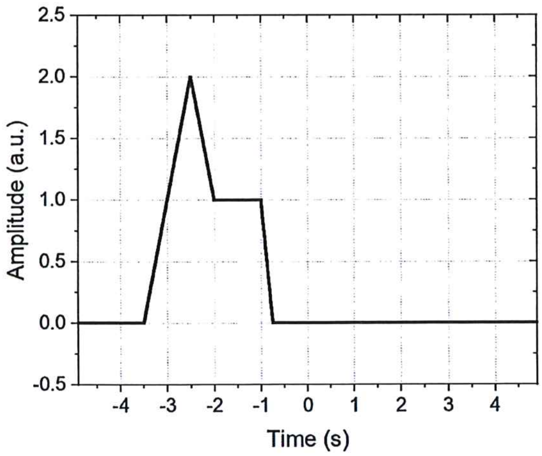


4



Consider the following signal x(t):

Pair the time-translation of x(t) with the corresponding plot:

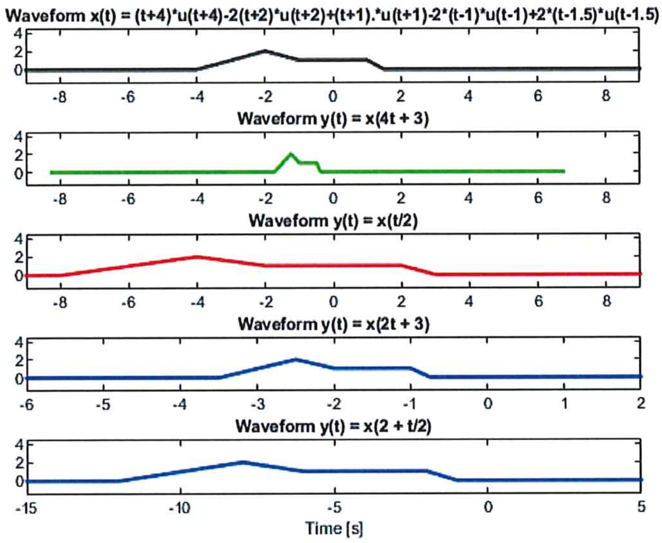
	$y(t) = x(4t+3)$ x(4t+3)	$y(t) = x(t/2)$ x(t/2)	$y(t) = x(2t+3)$ x(2t+3)	$y(t) = x(2+t/2)$ x(2+t/2)
	○	○	○	✓
	○	○	✓	○

AS FOR (3) →

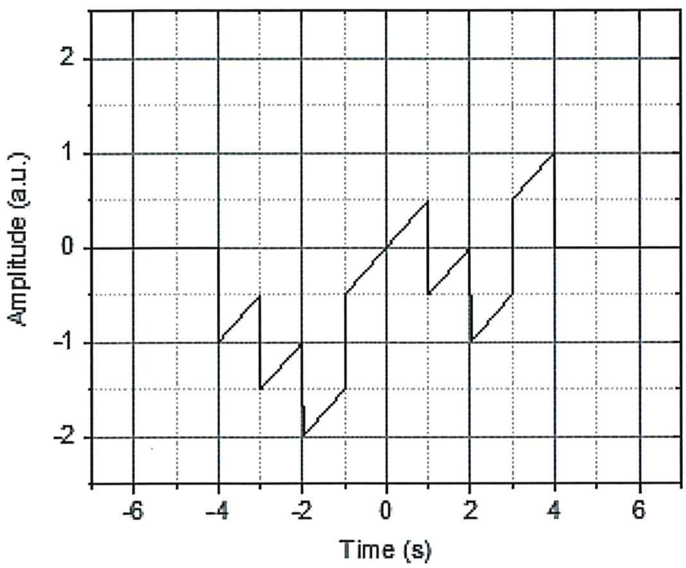
Maximum marks: 4

```
clear all; close all;
% LF Oppgave 4 June 8th 2022 script for waveform + time-transforms
% Mikael Lindgren, June10th, 2022
t = -30:0.001:30;       % Time axis from -33 to 33 step of 0.01, assumed unit seconds 's'
% 'u(t)' has call name 'heaviside(t)' in Matlab.
% Waveform x(t)
xt = (t+4).*heaviside(t+4)-2*(t+2).*heaviside(t+2)+(t+1).*heaviside(t+1)-2*(t-1).*heaviside(t-1)+2*(t-1.5).*heaviside(t-1.5);
% Time-transform 1: tao = 4t + 3 -> tnew6 = tao/4 - 3/4
tnew1 = t/4 - 3/4;
% Time-transform 2: tao = t/2 -> tnew2 = 2*tao
tnew2 = 2*t;
% Time-transform 3: tao = 2t + 3 -> tnew3 = tao/2 - 3/2
tnew3 = t/2 - 3/2;
% Time-transform 4: tao = 2 + t/2 -> tnew4 = 2*(tao - 2)
tnew4 = 2*(t - 2);

subplot(5,1,1),plot(t,xt,'k.'), axis([-9 9 -1.5 +4.5])
title('Waveform x(t) = (t+4)*u(t+4)-2(t+2)*u(t+2)+(t+1).*u(t+1)-2*(t-1)*u(t-1)+2*(t-1.5)*u(t-1.5)')
subplot(5,1,2),plot(tnew1,xt,'g.'), axis([-9 9 -1.5 +4.5])
title('Waveform y(t) = x(4t + 3)')
subplot(5,1,3),plot(tnew2,xt,'r.'), axis([-9 9 -1.5 +4.5])
title('Waveform y(t) = x(t/2)')
subplot(5,1,4),plot(tnew3,xt,'b.'), axis([-6 2 -1.5 +4.5])
title('Waveform y(t) = x(2t + 3)')
subplot(5,1,5),plot(tnew4,xt,'b.'), axis([-15 5 -1.5 +4.5])
title('Waveform y(t) = x(2 + t/2)')
xlabel('Time [s]')
```



