Norges teknisk–naturvitenskapelige universitet NTNU

Institutt for fysikk
Fakultet for naturvitenskap og teknologi

Eksamen i TFY4305 IKKELINEÆR DYNAMIKK

Fredag, 8. desember, 2006
09:00–13:00

Tillatte hjelpemidler: Alternativ B
Godkjent lommekalkulator.
K. Rottman: *Matematisk formelsamling* (alle sprogutgaver).
O.H. Jahren og K.J. Knudsen: *Formelsamling i matematikk*.

Dette oppgavesettet er på 2 sider.

Oppgave 1

Consider the differential equations

$$\dot{x} = y \, , \tag{1}$$

$$\dot{y} = -\left(\sqrt{x^2+y^2}-\mu\right) y - x \, . \tag{2}$$

a) Show that $(x, y) = (0, 0)$ is a fixed point of Equations (1) and (2) for all values of μ. Find the eigenvalues of this trivial fixed point and show that the real parts vanish when $\mu = 0$.

b) What is the character of the solutions that are expected to bifurcate at $\mu = 0$ (i.e., what *kind* of solutions)?

c) Change to polar coordinates (r, θ). Construct the resulting dynamical system $\dot{r} = h_1(r, \theta)$, $\dot{\theta} = h_2(r, \theta)$.

d) Show that in the case $\mu \ll 1$ and $r \ll 1$, the equation for $\dot{\theta}$ can be integrated to $\theta(t) = -t + \theta_0$ (plus terms of order μ and of order r).

e) Apply averaging theory to derive an effective equation $\dot{r} = h(r)$. Use this equation to find the fixed points r^* of r.

f) Investigate the stability of the trivial fixed point $r^* = 0$ as well as the nontrivial fixed point in a neighbourhood of $\mu = 0$.

g) Plot the bifurcation diagram, i.e., (μ, r^*).

h) Is the bifurcation around $\mu \approx 0$ a *generic* Hopf bifurcation?

Oppgave 2

Consider the disrete mapping

$$x_{n+1} = f_r(x_n) = rx_n + x_n^3 \; . \tag{3}$$

a) Determine the fixed points x_n^* of Equation (3).

b) Show that $x_n^* = 0$ is a stable fixed point in an interval $[r_-, r_+]$ of the parameter.

c) Draw the bifurcation diagram (r, x_n^*) using a full line for stable fixed points and dotted line for unstable fixed points.

d) What kind of bifurcation takes place for $x_n^* = 0$ around $r \approx r_+$?

e) What kind of bifurcation takes place for $x_n^* = 0$ around $r \approx r_-$?

f) Show that a two-cycle with elements $x_\pm^c$ exists for $r = r_- - \epsilon$ where ϵ is a small parameter. Express $x_\pm^c$ to lowest order in ϵ. (Hint: Start with the equation for the two-cycle elements, $x^c = f_r^2(x^c)$, and keep terms up to order ϵ and to order $(x^c)^3$.)