

## Some useful expressions and values.

$$\begin{aligned}
 I &= NE \\
 I &= I_0 e^{-\int \mu(x) dx} \\
 S &= \varepsilon A (I_P + I_S) \\
 C &= \frac{I_1 - I_2}{I_1} \\
 k &= \frac{S_1 - S_2}{STD_s} \\
 D &= EN_0 \left( \frac{\mu_{en}}{\rho} \right) \\
 \varepsilon &= \frac{N_{out}}{N_{in}} \\
 p(s, \theta) &= -\ln \frac{I}{I_0} = \int_A^B \mu(s, t) dt \\
 HU &= 1000 \cdot \frac{\mu - \mu_w}{\mu_w} \\
 R_c &= \frac{d(L+z)}{L} \\
 g &= \frac{d^2}{4\pi L^2} \frac{d^2}{(d+t)^2} \\
 t &= \frac{6d}{\mu L - 3} \\
 S &= S_0 (1 - e^{-TR/T_1}) e^{-TE/T_2} \\
 S &= \frac{S_0 (1 - e^{-TR/T_1}) \sin \theta e^{-TE/T_2}}{1 - \cos \theta e^{-TR/T_1}} \\
 f\lambda &= c \\
 c &= c_0 + v\beta \\
 \beta &= 1 + B/2A \\
 f_d &= \frac{\pm 2f_0 v \cos \theta}{c}
 \end{aligned}$$

## Problem 1: X-ray and CT Imaging

### 1a

The following three detector materials are available:

1. Unstructured CsI.
2. Structured CsI.
3. Amorphous Se.

Mass attenuation coefficients are found in figure 1. Density is around  $4.5 \text{ g/cm}^3$  for both materials.

- Which detector material would you choose for use in skeletal X-ray at an effective energy of 50 keV? Assume a fixed detector thickness should be used. Justify your answer.
- Estimate the detector thickness required to achieve 99% detector efficiency for your chosen detector material?
- Calculate the image SNR if the incoming X-ray intensity is  $8000 \text{ MeV/cm}^2$  and the pixel size is  $0.2 \times 0.2 \text{ mm}^2$ .

### 1b

Suggest a computer program (in pseudo-code) for a filtered back-projection reconstruction algorithm for two different cases:

- 1D filtering in the image domain.
- 2D filtering in the Fourier domain.