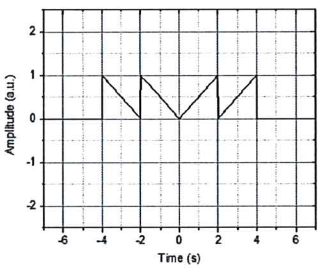
5 The following waveform is composed by adding two signals.

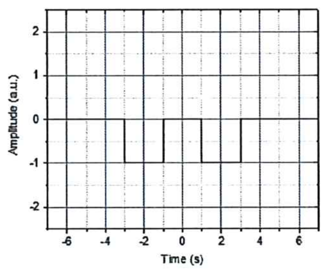


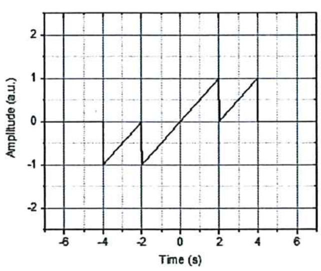
TRIAL & ERROR TESTING OF EVEN & ODD VARIANT WILL GIVE ONLY ONE POSSIBILITY

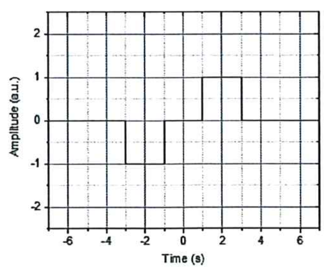
Select the two signals from the alternatives below.

Select two alternatives:



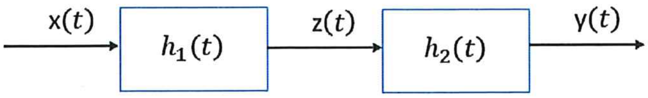




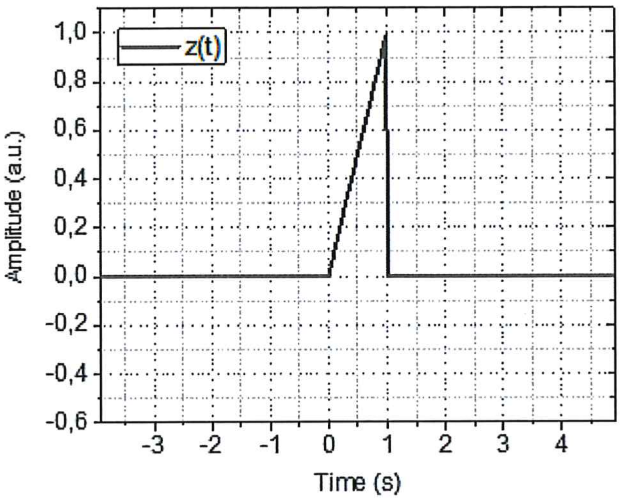


Maximum marks: 6

6 Consider two cascaded LTI-systems as shown below:



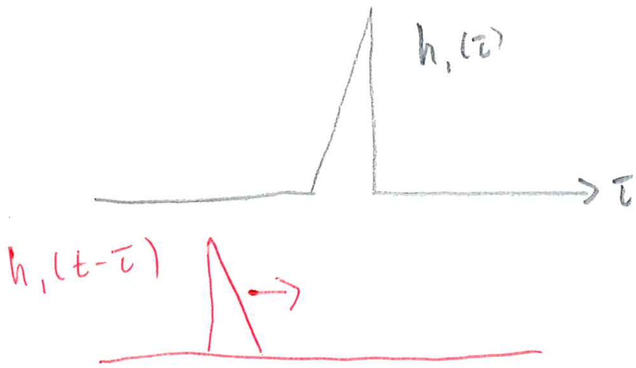
The impulse responses of the two systems are related by $h_2(t) = -h_1(t)$. If the input x(t) is the impulse response, the following signal z(t) is obtained:



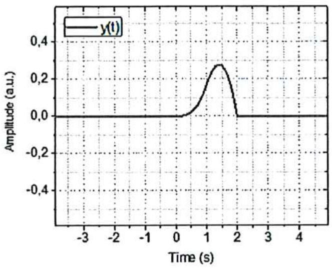
$$y(t) = -h_1(t) \otimes h_1(t)$$

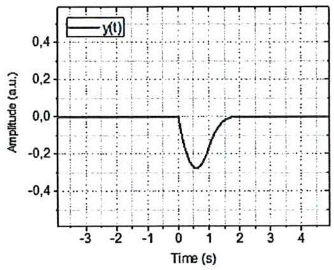
$$= -\int_{-\infty}^{\infty} h_1(\tau) \cdot h_1(t-\tau)\, d\tau$$

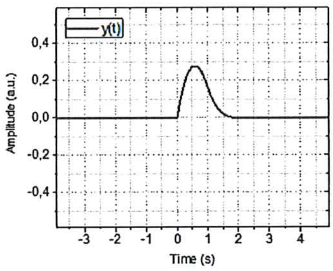
What will then be the signal y(t)?

