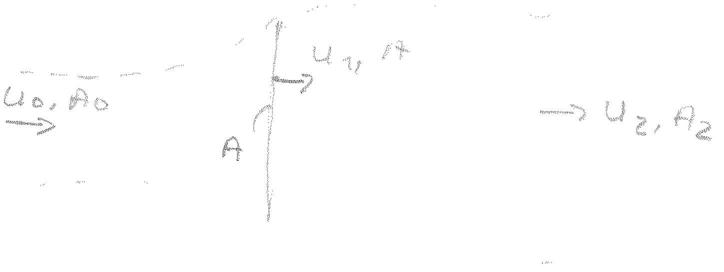
Solution problem 1. (1)

a)

Power through area A per unit time

$$P_0 = \underbrace{A \cdot u_0 \cdot \Delta t}_{\Delta V} \cdot \rho \cdot \frac{1}{2} u_0^2 \cdot \frac{1}{\Delta t} = \frac{A \rho u_0^3}{2} \qquad (1)$$

Assume a turbine to be an ideal "power extractor" within a stream tube:



Mass flux conservation: $u_0 A_0 = u_1 A = u_2 A_2 = \dot{m}$ (2)

Incompressible fluid assumed

Power extracted = power in − power out

$$P = \frac{\rho}{2}\left[A_0 u_0^3 - A_2 u_2^3\right] = \frac{\dot{m}}{2}\left[u_0^2 - u_2^2\right] \qquad (3)$$

Axial Force F on turbine given momentum difference:

$$F_A = \dot{m}\left[u_0 - u_2\right] \qquad (4)$$

For ideal system $P = F_A \cdot u_1$ (5)

$$(3), (4) \,\&\, (5) \Rightarrow \frac{\dot{m}}{2}\left[u_0^2 - u_2^2\right] = \dot{m} u_1 \left[u_0 - u_2\right] \Rightarrow u_1 = \frac{u_0 + u_2}{2}. \qquad (6)$$

Define axial inference factor a

$$a = (u_0 - u_1)/u_0 \Rightarrow u_1 = (1-a)u_0 \,\&\, \overset{(6)}{\downarrow} u_2 = (1-2a)u_0 \qquad (7)$$

From (3) it then follows

$$P = \frac{A \rho u_0^3}{2}(1-a)\left[1 - (1-2a)^2\right] = 4 P_0 \left[a^3 - 2a^2 + a\right]$$

$\frac{\partial P}{\partial a} = 0 \Rightarrow 3a^2 - 4a + 1 = 0 \Rightarrow a = \frac{2 \pm 1}{3} = \begin{cases} 1 \\ \frac{1}{3} \end{cases}$ (8)

1 corresponds to reflection (unphysical)

$\frac{1}{3}$ gives maximum $\Rightarrow$

$P = P_{max} = \frac{16}{27} P_0 \equiv C_{TB} \cdot P_0$ $C_{TB} = \frac{16}{27}$

Generally (non ideal systems) (9)

$P < C_{TB} \cdot P_0$

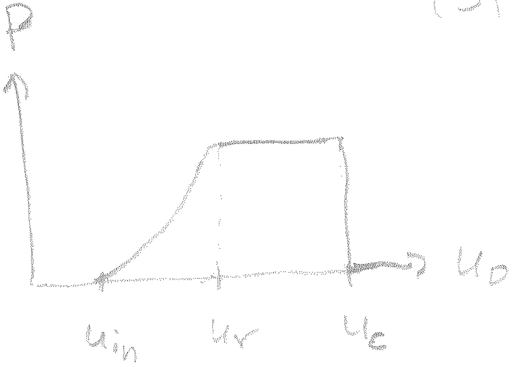
Good 2 or 3 bladed turbines obtain ~ 80% of Betz power.

