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ENGLISH

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EXAM IN TFY 4300 Energy and environmental physics

Monday 7 December 2009
Duration: 4 hours

Number of pages: 15

Permitted aids:
1) One side of an A5 sheet with printed or handwritten formulas
2) Dictionary (ordinary or bi-lingual)
3) A calculator meeting NTNU examination criteria

Physical parameters and lists of equations are given in the appendix.

You must answer only 5 of the 6 questions.

Mark the question numbers you have answered on the top of the first page of your examination answers

Remember, most equations use SI units (i.e. Kelvin, metres seconds, Joules, Watts etc.)

SOLUTIONS

For graders reference:

$27.5\ TW \cdot h \cdot yr^{-1} = 2.75x10^{13}\ W \cdot h \cdot yr^{-1} = 3.14x10^{9}\ W = 3.14x10^{9}\ J \cdot s^{-1} = 9.90x10^{16}\ J \cdot yr^{-1}$

$110\ TW \cdot h \cdot yr^{-1} = 1.10x10^{14}\ W \cdot h \cdot yr^{-1} = 1.26x10^{10}\ W = 1.26x10^{10}\ J \cdot s^{-1} = 3.96x10^{17}\ J \cdot yr^{-1}$

IMPORTANT, THE CONVERSION IN THE APPENDIX WAS INCORRECTLY PRINTED ON THE TEST. AN ERRATA SHEET WAS PROVIDED BUT 2 STUDENTS DID NOT RECEIVE THIS. THUS THEY USED

$$1TW \cdot h \cdot yr^{-1} = 10^{12} W \cdot h \cdot yr^{-1} = 10^{12}\ W \cdot h \cdot yr^{-1} \div (24 \cdot 365 h \cdot yr^{-1}) = 8.76x10^{15} W$$

I HAVE THEREFORE ACCEPTED ANSWERS BASED ON

$$27.5\ TW \cdot h \cdot yr^{-1} = 2.49X10^{17} W$$

Problem 1. Wind turbine power (20%)

a) Give and expression for the power generated by a wind turbine with wind speed, u_o, the air density, ρ, and the cross-sectional area, A, of the wind front that is intercepted. What does the power index, C_p, tell us? (5%)

Taking into account the fraction of time, η, that a wind, u_o is blowing, the expression would be (2%):

$$Pt := \frac{1}{2} Cp\, A\, \rho\, Uo^3$$

The term $C_p=4*a(1-a)^2$ tells us how much of the incoming power in the wind, P_o, can be extracted by a wind turbine when we take into consideration that the turbine perturbs the flow, u_o. (3%)

b) What is the maximum value of C_p and why is it not 100%? That is, why can we not extract 100% of the wind energy? (5%)

The maximum for C_p is reached for $a=1/3$ and equals $C_p^{max}= 0.59$. C_p can not be 100% since that would imply that all the power is taken out of the wind, and the wind speed at the down-steam side u_2 would equal zero and the air at the down-stream side will not have any energy to move away from the turbine region.

c) The electric consumption in Norway is ~110 TW·h·yr^{-1}. How many windmills with a diameter of 80 m are needed to supply 25% (or 27.5 TW·h·yr^{-1}) of this consumption? Assume the wind speed is 16 m·s^{-1}, but it blows only 50% of the time (the wind speed is zero at other times). Take the density of air to be 1.2 kg·m^{-3} and the efficiency of each windmill to be 50% of the maximum theoretical value. Is it realistic to generate this amount of power this way? (10%)

Here we take $C_p = 0.5*C_p^{max}$, and are given all the other information to calculate the total power generated by each windmill if the wind were constant as:

$$P_t = ½ * [(0.5)*(0.59)] * [\pi*¼*(80\ m)^2]*1.2\ kg\cdot m^{-3}*(16\ m\cdot s^{-1})^3 = 3.6x10^6\ W\cdot windmill^{-1}$$

But the wind is only blowing ½ the time, so we only generate 50% of this:

$$P_{gen}=P_t*(0.5)=1.8x10^6\ W\cdot windmill^{-1}$$

The power we need to generate in watts can be found from:

$$P_{needed}=27.5\ TW\cdot h\cdot yr^{-1}*(10^{12} W\cdot TW^{-1})\div(365\ d\cdot yr^{-1}*24\ h\cdot d^{-1})=3.1x10^9\ W$$

Dividing the two gives me the number of windmills needed as

$$Windmills= P_{needed}/P_{gen} =3.1x10^9\ W/1.8x10^6\ W\cdot windmill^{-1}= \underline{1723\ windmills}$$

~2000 windmills spread out over Norway seems reasonable. In terms of area, one could give them 500x500 m each or 0.25 km^2. That would mean a wind park of about 431 km^2, or about 21x21 km. Maintenance would be easy, however, one could argue about the estimation of the wind speed and the fact that what do you do for power 50% of the time!