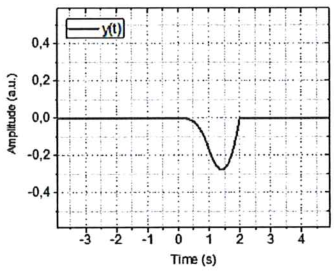


Select one alternative:









FROM THE GRAPHICAL CONSTRUCTION OF THE CONVOLUTION WE SEE ONLY ONE IS POSSIBLE (TAKING INTO ACCOUNT THE SIGN).

Maximum marks: 3

7 Along the column to the left are some time-functions listed (*u(t)* is as usual the unit step function). Note also that the function *r(t)* is a function of *p(t),* and *h(t)* is a function of *g(t)*. Match the 4 functions with their Fourier transforms found along the row on the top.

Please match each of the time-domain functions with their Fourier transform.

	$\frac{15+4i\omega}{12-\omega^2+7i\omega}$	$\frac{(3+2i\omega)}{\omega^2-3i\omega-2}$	$\frac{2}{2+i\omega}+\frac{1}{1+i\omega}$	$\frac{0.25(19+5i\omega)}{\omega^2-7i\omega-12}$
$p(t)=(e^{-4t}+3e^{-3t})u(t)$	✓	○	○	○
$r(t)=\int_{-\infty}^{t}p(\tau)d\tau$	○	○	○	✓
$h(t)=\int_{-\infty}^{t}g(\tau)d\tau$	○	✓	○	○
$g(t)=(e^{-t}+2e^{-2t})u(t)$	○	○	✓	○

Maximum marks: 8

DETAILS

$\longrightarrow$

LF 7

$$p(t) = \left(e^{-4t} + 3e^{-3t}\right) u(t)$$

$$P(\omega) = \frac{1}{4+i\omega} + \frac{3}{3+i\omega} = \frac{(3+i\omega) + 3(4+i\omega)}{12+7i\omega-\omega^2} = \frac{15+4i\omega}{12+7i\omega-\omega^2}$$

$$r(t) = \int_{-\infty}^{t} p(\tau)\,d\tau = -\frac{1}{4}\cdot e^{-4t} - e^{-3t}$$

$$R(\omega) = -\frac{1}{4}\frac{1}{(4+i\omega)} - \frac{1}{(3+i\omega)} = \frac{-\frac{1}{4}(3+i\omega) - (4+i\omega)}{12+7i\omega-\omega^2}$$

$$= \frac{-\frac{3}{4} - 4 - \frac{i\omega}{4} - i\omega}{} = \frac{-\frac{19}{4} - \frac{5i\omega}{4}}{} = -\frac{1}{4}\frac{(19+5i\omega)}{(12+7i\omega-\omega^2)}$$

$$g(t) = \left(e^{-t} + 2e^{-2t}\right) u(t)$$

$$G(\omega) = \frac{1}{1+i\omega} + \frac{2}{2+i\omega} = \frac{(2+i\omega) + 2(1+i\omega)}{2+3i\omega-\omega^2} = \frac{4+3i\omega}{2+3i\omega-\omega^2}$$

$$h(t) = \int_{-\infty}^{t} g(\tau)\,d\tau = -e^{-t} - e^{-2t} = \overbrace{-\frac{1}{1+i\omega} - \frac{1}{2+i\omega}}^{H(\omega)} = \frac{-2-i\omega-1-i\omega}{2+3i\omega-\omega^2}$$

$$\frac{-3-2i\omega}{2+3i\omega-\omega^2}$$

8 A unilateral Laplace transform is given by:

$$\frac{(s+1)}{(s^2+13s+36)}$$

What is the corresponding time-function? (u(t) is as usual the unit step function)

Select one alternative:

- $[6e^{-9t} - 3e^{-4t}]u(t)$
- $[\sin 3t - \cos 2t]u(t)$
- $\frac{1}{5}[8e^{-9t} - 3e^{-4t}]u(t)$ ✓
- $[\sqrt{2}\cos(3t + \pi/4)]u(t)$
- $\left[\frac{1}{\sqrt{2}}\sin(3t - \pi/4)\right]u(t)$
- $[\sin 3t - 2\cos 2t]u(t)$

Maximum marks: 2

WE NOTE THAT

$$\frac{s+1}{(s^2+13s+36)} = \frac{s+1}{(s+4)(s+9)} = \frac{1}{5}\left[\frac{8}{(s+9)} - \frac{3}{(s+4)}\right]$$

HENCE,

$$\frac{1}{5}\,8e^{-9t} - \frac{1}{5}\cdot 3\cdot e^{-4t}$$

9 A unilateral Laplace transform is given by:

$$\frac{(s-3)}{(s^2+9)}$$

What is the corresponding time-function? (u(t) is as usual the unit step function)

Select one alternative:

- $[\sin 3t - \cos 2t]u(t)$
- $\frac{1}{5}[8e^{-9t} - 3e^{-4t}]u(t)$
- $[\sin 3t - 2\cos 2t]u(t)$
- $[6e^{-9t} - 3e^{-4t}]u(t)$
- $[\sqrt{2}\cos(3t+\pi/4)]u(t)$ ✓
- $\left[\frac{1}{\sqrt{2}}\sin(3t-\pi/4)\right]u(t)$

HERE IT IS EASIEST TO EXAMINE THE LAPLACE TRANSFORMS OF THE TIME FUNCTIONS.