b) from (4), (7) & (8) it follows

$F_A = F_{AB} = A\rho u_1 [u_0 - u_2] = A\rho \frac{u_0^2}{2} \cdot \frac{8}{9} \equiv C_D \cdot A\rho \frac{u_0^2}{2}$ (10)

$C_D = \frac{8}{9}$ are conventionally called the drag coeff $C_D \cong 1.2$ for circular plate; ~ 70% of this value.

c) Modern large wind turbines have normally 2 or 3 blads. They are fast rotating, blad tip speed 9 or 6 times rated wind speed. To reduce thrust or axial force, power is kept constant above rated ~~no~~ wind speed u_R; from (5) it then follows that F_A decrease with increasing $u_0 > u_R$



$u_{in} \sim 4$ m/s "cut in"

$u_R \sim 14$ m/s "rated"

$u_E \sim 25 \frac{m}{s}$ "cut out"

Power curve.

Principles of control are either "stall"; turbine is then locked to grid frequency. By increasing wind $u_0 \geq u_R$ turbulence is formed behind blades, reducing power. Or "pitch"; blades are turned to reduce force and power. Now often combined with free speed of turbine, and AC – DC – AC el. power conversion.

d) Drag machines. Power per flap of Area A comp. (5) & (10)

$$P = A \cdot u \cdot C_D \rho \frac{(u_0-u)^2}{2}$$

$$\frac{\partial P}{\partial u} = 0 \Rightarrow P_{max} = \frac{4}{27} C_D \cdot P_0 \text{ for } u = \frac{u_0}{3}.$$

Concave flats can give $C_D \approx 1.5$; thus max power ≈ 22% of P_0 compared ~~to~~ 59% of P_0 for Turbines. Whereas A is largely empty (except for slender blades) for a turbine, it is fully stuffed for drag machine.

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Suggested solution to exam Problem 2

a) Show that the energy received from the sun earth is on average equal to the solar constant S given by

$$\frac{r_s^2 \sigma T_s^4}{d_{se}^2} = S \approx 1400[\mathrm{W/m^2}].$$

Here S is the solar constant, r_s is the sun radius ($r_s = 6.96 \times 10^8$ [m]), d_{se} is the sun earth distance ($d_{se} = 1.49 \times 10^{11}$ [m]) and σ is the Stefan Booltzmann's constant ($\sigma = 5.672 \times 10^{-8}$ [$\mathrm{W/m^2K^4}$]).

Solution: If the sun behaves like a black body the output from the surface is given by

$$I(T) = \sigma T_s^4 \quad [W/m^2]$$

and multiplying this with the total area of the sun surface we get the total radiation from the sun

$$\begin{aligned} I(T) &= \sigma T_s^4 \cdot A \\ &= \sigma T_s^4 4\pi r_s^2 \quad [W] \end{aligned}$$

From geometrical points of view, the earth intercepts a fraction

$$\frac{\pi r_e^2}{4\pi d_{se}^2}$$

out of the radiation at a sphere with radius d_{se} the mean distance from sun to earth. So that the total energy the earth receives is

$$I = \sigma T_s^4 4\pi r_s^2 \frac{\pi r_e^2}{4\pi d_{se}^2} \quad [W]$$

and dividing by the area receiving we get the average value, the solar constant

$$S = \sigma T_s^4 4\pi r_s^2 \frac{\pi r_e^2}{4\pi d_{se}^2} \cdot \frac{1}{\pi r_e^2}$$

or

$$\underline{\underline{S = \frac{\sigma T_s^4 r_s^2}{d_{se}^2}\ [W/m^2]}}$$

b) Balance the incoming solar radiation with the total radiation emitted from the earth (assume that the earth behaves like a black body). Also assume that a fraction of the incoming radiation a is reflected back to space. Show that an estimate of the temperature of the earth can be found by

$$T_e^4 = \frac{(1-a)r_s^2 T_s^4}{4d_{se}^2}.$$

What is the temperature on earth calculated from this estimate if we assume that the earth albedo is 34%?