Problem 2. Geothermal and Tidal Energy (20%)

Normally the Earth's crust is ~30 km thick, and there is a 30 K·km^{-1} temperature gradient between the constant mantle temperature of 1200 K and the constant surface temperature of 300 K. But in a hyper-thermal area the gradient is 90 K·km^{-1}. If the crustal rocks have the same thermal conductivity, k= 2W·m^{-1}·K^{-1}:

a) Calculate the crustal thickness in the hyper-thermal region. (3%)

Here we just use the definition of gradient:

$$\Gamma := \frac{T2 - T1}{dx}$$

We are given the two temperatures and the gradient for the hyperthermal area and just need to solve for $dx = \Delta T/\Gamma$ = (1200-300)K / 90K·km^{-1} to get dx=<u>10 km = 10x10^{3}m</u>

b) Calculate the conductive geothermal heat flux (W·m^{-2}) in the hyper-thermal area. How does this compare to the heat flux in a normal area? (7%)

Here we use the heat transfer equation as we are given the temperature gradient and thermal conductivity in the hyperthermal area. Thus:

$$q := -\frac{k\,(T2 - T1)}{dx}$$

And we calculate q= -2 W·m^{-1}·K^{-1}·90 K·km^{-1}·10^{-3} km·m^{-1}=<u>-0.18 W·m^{-2}</u> from the hotter to colder temperature. The normal area has a heat flux of ~0.06 W·m^{-2}, so about <u>3x less.</u>

The semi-diurnal (12.5 hr) tidal range in Norway typically varies between 1 m and 2.5 m. An area, A, of seawater with a density of 1.03x10^3 kg·m^{-3} is used to generate electricity using this tidal range:

c) Calculate the average power per unit area available from the 12.5 hour tide. (7%)

There are several equations which one could use to calculate the average period. Since it is not specified, and I am not a masochist, I would use the simplest one:

$$q := \frac{1}{4}\,\frac{\rho\, g\,(Rmax^2 + Rmin^2)}{\tau}$$

Putting in all the information to calculate the power per unit area, I calculate:
*q= ¼ *1.03x10^{3}kg·m^{-3}*9.8 m·s^{-2}*[(2.5 m)^2+(1m)^2]/(12.5 hr*3600 s·hr^{-1})=<u>0.41 W·m^{-2}</u>*

This problem could also be approached using the approximation that the average power per unit area is given by:

$$q := \frac{1}{2}\,\frac{\rho\, g\, Ravg^2}{\tau}$$

Where Ravg=(Rmax+Rmin)/2. Here we have that Ravg=1.75 m, so:

*q=1/2*1.03x10^{3}kg·m^{-3}*9.8m·s^{-1}*(1.75m)^2/(12.5*3600s)=0.34 W·m^{-2},*

d) **The electric consumption in Norway is ~ 110 TW·h·yr^{-1}. What tidal area would have to be dammed to supply 25% (or 27.5 TW·h·yr^{-1}) of this consumption with tidal power? Assume that the conversion efficiency of the power plant is 100%. Is it realistic to generate this amount of power this way? (3%)**

Well, given the power per unit area above, I just need to convert this 27.5 TW·h·yr^{-1} to watts. This I do as:

$$P_{needed}=27.5\ TW\cdot h\cdot yr^{-1}*(10^{12}W\cdot TW^{-1})\div(365\ d\cdot yr^{-1}*24\ h\cdot d^{-1})=3.1x10^{9}\ W.$$

Just divide this by my power per unit area and I get the area of tidal water I need sequester to be:

$$A=P_{needed}/q=3.1x10^{9}W\ /\ 0.41\ W\cdot m^{-2}=7722x10^{6}\ m^{2}=\underline{7.722x10^{9}\ m^{2}}$$
$$\text{Or at }(10^{-6}\ km^{2}\cdot m^{-2})\text{ this becomes }=\underline{7722\ km^{2}}$$

If the second tidal approximation was used, one has $q=0.34\ W\cdot m^{-2}$ and the area would be:

$$A=P_{needed}/q=3.1x10^{9}W\ /\ 0.34\ W\cdot m^{-2}=9140x10^{6}\ m^{2}=\underline{9.14x10^{9}\ m^{2}}$$
$$\text{And again at }(10^{-6}\ km^{2}\cdot m^{-2})\text{ this becomes }=\underline{9140\ km^{2}}$$

This is an area about the size of Puerto Rico (about 90x90 km), which is enormous dam. The associated environmental effects would be extensive. So, this is probably not realistic.

Problem 3. Weather, Climate and Ozone (20%)

a) What is the temperature and scale height of an isothermal atmosphere in which the pressure is 1000 hPa at the surface and 500 hPa at a height of 5 km? (7%)

Here we use the hydrostatic equation and the perfect gas law (solved for ρ):

$$\frac{\partial}{\partial z}P = -\rho\, g \qquad\qquad \rho := \frac{P}{R\,T}$$

Substitute for ρ into the first equation to get:

$$\frac{dP}{P} = -\frac{g\,dz}{R\,T}$$

For an isothermal atmosphere, this can be integrated to give:

$$P = Po\,\mathbf{e}^{\left(-\frac{z\,g}{R\,T}\right)}$$

This can then be solved for T to give $T=ln(P/P_o)*R/(g*z)$. By substitution of the values:

$T=ln(500\ hPa/1000\ hPa)*287\ J\cdot K^{-1}\cdot kg^{-1}*/(9.8\ m\cdot s^{-2}*5x10^3 m)$ = <u>246K</u>.