Maximum marks: 2

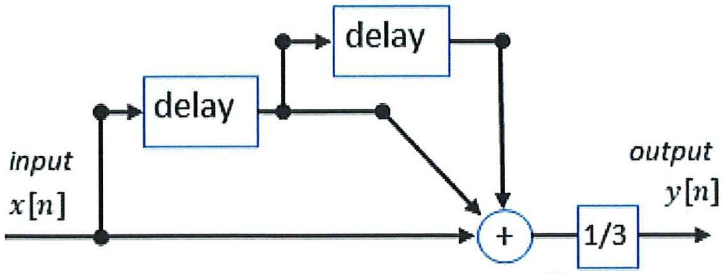
E.G.;

$$\mathcal{L}\left\{\sqrt{2}\cdot\cos\left(3t+\frac{\pi}{4}\right)u(t)\right\} = \mathcal{L}\left\{\sqrt{2}\cos 3t\underbrace{\cos\frac{\pi}{4}}_{\frac{1}{\sqrt{2}}} - \sqrt{2}\sin 3t\underbrace{\sin\frac{\pi}{4}}_{\frac{1}{\sqrt{2}}}\right\} =$$

$$= \mathcal{L}\{\cos 3t - \sin 3t\} \longrightarrow \text{TABLE} \longrightarrow$$

$$\frac{s}{s^2+9} - \frac{3}{s^2+9} = \frac{(s-3)}{(s^2+9)} \quad \text{Q.E.D.}$$

10 What is the output of the system below if the input is the discrete impulse function?



DIFFERENCE EQn BY INSPECTION $y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$

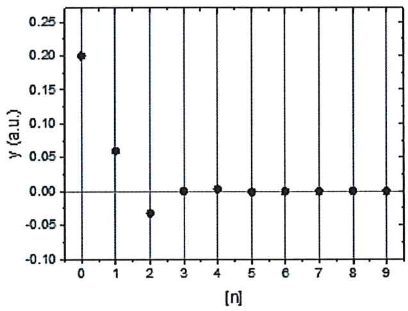
Select one alternative:

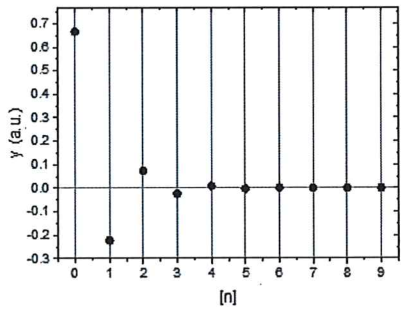
Z-TRANSFORM: $Y = \frac{1}{3}X + \frac{1}{3}Xz^{-1} + \frac{1}{3}Xz^{-2}$

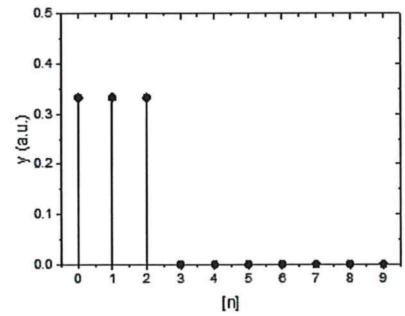
$\Rightarrow Y = \frac{1}{3}\left(1 + z^{-1} + z^{-2}\right)X$

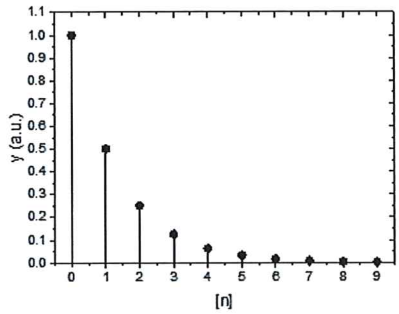
SET $X = 1$

$\Rightarrow \frac{1}{3}(1,1,1)$







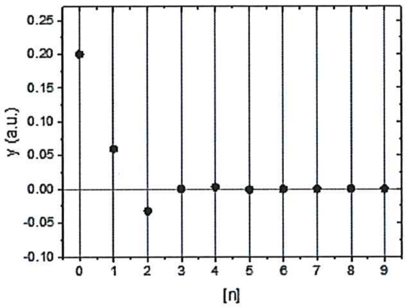


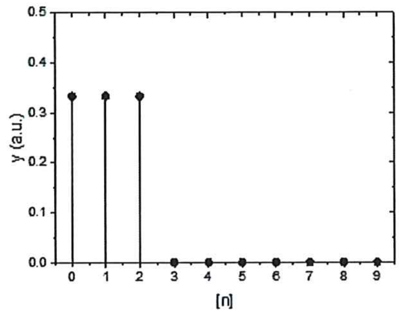
Maximum marks: 5

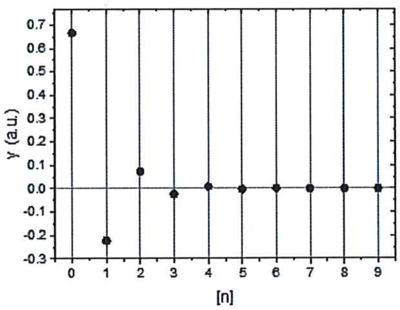
11 What is the output of the system below if the input is the discrete impulse function? Input is x[n] and output is y[n].