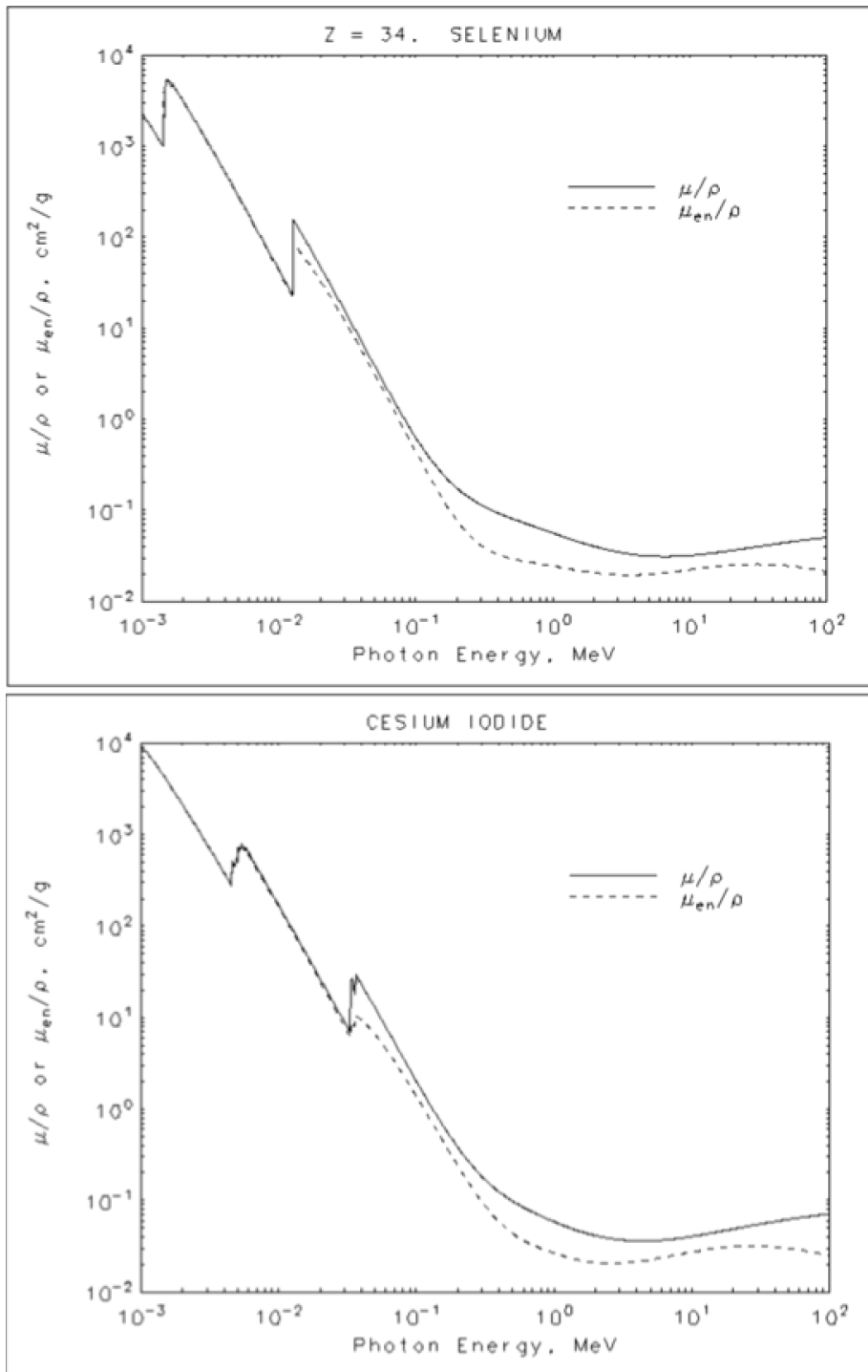


Figure 1: Mass attenuation coefficients, problem 1a.

## Problem 2: Nuclear Medicine Imaging

### 2a

Draw and explain the following three energy spectra in SPECT imaging, assuming initial emission of mono-energetic gamma-photons at energy  $E_\gamma$  inside a human body:

- Energy spectrum of the radiation emerging from the body.
- Spectrum of absorbed energy in the scintillator.
- Final spectrum as measured by the gamma camera.

### 2b

Given the following two parallel hole collimators:

1. Hole length = 36 mm, hole diameter = 3.4 mm, septal thickness = 0.5 mm.
  2. Hole length = 40 mm, hole diameter = 1.8 mm, septal thickness = 0.3 mm.
- Which collimator would you choose for single-photon emission imaging in an application where high spatial resolution is the highest priority. Justify your answer with calculations.
  - What is the disadvantage of your chosen collimator?

### 2c

- Describe the typical detector design for PET, including the scintillator crystals and photo detector geometry.
- In particular, explain how localization is achieved.

### Problem 3: Magnetic Resonance Imaging

#### 3a

- Describe the difference between  $R_2$  and  $R_2'$  in terms of their physical origin.
- Suggest a measurement procedure in order to estimate  $R_2'$ .

#### 3b

Derive the signal equation for the FID sequence in the case of arbitrary flip angle  $< 90^\circ$ . Make sure to explain the concept of steady-state as part of the derivation.

#### 3c

- Describe the effect of applying a linear magnetic field gradient during signal acquisition.
- Express the relation between the gradient  $G$  and wavenumber  $k$ .

### Problem 4: Ultrasound Imaging

#### 4a

Starting from the following relation:

$$p = -\kappa \frac{\partial u}{\partial x}$$

where  $p$  is the pressure and  $u$  is the longitudinal displacement, derive the 1D acoustic wave equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

Define the wave velocity  $c$ .

#### 4b

Explain the physical origin of speckle in pulse-echo imaging.