The scale height is then just:

$H=RT/g=287\ J\cdot K^{-1}\cdot kg^{-1}*246\ K/9.8\ m\cdot s^{-2}$ = <u>7214 m</u> =<u>7.2 km</u>

Solving for either H or T from the hydrostatic equation is 5%; calculating either H or T from scale height equation is 2%.

b) On a global basis, how has the amount of stratospheric ozone changed since 1970, and what has caused this change? (2%).

On a <u>global</u> basis, the ozone amount has <u>decreased by ~4%.</u> In the Antarctic, the change is ~10% (though this was not asked) The cause of this change is the <u>catalytic reactions of Cl that destroy ozone</u>.

You will see students who have used an incorrect number they got from an earlier exam of 10% globally and 40% in the Antarctic. I have deducted 1 point for that as I had been very clear that it was incorrect..

c) If the earth's temperature is 15 C, at what wavelength does it radiate the most energy? (2%)

To get the wavelength in microns, just substitute T=15+273 = 288K into Wiens Displacement law: $\lambda := 2898\,\frac{1}{T}$, *to get that λ=10 microns.*

d) What is the greenhouse effect and what properties of a gas allow it to create a greenhouse effect in the Earth's atmosphere? (2%)

The greenhouse effect is caused by the ***atmosphere absorbing*** *the emission radiated from the planet towards space and warming up. As it has a warm temperature, the* ***atmosphere radiates*** *and at least ½ of that radiation will be directed back towards the planet's surface, warming the planet further*

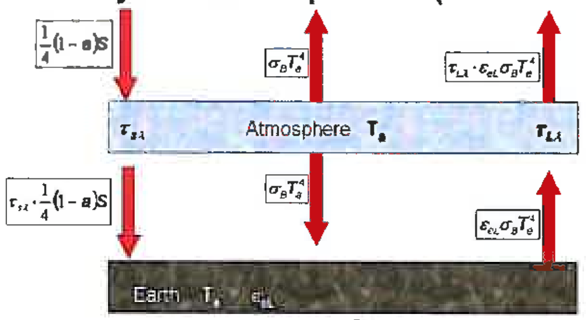
To have a significant greenhouse effect on Earth, a gas should have a low absorption, $\alpha_{S\lambda}$ (or high transmission $\tau_{S\lambda}$) in the visible and near infrared (wavelengths below ~1 micron). At the same time, it should have a high absorption, $\alpha_{L\lambda}$ (or low transmission $\tau_{L\lambda}$) at infrared wavelengths above ~4 micron.

e) How do you calculate the radiative equilibrium temperature of the Earth without an atmosphere, and how do you add the effect of an atmosphere? (note: you don't have to calculate the temperatures, but show or describe briefly how you set up the calculations) (7%)

A radiative equilibrium model balances the flux of (energy in) = (energy out) at any surface. For no atmosphere, the only surface is the Earth's surface, so the energy in from the sun that gets absorbed by the earth $=S(1-A)*\pi*R_e^2$, as the Earth catches the cross-sectional area $\pi*R_e^2$ of the solar flux, S. Here A is the albedo, or the amount of radiation reflected away from the earth. The energy out is what is radiated according over the entire globe according to the Steffan-Boltzmann Law $= 4*\pi*R_e^2 *\sigma_B*T_e^4$. Equating these two lets us solve for the temperature of the earth, T_e from: $S/4(1-A) = \sigma_B*T_e^4$.

With an atmosphere, one needs to balance the energy flux at the surface of the atmosphere and the surface of the earth, take into account the short and long wavelength transmission of the atmosphere, and that the atmosphere has a temperature, T_a and so radiates to space and to the surface of the Earth. The simple picture is:

Radiative Forcing:
Thin Layer Atmosphere (Introduction)



At Earth: $\tau_{S\lambda}\cdot\frac{1}{4}(1-a)\cdot S+\sigma_B\cdot T_a^4=\varepsilon_{eL}\cdot\sigma_B\cdot T_e^4$

At topside: $\frac{1}{4}(1-a)\cdot S=\sigma_B\cdot T_a^4+\tau_{L\lambda}\cdot\varepsilon_{eL}\cdot\sigma_B\cdot T_e^4$

Gives: $\varepsilon_{eL}\cdot\sigma_B\cdot T_e^4=\frac{1}{4}(1-a)\cdot S\cdot\frac{1+\tau_{S\lambda}}{1+\tau_{L\lambda}}$

If $\tau_{S\lambda}\approx0.85$, $\tau_{L\lambda}\approx0.15$ and $\varepsilon_{eL}=1$

Gives $T_e \approx 288$ K=15°C

So, either the picture ***or*** *the description of the fluxes is acceptable* ***and*** *the concept that they must be balanced at each surface is acceptable.*

Problem 4. Solar Energy (20%)