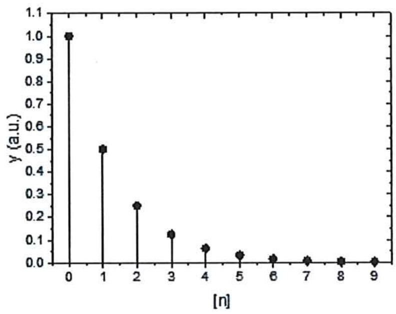
$$y[n-2] + 2y[n-1] + 10y[n] =$$
$$= x[n-1] + 2x[n]$$

Select one alternative:









DETAILS

Maximum marks: 5

LF 11.

$$y[n-2] + 2y[n-1] + 10y[n] = x[n-1] + 2x[n]$$

$$Y(z^{-2} + 2z^{-1} + 10) = (z^{-1} + 2)X$$

$$Y = \left(\frac{z^{-1} + 2}{z^{-2} + 2z^{-1} + 10}\right) X$$

$$Y = \frac{(z + 2z^2)}{(1 + 2z + 10z^2)} X$$

$$= \frac{(2z^2 + z)}{(10z^2 + 2z + 1)} X$$

X = 1 GIVES IMPULSE RESPONSE

LONG DIVISION

$$\frac{1}{5} + \frac{3}{50} z^{-1} - \frac{16}{500} z^{-2} \ \ldots$$

$$10z^2 + 2z + 1 \ \big) \ 2z^2 + z$$

$$2z^2 + \frac{2}{5}z + \frac{1}{5}$$

$$0 \quad \frac{3}{5}z - \frac{1}{5}$$

$$\frac{3}{5}z + \frac{6}{50} + \frac{3}{50}z^{-1}$$

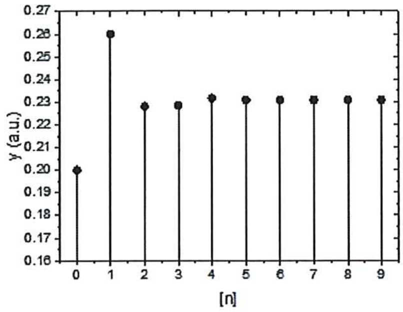
$$0 \quad -\frac{16}{50} - \frac{3}{50}z^{-1}$$

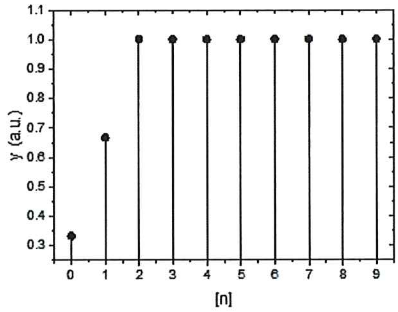
WE SEE HERE ONLY LEFT-TOP CAN BE RIGHT

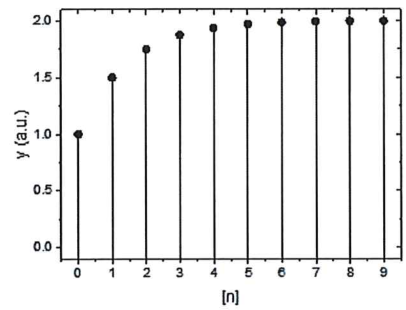
12 . What is the output of the discrete system transfer function as shown below if the input $x[n]$is the discrete unit step function? (as usual; the output $Y(z) = H(z)X(z)$)

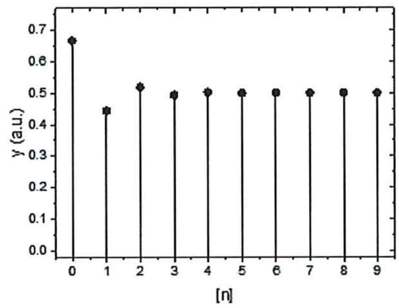
$$H(z) = \frac{(z^2 + z + 1)}{3z^2}$$

Select one alternative:









DETAILS

Maximum marks: 4

LF 12

DISCRETE UNIT STEP

$$\Rightarrow \quad X(z) = \frac{z}{z-1}$$

$$\Rightarrow \quad Y(z) = \frac{(z^2+z+1)}{3z^2} \; \frac{z}{(z-1)} = \frac{z^3+z^2+z}{3z^3-3z^2}$$

HERE, FINAL VALUE THEOREM CAN BE USED

$$\lim_{z\to 1} (z-1)\cdot Y(z) = \left.\frac{z^3+z^2+z}{3z^2}\right|_{z\to 1} = \frac{3}{3} = 1$$

CHECK:

