Norges teknisk–naturvitenskapelige universitet NTNU

Institutt for fysikk
Fakultet for naturvitenskap og teknologi

Exam in TFY4305 IKKELINEÆR DYNAMIKK
Fredag, 12. desember, 2008
09:00–13:00

Allowed help: Alternativ B
    Godkjent lommekalkulator.
    K. Rottman: *Matematisk formelsamling* (alle sprogutgaver).
    O.H. Jahren og K.J. Knudsen: *Formelsamling i matematikk*.
This problem set consists of 2 pages.

Problem 1
The map
$$x_{n+1} = x_n + a + b \cos x_n \tag{1}$$
where $b > 0$ and $x_n \in [-\pi, +\pi]$, has fixed points in a certain region of the parameter plane (i.e., the (a, b) plane).

a) Find this region.

b) Describe the bifurcations on the edge of this region.

c) Locate any bifurcations of fixed points in the interior of the region.

Problem 2
Consider the system of differential equations

$$\dot{x} = y, \tag{2}$$
$$\dot{y} = -\frac{2x}{x^2 + a} - by + c. \tag{3}$$
$$\tag{4}$$

a) Find all fixed points for all values of the parameters a, b and c, which are all assumed to be positive.

b) Find the stability of each fixed point.

c) Find the bifurcation point in the parameter space and the type of bifurcation which occurs.

d) Show for $b = 0$ that the motion is that of a particle moving in a one-dimensional potential. What is the potential?

e) Discuss the physical meaning of the parameters a, b and c and of the bifurcation.

f) Let $b = c = 0$. Find the frequency of the periodic motion for a given amplitude r to first order in the small quantitiy r^2/a.