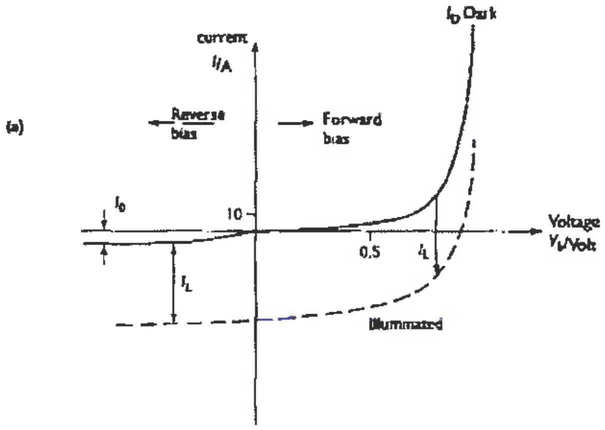
a) What is a p-n junction, and why do we need p-n junctions in a solar cell? (7%)

A pn-junction (or a diode) is made of a semiconductor where one part is n-doped with donor atoms to give excess of free electrons and the other side is p-doped with acceptor atoms to give excess of free holes. Initially the free charges associated with the donors and acceptors diffuse over the metallurgical junction where the p- and n- materials meet. The electrons diffuse from the n-side where there are many electrons to the p-side where there are no (free) electrons, and the holes diffuse in the other direction. As this diffusion progresses the ionized acceptors and donors are left behind, and since they are charged they begin to set up an internal electric field. The diffusion process will end when the electric field is strong enough to oppose further diffusion.

The pn-junction with the internal electric field is needed for efficient collection of the photo-generated charge carriers (electrons and holes). Without the electric field, the electron would most likely recombine with the hole again; i.e. it would "jump" down to the valence band again, and emit a photon in the process, and no or little current would be extracted. The internal electric field separates the photo-generated electron hole pairs and forces them out of the depletion region towards the contacts.

During the solar cell operation, the electric field forces the photo-generated electrons to enter the n-region of the solar cell where there are few holes to recombine with, so the probability for recombination is smaller. The holes are forced to move into the p-region with few electrons.

b) Draw a graph of the current-voltage characteristic of a solar cell when it is in the dark and when it is illuminated. Label the dark current and the photo-current. (6%)



I_D= dark current (solid line), and I_L = photo current.
The dashed line represents I_D-I_L = total current. Note, I_D looks like a standard diode characteristic curve, and the illuminated curve is the standard curve - the photo current which depends only on the illumination of the device (not the bias). A drawing of the potential energy diagram with currents noted will give 2 points.

c) **How large of an area needs to be covered with silicon solar cells to generate 27.5 TW·h·yr^{-1} (25% of Norway's yearly electricity consumption) if the annual solar irradiance is 900 kW·h·m^{-2}·yr^{-1} and the solar cells have a typical efficiency? Is it realistic to generate this amount of power this way? (7%).**

Here we can calculate the power generated per m^2 by multiplying the solar irradiation by the efficiency of the typical solar cell, 15%. Thus,

$$P'_{gen} = 900\ kW{\cdot}h{\cdot}m^{-2}{\cdot}yr^{-1} * 0.15 = 135\ kW{\cdot}h{\cdot}m^{-2}{\cdot}yr^{-1}$$

Equating this with the power required, 27.5 TW·h·yr^{-1} will give us the area:

$$A = 27.5x10^{12}\ W{\cdot}h{\cdot}yr^{-1} / 135x10^{3}\ W{\cdot}h{\cdot}m^{-2}{\cdot}yr^{-1} = \underline{2.03x10^{8}\ m^2 = 204\ km^2}$$

With an efficiency of 0.2, this becomes:

$$P'_{gen} = 180\ kW{\cdot}h{\cdot}m^{-2}{\cdot}yr^{-1}$$

and

$$A = 27.5x10^{12}\ W{\cdot}h{\cdot}yr^{-1} / 180x10^{3}\ W{\cdot}h{\cdot}m^{-2}{\cdot}yr^{-1} = \underline{1.52x10^{8}\ m^2 = 152\ km^2}$$

Or, you can convert P'_{gen} to Watts/m^2, which is 103 W/m^2, multiply by the efficiency and and equate this with the power required, 27.5 TW·h·yr^{-1} = 3.14x10^9 W

This is not too large of an area, about 14x14 km. However, it would be expensive to install and operate as it must be kept free of snow and dirt. Also, this is an average yearly number, and the peak power demand will occur in the winter when the plant is producing the least!

Problem 5. Bio Energy (20%)

a) Can bio-energy <u>always</u> be classified as a renewable energy? Why or why not. (3%)
No, to be classified as a renewable energy, the bio-mass harvested for energy must be replaced by new-growth crops that themselves can be harvested after some time (from less than a year to tens of years)

b) Make a list of the 5 most important advantages and disadvantages of bio-energy? (7%)
So, I will accept any 5 from the list below or reasonable arguments from the students.