LONG DIVISION GIVES

$$\frac{1}{3} + \frac{2}{3}z^{-1} + z^{-2} + z^{-3} + \ldots$$

$3z^3 - 3z^2 \,\big)\; z^3 + z^2 + z$

$z^3 - z^2$

$0 \quad 2z^2 + z$

$2z^2 - 2z$

$3z$

$3z - 3$

3

OK

13 Given the digital system described by the difference equation:

$$y[n] = \frac{1}{3}\{x[n] + x[n-1] + x[n-2]\}$$

What is the digital-time Fourier transform of the digital impulse response, i.e., $H(\Omega)$?
(x[n] is as usual the input and y[n] the output)

Select one alternative:

- $H(\Omega) = \frac{2}{3}e^{-i\Omega}(1 + \cos\Omega)$
- $H(\Omega) = \frac{1}{1 - 0.5e^{-i\Omega}}$
- $H(\Omega) = \frac{1}{3}e^{-i\Omega}(1 + 2\cos\Omega)$ ✓
- $H(\Omega) = \frac{1}{1 + 0.5e^{-i\Omega}}$

Let $x[n] = \delta[n]$

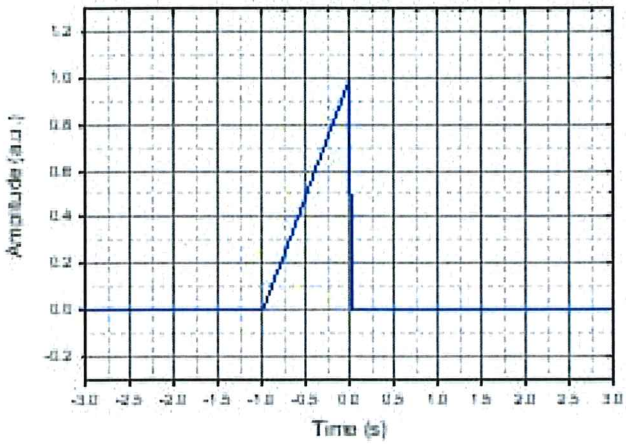
$$\Rightarrow H(\Omega) = \frac{1}{3}\left(e^{-i\Omega} + e^{-2i\Omega} + 1\right)$$

$$= \frac{1}{3} e^{-i\Omega}\left(1 + e^{i\Omega} + e^{-i\Omega}\right)$$

$$= \frac{1}{3} e^{-i\Omega}\left(1 + 2\cos\Omega\right)$$

Maximum marks: 3

14 Consider the wave-form below, what is the appropriate Fourier transform?



LINE SEGMENT

$(1+t) \quad t \in [-1,0]$

USE DEFINITION OF FT. $\Rightarrow F(\omega) = \int_{-1}^{0} (1+t)\, e^{-i\omega t}\, dt$

USE PARTIAL INTEGRATION

$$\int_a^b \underset{(1+t)}{u(x)}\, \underset{e^{-i\omega t}}{v'(x)}\, dx = [u(x)v(x)]_a^b - \int_a^b u'(x) v(x)\, dx$$

$$\Rightarrow F(\omega) = \left[(1+t) \frac{e^{-i\omega t}}{(-i\omega)} \right]_{-1}^{0} - \int_{-1}^{0} \frac{e^{-i\omega t}}{(-i\omega)}\, dt =$$

$$= \frac{1}{(-i\omega)} - \frac{1}{(-i\omega)^2} \left[e^{-i\omega t} \right]_{-1}^{0} = -\frac{1}{i\omega} - \frac{1}{(-i\omega)^2} \left[1 - e^{i\omega} \right] =$$

$$= -\frac{1}{i\omega} + \frac{1}{\omega^2} \left(1 - e^{i\omega} \right) = \frac{i}{\omega} + \frac{1}{\omega^2} (1 - \cos\omega - i\sin\omega)$$

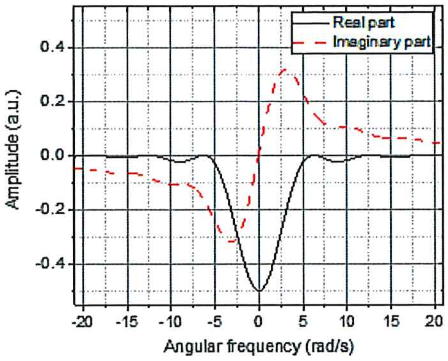
$$Re[F(\omega)] = \frac{1}{\omega^2} \cdot (1 - \cos\omega) \qquad \text{USE } \sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

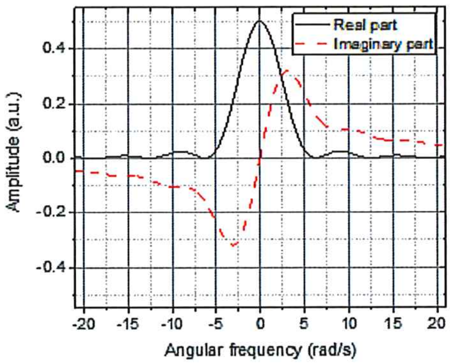
$$\Rightarrow Re[F(\omega)] = \frac{2}{\omega^2} \cdot \sin^2\left(\frac{\omega}{2}\right) = \frac{1}{2} \frac{\sin^2\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)^2}$$