Solution: Solving this we have to balance the earth area receiving the solar constant with the area of the total earth area radiating thermal radiation out. This give the balancing equation

$$\pi r_e^2(1-a)S = 4\pi r_e^2 \sigma T_e^4$$

and rearranging

$$T_e^4 = \frac{(1-a)S}{4\sigma}$$

or if we insert from the results from the previous final result

$$T_e^4 = \frac{(1-a)\sigma T_s^4 r_s^2}{s_{se}^2 4\sigma}$$

or

$$\underline{\underline{T_e^4 = \frac{(1-a)T_s^4 r_s^2}{4d_{se}^2}}}$$

Inserting the values into this equation should yield:

$$T_e = \left[\frac{(1-0.34)\cdot 5800^4[K^4]\cdot 6.96\times 10^8[m^2]}{4\cdot(1.49\times 10^{11})^2[m^2]}\right]^{\frac{1}{4}}$$

that should give

$$\underline{\underline{T_e = 252.6449[K]}}$$

c) The global average temperature on earth is measured to be 288 [K]. Explain why our estimate from b) is to low?

Solution: Our estimate is the temperature of the earth as seen from a distant observer. This is the top of the atmosphere estimate (correct value is 255K). We have not taken any greenhouse effect from the atmosphere into account and hence our estimate is to low.
The greenhouse effect traps thermal radiation and hence the thermal cooling of the earth is less.

d) A doubling of the CO_2 in the atmosphere is estimated to give a radiative forcing of 4.6 [W/m^2] or as a consequence, a temperature increase of 1.37 [K]. What is meant with radiative forcing?

Solution: The book defines radiative forcing on page 35. (Boeker and Grondelle). The answer is more or less the same as last years exam question.

MNFFY 221/SIF 4082 Energy and Environmental Physics. 2002

Suggested solution to exam Problem 3

a) What is meant by fission absorption cross section?

Solution:
Nuclear absorption cross sections are given in barns 10^{-28} $[m^2]$ and describe the effective area of a nucleus a neutron see given energy of the neutron. Due to quantum mechanical effects this area exceeds the geometrical area of the nucleus. Energy dependent.
Fission absorption cross section: the number of fissions reactions per second over the incident neutrons per second per $[\mathrm{m}^2]$.
We have not covered the quantum mechanical aspects of the subject.

b) Explain the function of a moderator in a nuclear fission reactor.
Solution:
The nuclear fission absorption cross section is strongly energy dependent. When the neutrons are released from a nuclear fission they have a high energy and the nuclear fission absorption cross section is low. By slowing down the neutrons to thermal energies where the fission absorption cross section is 500 times larger, the probability that one neutron will give at least one new fission reaction gets larger and it is possible to operate the nuclear reactor. The moderators like water and heavy water also often act as the coolant of the reactor for safety reasons.

c) The multiplication factor k is given by

$$k = \eta \varepsilon p f (1 - l_f)(1 - l_s)$$

What does this factor describe and what are processes involved?

Solution:
k is called the multiplication factor in a generator. kn gives the probability that one generation of n neutrons give kn neutrons in

the next generation in a nuclear reactor.
If k¡1 the reactor produces less neutrons than what is created in the previous generation and the reactor will cool down. If k¿1 the number of neutrons present in the next generation exceeds the previous generation and the reactor will heat up. Critical operation at k=1 is wanted for stable operation when the reactor has reached the operation level.
η is the fission yield from one fission process > 1. One slow neutron will in a fission process give 1.33 new neutrons for natural uranium and 2 neutrons in 5% enriched uranium.
ε gives the fast fission yield, some neutrons that are not slowed down can give fission reaction and hence give new neutrons in the process before the neutrons are moderated > 1.
p is the resonance absorption escape probability, a probability that a neutron will be absorbed by the fission material without causing fission < 1. As the neutron energy decrease there is several "absorption lines" in neutron absorption cross section of the fission material and in these lines there is a finite probability that the neutron will just be absorbed.
f is the thermal fission utilization probability < 1. When the neutron has been slowed down it still does not have to cause a fission reaction, despite the large fission absorption cross section at these energies.
Some fast neutrons will be lost from the reactor by leaving the reactor and entering the reactor walls. The factor $(1\text{-}l_f)$ describes the fraction of fast neutrons lost and $(1\text{-}l_s)$ gives a fraction of slow neutrons absorbed. For a infinite sized reactor this losses are not present because the neutrons never leave the reactor. This losses hare however present at all energies and breaking it down to a slow and fast neutron loss is a simplification.