- CO_2 neutral: Carbon dioxide, which is released when biomass fuel burns is returned to the natural carbon cycle. *- Waste from human activities can be utilized as an energy source* *- Biomass is readily available and can be continuously produced* *- the potential for bioenergy is largest in the third world countries* *- the biomass and biofuels have a large variety of uses* *- Biomass fuel from wastes may be a secondary product that adds value to the human activity (agriculture, aquaculture, industries etc).* *- Ethanol blends can be used in all petrol engines without modifications.*	*- compete with food production* *- increase in soil infertility and erosion.* *-land used for energy crops may be in demand for other purposes* *- deforestation in some regions* *- need labor and area for the processing of the sunflowers and for the heat power plant* *- some pollution and emissions possible* *- research is needed to reduce the costs of production of biomass-based fuels.*

c) If the energy content of the forests in Norway is sufficient to supply 100 GJ·ha^{-1}·yr^{-1} of thermal energy, how many km^2 of forest would be needed to generate 27.5 TW·h·yr^{-1} of electricity in a steam-turbine power plant? Assume that the power plant has an overall efficiency of 36%. Is it realistic to generate this amount of power this way? (10%)
Here we will calculate the Energy needed to produce 27.5 TW·h·yr^{-1} of electricity. First:

$$P_{electricity}=27.5\ TW\cdot h\cdot yr^{-1}*(10^{12} W\cdot TW^{-1})\div(365\ d\cdot yr^{-1}*24\ h\cdot d^{-1})=3.1x10^{9}\ W=3.1x10^{9} J_{el}\cdot s^{-1}$$

*In one year, there are (365 d*24 h·d^{-1}*3600 s·h^{-1})=3.15x10^7 s, so the electric energy produced in one year is:*

$$E_{elec}=3.1x10^{9} J_{el}\cdot s^{-1}*3.15x10^{7}\ s\cdot yr^{-1}=9.8x10^{16}\ J_{el}\cdot yr^{-1}$$

(Note, the calculation can also be done by taking:

$$27.5\ TW\cdot h\cdot yr^{-1} \text{ to be } 27.5x10^{12}\ J_{el}\cdot s^{-1}\cdot h\cdot yr^{-1}*3600\ s\cdot h^{-1}=9.9x10^{16}\ J_{el}\cdot yr^{-1})$$

To make this much electricity, we must divide by our efficiency to get J_{th}:

$$E_{th}=9.9x10^{16}\ J_{el}\cdot yr^{-1}/0.36 = 2.8x10^{18}\ J_{th}\cdot yr^{-1}$$

The forest can supply $E'_{for}=100x10^{9}\ J_{th}\cdot yr^{-1}\cdot ha^{-1}$, and to find the area, we divide E_{th} by E'_{for} to get

Area (ha) = E_{th}/E'_{for} = <u>2.75x10^6 ha</u>, or at 10^4 m^2·ha^{-1} we have A (m^2)=2.75x10^{10} m^2

Finally, at 10^{-6} km²·m⁻² we have

$$A\ (km^2) = \underline{2.75\times10^{4}\ km^2} = \underline{27{,}500\ km^2} \text{ or a } 166\times166 \text{ km patch!}$$

Any unit of area is acceptable.

Norway is 323000 km^2, and only about 1/3, or 110000 km^2 is forested. So this would represent ¼ of Norwegian forest to be cut every year. If the grow-back time is <4 years, it could be done, as the first forest you cut will be ready to harvest again by the time you cut the last one. But this is a high environmental cost and probably not a realistic way to generate the power. Plus you have the pollution from burning the wood.

Problem 6. Nuclear Energy (20%)

a) What is the difference between fission and fusion? Give an example of each process and explain how energy is released in these processes. (3%)

Fission is when a nucleus becomes or is made unstable and breaks into two smaller nuclei. An example of this is when $^{235}U+n \rightarrow ^{236}U \rightarrow X+Y+n\nu$ + ~200 MeV (here they can leave out the intermediate ^{236}U state and don't need the energy).(1%)

Fusion occurs when two small nuclei are brought together to form a larger nuclei.(1%) An example of this process would be the fusion of deuterium and tritium: $^2D+^3T \rightarrow ^4He + n$ + 17.6 MeV (here, any process would do and they don't need the energy).(1%)

The energy results because the binding energy of the products is different from the initial reagents. In fission, it takes less energy to break the bond than the binding energy released when the products are formed. In fusion, the binding energy released when the product is formed is more than the binding energies of the individual reagents. One could also put this in terms of the mass of the products is less than the mass of the reagents and the difference is energy=$\Delta m \cdot c^2$ (1%)

b) How does the energy released in each <u>fusion</u> of deuterium and tritium compare to the combustion energy of a methane molecule (CH_4) at ~55 MJ/kg? (5%)

Each fusion of deuterium and tritium releases 17.6 MeV of energy. If we convert the combustion energy of methane to eV, we can compare these:

*$55x10^6$ J/kg * 16 kg/kmole / $6.2x10^{26}$ molecules/kmole = $1.5x10^{-18}$ J/molecule of CH_4.*

$1.6x10^{-19}$ J/eV means that this is 9.4 eV released for each molecule of CH_4 burned, or

$17.6x10^6/9.4 = 1.8x10^6$ times more energy per fusion event.