$\longrightarrow$ CONT

Select one alternative:

WE NOTE THAT $\mathrm{Re}[F(\omega)]$ CAN BE ONLY EVEN POSITIVE FUNCTION





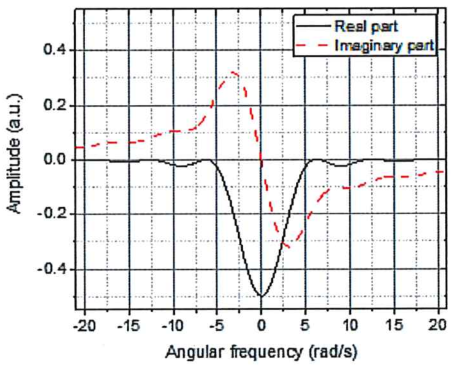
IMAGINARY PART

$$\mathrm{Im}[F(\omega)] = \frac{1}{\omega} - \frac{1}{\omega^2}\sin\omega = \frac{1}{\omega}\left(1 - \frac{\sin\omega}{\omega}\right)$$

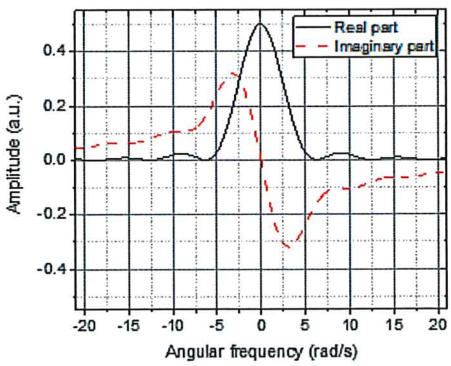
THIS IS AN ODD FUNCTION

$\lim_{\omega \to +\text{LARGE VALUE}} \Rightarrow$ SMALL POSITIVE VALUE

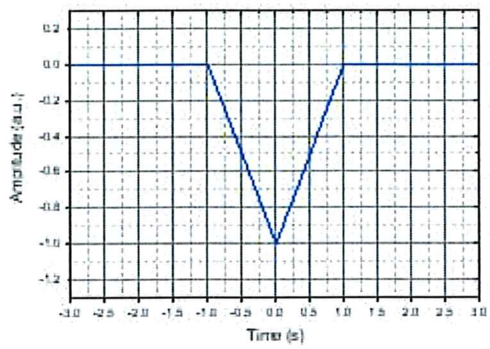
$\lim_{\omega \to -\text{LARGE VALUE}} \Rightarrow$ SMALL NEGATIVE VALUE



THE COMPLEX $F(\omega)$ CAN ALSO BE PLOTTED USING MATLAB AS DONE IN THE GRAPHS

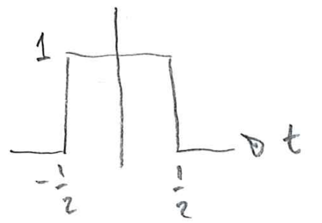


15 Consider the wave-form below, what is the appropriate Fourier transform?

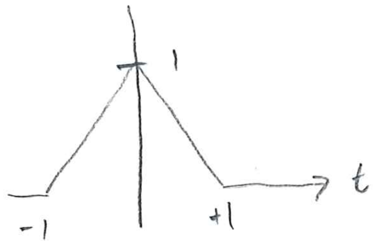


ALSO HERE WE CAN GET FT FROM THE FT INTEGRAL, BUT ALSO OTHER TRICKS

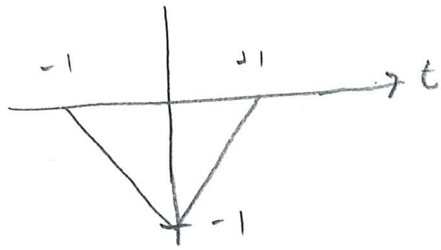
N.b. $\mathrm{rect}(t) \rightarrow \mathrm{RECT}(\omega) = \dfrac{\sin(\frac{\omega}{2})}{(\frac{\omega}{2})}$ FROM TABLE



$\mathrm{rect}(t) \otimes \mathrm{rect}(t) \Rightarrow$



$-\mathrm{rect}(t) \otimes \mathrm{rect}(t) \Rightarrow$

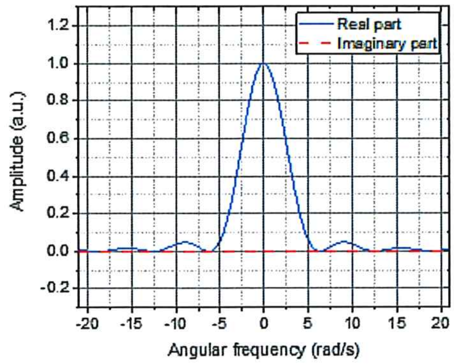


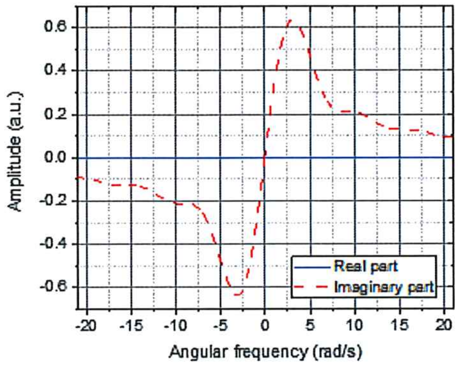
$\mathrm{FT}\{a(t) \otimes b(t)\} \rightarrow A(\omega) \cdot B(\omega)$ IN FOURIER DOMAIN