d) Discuss the main arguments for and against nuclear energy?

Solution:
For:
Stable energy source. Large power plants and the energy fuel has high energy density.

Climate change related arguments. No CO_2 emission, a simple solution to CO_2 emission demands from Kyoto.
Against:
Nuclear waste problem. Nuclear waste has a large economical consequence and and this problem will grow in the future.
An increasing pollution problem and available nuclear waste has become a risk problem. (May be expected to be terror targets)
Public opinion and general concern.
Comment:
There is in general an ongoing discussion about nuclear power and the use as a energy source that has not been covered in the lectures. Some students may bring in other arguments than these that has been to variable extent been covered in the lectures.

Exam 2002, MNF FY 221/SIF4082 Solution Problem 4

a) What are the three main heat transfer mechanisms, and how can each of them be expressed by a simple model (equation) ?

Conduction: $q=-k\Delta T$, k=conduction coefficient

Convection: $q=-h_c\Delta T$, h_c= convection coefficient, depends on orientation and size of surface, free or forced convection.

Radiation: $q=-h_r\Delta T$, $h_r = 4\,\varepsilon_{12}\,\sigma\,\underline{T}^4$

b) A commonly used expression for heat transport is U-value, for a typical insulated wall one can find a U-value of 0.3 $Wm^{-2}K^{-1}$. How do you define the U-value for a wall made out of bricks on the outside and wood on the inside with an isolation layer and an air layer between? (You are not supposed to calculate any numbers, just show how the value can be found and clearly explain the parameters needed to find the value)

U-value = $1/R_{total}$, R_{total}: total heat resistance for the wall
$R_{total} = 1/h_{outside} + d_1/k_1A + 1/h_r + d_2/k_2A + d_3/k_3A + 1/h_{innside}$
k_1: conduction coefficient brick, d_1 = thickness of the brick layer
k_2: conduction coefficient isolation, d_2 = thickness of the isolation layer
k_3: conduction coefficient wood, d_3 = thickness of the wood layer
A: area of the wall
$h_{innside/outside}$: forced convection coefficient.

c) Direct gain passive solar heating of a house can be of great value. Explain how this can be done in practice. Calculate the solar irradiance required to maintain room temperature 20°C above ambient. A typical value for thermal resistivity for windows can be 0.07 m^2KW^{-1} ? Make your own assumptions for the other factors needed for calculating the value?

If the solar radiation should keep a stable room temperature 20 degrees higher than the ambient temperatur, both incoming and outgoing heat fluxes should be the same:

$\tau\alpha G=(T_r - T_a)/r$

T_r (room temp), T_a (ambient temp), r=window resitivity, τ=window transmission, α= wall absorptance, G = total solar radiation .

$\tau=0.9$, $\alpha=0.8$

G= $20^oC/(0.07 \cdot 0.9 \cdot 0.8)$ = 400 Wm^{-2} The value may be expected on a vertical Sun facing window summertime in Norway, even spring and autumn. Not in the winter.

d) A solar collection will be more efficient with a selective surface. Explain why

A solar collector should maximize its energy gain and minimize its losses. Having a high absorption in the visible range and a low emitance in the infrared. Semiconductor materials have these properties to some extend. Metal but less efficiency in the visible/red.

(fig. 5.9)