Or say $17.6x10^6$eV/fusion$1.6x10^{-19}$J/eV=$2.8x10^{-12}$J/fusion compared to $1.5x10^{-18}$ J/CH_4*

Or, the tritium and deuterium are ~ 5 amu (kg·k-mole^{-1}), thus
$6.02x10^{26}$ fusions·k-moles^{-1} gives $8.31x10^{-27}$ kg·fusion^{-1},

At $2.8x10^{-12}$ J·fusion^{-1}, we get: $2.8x10^{-12}/8.31x10^{-27}$ J/kg= ~$3.37x10^{14}$ J/kg, or:
$3.37x10^8$MJ/kg compared to 55 MJ/kg, or $6x10^6$ times more energy.

c) **A typical pressurized water nuclear reactor can produce 27.5 TW·h·yr^{-1} of electrical energy by the fission of Uranium 235. If the overall efficiency of electricity production from the fission power plant is 40%, what mass of ^{235}U is burned over a three-year period? Assume 200MeV/fission event and a mass for ^{235}U of 235.04394 atomic mass units. (12%).**
We want to know how many joules of thermal energy that represents per year, so that we can divide by the energy of each fission event (giving us fissions per year). Then since each fission events consumes one ^{235}U, and we know the mass of a single ^{235}U, we can calculate the mass of ^{235}U consumed each year.

So, for our $27.5x10^{12}$ $W{\cdot}h{\cdot}yr^{-1}$, the calculation goes as follows:
$P_{el}=27.5\ TW{\cdot}h{\cdot}yr^{-1}*(10^{12}W{\cdot}TW^{-1})\div(365\ d{\cdot}yr^{-1}*24\ h{\cdot}d^{-1})=3.1x10^9\ W=3.1x10^9\ J_{el}{\cdot}s^{-1}$.

This gives us the $J_{el}{\cdot}s^{-1}$. To convert this to $J_{th}{\cdot}s^{-1}{}_{t}$, we need to use the efficiency of 40%. So $J_{th}{\cdot}s^{-1}{}_{t} = J_{el}{\cdot}s^{-1}/0.4$, or the plant has to produce $\underline{7.9x10^9\ J_{th}{\cdot}s^{-1}}$ in order to produce 27.5 TW-hr of electricity each year.

Now, we know that there is 200MeV/fission event, and that 1 eV =$1.6x19^{-19}$ J. So we can calculate that there are $200x10^6eV*1.6x10^{-19}\ J{\cdot}eV^{-1}=3.2x10^{-11}\ J_{th}$/fission-event. So, the $7.9x10^9\ J_{th}{\cdot}s^{-1}$ gets divided by $3.2x10^{-11}\ J_{th}$/fission-event, and we see that there are $\underline{2.4x10^{20}\ fissions{\cdot}s^{-1}}$ in the plant.

Now each fission event consumes 235.04394 amu of ^{235}U. However, an amu is the number of kg/k-mole of ^{235}U. Thus there are $6.02x10^{23}$ ^{235}U atoms / mole-^{235}U (Avogadro's number). So:

(235.04395 kg / k-mole-^{235}U) / ($6.02x10^{26}$ ^{235}U-atoms / k-mole-^{235}U)

This says that there are $3.9x10^{-25}$ kg / ^{235}U atom. Then since each fission event consumes one ^{235}U atom, we have $3.9x10^{-25}$ kg /fission-event.

Multiply this by the number of fission events going on every year at the plant to produce the 27.5 TW-hr, and we see that we consume:

$2.4x10^{20}\ fissions{\cdot}s^{-1}*3.9x10^{-25}$ *kg of* ^{235}U*/fission* = $\underline{9.6x10^{-5}\ kg\ ^{235}U{\cdot}s^{-1}}$.

Now, in one year there are $(365\ d*24\ h{\cdot}d^{-1}*3600\ s{\cdot}h^{-1})=3.15x10^7\ s$
So in one year we consume $9.6x10^{-5}\ kg\ ^{235}U{\cdot}s^{-1}*3.15x10^7\ s=\underline{3020\ kg{\cdot}yr^{-1}}$

Multiply this by 3 years (a typical fueling cycle for a reactor) and we find we have consumed $\underline{9059\ kg\text{-}^{235}U}$ during that period.