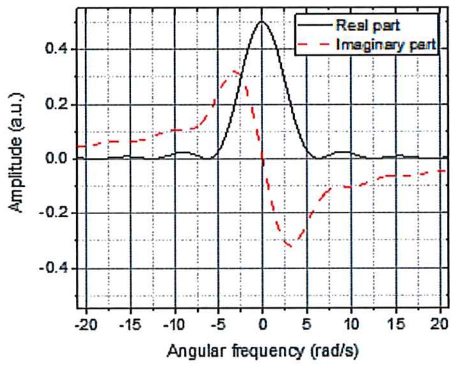
$\therefore$ THE FT WE WANT IS $-\dfrac{\sin^2(\frac{\omega}{2})}{(\frac{\omega}{2})^2}$

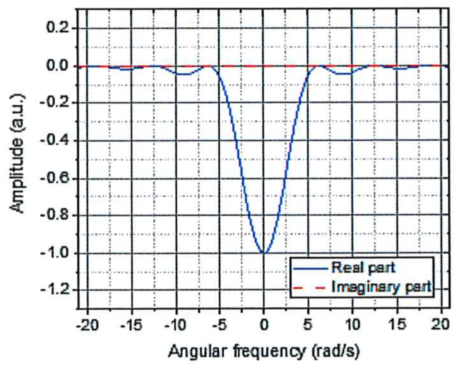
N.b. EVEN FUNCTION GIVES EVEN REAL FT AS EXPECTED $\longrightarrow$

Select one alternative:

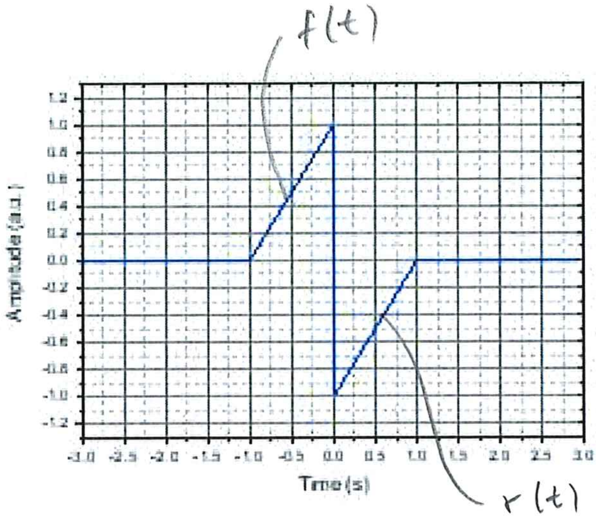
○ 

○ 

○ 

○ 

16 Consider the wave-form below, what is the appropriate Fourier transform?



WE ALREAD KNOW

$$F(\omega) = -\frac{1}{i\omega} + \frac{1}{\omega^2}\left(1 - e^{i\omega}\right)$$

$R(\omega)$ IS CALCULATED SIMILARLY, GIVING

$$R(\omega) = -\frac{1}{i\omega} + \frac{1}{\omega^2}\left(e^{-i\omega} - 1\right)$$

SO THE ANSWER IS $F(\omega) + R(\omega)$ AS FT IS A LINEAR OPERATION

THUS $$F(\omega) + R(\omega) = -\frac{2}{i\omega} + \frac{1}{\omega^2}\left(e^{-i\omega} - e^{i\omega}\right)$$

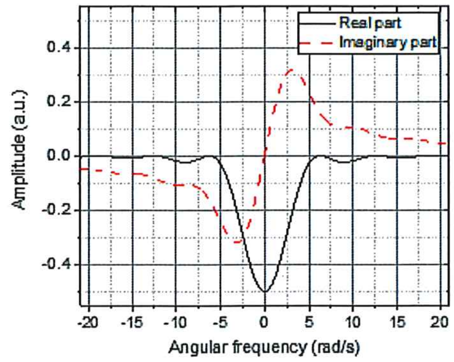
$$= \frac{2i}{\omega} - \frac{2i}{\omega^2}\sin\omega$$

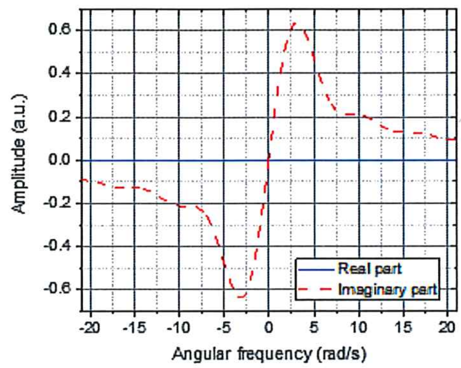
$$= i\,\frac{2}{\omega}\left(1 - \frac{\sin\omega}{\omega}\right)$$

THIS IS (PURELY) IMAGINARY, AS EXPECTED FROM FT OF AN ASYMMETRIC FUNCTION

$\longrightarrow$

Select one alternative:





This fits well with plot of imaginary part of (14) ×2 in amplitude