Can be done on a per year basis as well:
$P_{el} = 27.5\ TW{\cdot}h{\cdot}yr^{-1}$ *to be* $27.5x10^{12}\ J_{el}{\cdot}s^{-1}{\cdot}h{\cdot}yr^{-1}*3600\ s{\cdot}h^{-1} =9.9x10^{16}\ J_{el}{\cdot}yr^{-1}$*, and taking into account the efficiency of the plant (40%), this means that we have to generate*
$P_{th}=P_{el}/\ 0.4 = 2.4x10^{17}\ J_{th}{\cdot}yr^{-1}$

There are still $200x10^6$ *eV/fission* * $1.6x10^{-19}\ J_{th}/eV=3.2x10^{-11}\ J_{th}$*/fission-event, so the number of fission events per year at the plant =*
$Fissions/yr=2.4x10^{17}\ J_{th}{\cdot}yr^{-1}/\ 3.2x10^{-11}\ J_{th}{\cdot}fission^{-1} = 7.7x10^{27}\ fissions{\cdot}yr^{-1}$

We still consume $3.9x10^{-25}$ kg·fission^{-1}, which gives:
*$7.7x10^{27}$ fissions·yr^{-1}*$3.9x10^{-25}$ kg·fission^{-1}=3020 kg·yr^{-1}*
Or, over 3 years = 9059 kg

*Or on a 3yr basis we have to generate $2.4x10^{17}$ J_{th}·yr^{-1}*3yr = $7.43x10^{17}$ J_{th}.*
at $3.2x10^{-11}$ J_{th}/fission-event we have a total of $7.43x10^{17}$ J_{th}/$3.2x10^{-11}$ J_{th}/fission-event, or a total of $2.32x10^{28}$ fissions over 3 years. At $3.9x10^{-25}$ kg·fission^{-1} gives 9059 kg ^{235}U.

APPENDIX

Physical constants and conversions

Planck's constant: $h = 6.626\times10^{-34}\,\text{Js}$

The speed of light: $c = 2.998\times10^{8}\,\text{m/s}$

Boltzmann's constant: $k = 1.38\times10^{-23}\,\text{J/K}$

Stefan-Boltzmann's constant: $\sigma = 5.672\times10^{-8}\,\text{W/(m}^2\text{K}^4)$

Avogadro's number: $N_A = 6.022\times10^{23}\,\text{mol}^{-1}$

The electron charge: $e = 1.602\times10^{-19}\,\text{C}$

The radius of the sun: $r_s = 6.96\times10^{8}\,\text{m}$

The radius of the earth: $r_e = 6.4\times10^{6}\,\text{m}$

The sun-earth distance: $\text{d}_{se} = 1.49\times10^{11}\,\text{m}$

Gas constant for dry air: $\text{R}_{\text{air}} = 287\ \text{J}\cdot\text{K}^{-1}\cdot\text{kg}^{-1}$

1 ha = $10^4\ \text{m}^2$ 1TW·h·yr^{-1} = 10^{12}W·h·yr^{-1} = 10^{12} W·h·yr^{-1}÷(24 · 365h·yr^{-1}) = 1.14x10^8W

List of equations

$pV = nRT$	$P_{ij} = \frac{\Delta T}{R_{ij}}$	$\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$
$p = \rho R_{air} T$		$P_r = A\varepsilon\sigma T^4$
$\frac{\partial p}{\partial z} = -\frac{p}{H}$	$q = \frac{P}{A} = \frac{\Delta T}{r}$	$P_{12} = \sigma(T_1^4 - T_2^4)A_1F_{12}$
		$P_{12} = A_1F'_{12}\sigma(T_1^2+T_2^2)(T_1+T_2)(T_1-T_2)$
$\frac{\partial T}{\partial z} = -\frac{g}{c_p} = -\Gamma_d$	$r = \frac{1}{h} = RA$	$R_r = \left[A_1F'_{12}\sigma(T_1^2+T_2^2)(T_1+T_2)\right]^{-1}$
$H = \frac{R_{air}\cdot T}{g}$	$r_n = \frac{\Delta x}{k} \qquad R_n = \frac{\Delta x}{kA}$	
$E = h\upsilon$	$r_v = \frac{X}{N\ k} \qquad R_v = \frac{X}{N\ kA}$	$\eta = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$
$I_E dE = \frac{2\pi\upsilon^3}{c^2}\frac{1}{e^{h\upsilon/kt}-1}dE$	$r_r = \frac{(T_1-T_2)}{q} \qquad R_r = \frac{(T_1-T_2)}{P_{12}}$	
$I_\lambda = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kt}-1}$	$P_v = AN\,\frac{k(T_s-T_f)}{X}$	$\eta_{Carnot} = 1 - \frac{T_C}{T_H}$
$\lambda_{max}T = 2898[\mu m\ K]$	$R = \frac{uX}{\nu}$	$COP = \frac{\text{Useful Output}}{\text{Work Input}}$
$I(T) = \sigma T^4$		
$4\frac{dT_e}{T_e} = +\left\{\frac{dS}{S} + \frac{d\tau_{s\lambda}}{1+\tau_{s\lambda}}\right\}$	$\mathbf{A} = \frac{g\beta X^3\Delta T}{\kappa\nu}$	$= \frac{Q_C \text{ or } Q_H}{W_{in}}$
$-\left\{\frac{d\varepsilon_{eL}}{\varepsilon_{eL}} + \frac{da}{a} + \frac{d\tau_{L\lambda}}{1+\tau_{L\lambda}}\right\}$		
	$P_m = \frac{dm}{dt}c(T_3 - T_1)$	$B = (U-U_f) - p_o(V-V_f) - T_o(S-S_f)$
	$P_m = \frac{dm}{dt}\Lambda$	$B = Q\left(1 - \frac{T_o}{T_H}\right)$
$P_T = IV$	$-mc\frac{d}{dt}(T_1 - T_0) = \frac{(T_1-T_0)}{R_{10}}$	

$$c = 2c_g = \frac{g}{2\pi}T$$

$$E = k_E H^2; \quad k_E = \frac{\rho g}{8}$$

$$J = c_g E$$

$$J = k_J T H^2; \quad k_J = \frac{\rho g^2}{32\pi}$$

$$E = \rho g H^2 / 8$$

$$E = \rho g \int_0^\infty S(f)\,\mathrm{d}f \equiv \rho g H_s^2 / 16$$

$$^{235}\mathrm{U} + \mathrm{n} \rightarrow {}^{236}\mathrm{U} \rightarrow X + Y + \upsilon \mathrm{n} + \text{energy}$$

$$\eta = \nu \frac{N(235)\sigma_f(235)}{N(235)\left[\sigma_f(235) + \sigma_c(235)\right] + N(238)\sigma_c(238)}$$

$$\frac{dN}{dt} = \frac{\rho N}{l}$$

$$l^* = (1-\beta)l + \beta t_d$$

$$^2\mathrm{D} + {}^3\mathrm{T} \longrightarrow {}^4\mathrm{He} + {}^1\mathrm{n} + 17.6\mathrm{MeV}$$

$$P_L = \alpha n^2 \sqrt{kT} + 3n\frac{kT}{\tau_E}$$

$$P_{th} = \langle \sigma u \rangle E \frac{n^2}{4}$$

$$P_T = P_L + P_{th}$$

$$\eta P_T > P_L$$

$$F_g = \frac{G m_1 m_2}{R_{12}} \qquad F_c = mR\omega^2$$

$$\frac{GMM'}{D^2} = ML\omega^2 = M'L'\omega^2$$

$$\frac{\partial^2 z}{\partial t^2} = gh\frac{\partial^2 z}{\partial x^2} = c^2\frac{\partial^2 z}{\partial x^2}; \quad c = \sqrt{gh}$$

$$T_r = \frac{\lambda}{c} = \frac{4L}{jc} = \frac{4L}{j\sqrt{gh}}$$

$$\overline{\mathrm{P}} = \frac{\rho A R^2}{2} g \cdot \frac{1}{\tau} = \frac{\rho A g}{2\tau} R^2 \qquad \overline{\mathrm{P}} \approx \frac{\rho A g}{2\tau}\left(\frac{R_{\max}{}^2 + R_{\min}{}^2}{2}\right)$$

$$q = \frac{P}{A} = \frac{\rho u^3}{2}$$

$$\bar{q} = \eta \frac{\rho \overline{u^3}}{2} = \eta \frac{\rho}{2} \frac{u_0^3 \int_{t=0}^{t=\tau/4} \sin^3(2\pi t/\tau)dt}{\int_{t=0}^{t=\tau/4} dt} \approx 0.1\,\rho u_0^3$$

$$V_B = \frac{E_g}{e} - (\phi_n + \phi_p)$$

$$np = C = n_i^2$$

$$W \approx \sqrt{\frac{2\varepsilon_0 \varepsilon_r V_B}{e\sqrt{np}}}$$

$$I(V) = I_L - I_D = I_L - I_0\left[\exp\left(\frac{eV}{AkT}\right) - 1\right]$$

$$\Delta\mu = E_{Fn} - E_{Fp} = eV$$

$$\eta = \frac{I_m \cdot V_m}{P_{in}} = FF \cdot \frac{I_{sc} \cdot V_{oc}}{P_{in}}$$

$$FF = \frac{I_m \cdot V_m}{I_{sc} \cdot V_{oc}}$$

$$\rho = \frac{(n_0 - n_1)^2}{(n_0 + n_1)^2}$$

$$CO_2 + H_2\dot{O} \xrightarrow{light} \dot{O}_2 + [CH_2O] + H_2O$$

$$P_T = C_P A \frac{\rho u_0^3}{2}$$

$$P_o = A\frac{\rho u_0^3}{2}$$

$$F_A = \frac{\Delta \mathrm{p}}{\Delta t} = \frac{m(u_0 - u_2)}{\Delta t} = \rho A_1 u_1 (u_0 - u_2)$$

$$F_A = A_1(p_{1u} - p_{1d}) = A_1 \rho (u_0{}^2 - u_2{}^2)/2$$

$$P = u_1 F_A$$

$$P_T = u_1 F_A = u_1 \frac{dm}{dt}(u_0 - u_2)$$

$$P_T = \frac{1}{2}\frac{dm}{dt}(u_0{}^2 - u_2{}^2)$$

$$a = \frac{u_0 - u_1}{u_0}$$

$$P_T = \left[4a(1-a)^2\right] P_0$$

$$C_P \le \frac{16}{27}$$

$$\frac{1}{(D \pm r)^2} = \frac{1}{D^2}\left(1 \pm \frac{2r}{D}\right)$$