TFY4320 Physics of Medical Imaging
Exam 2016 Suggested Solutions

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1 Problem 1

1.1 Problem 1a

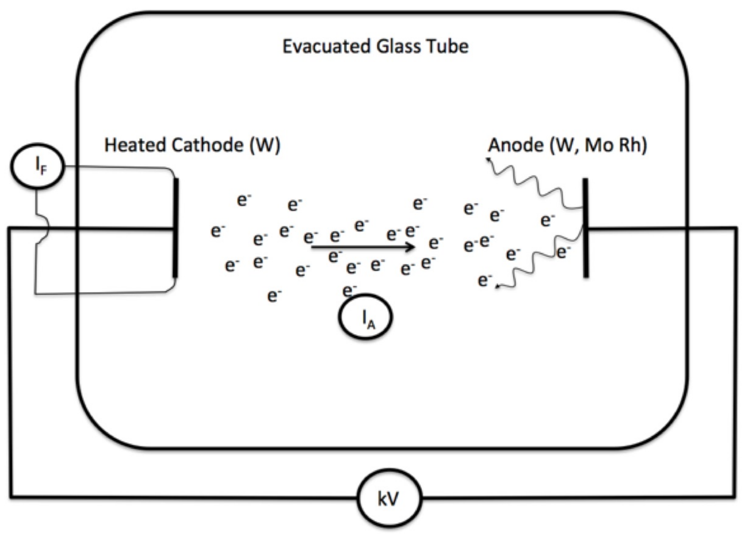

Figure 1: Principle sketch of the heated cathode X-ray tube. Electrons are escaping from the cathode, accelerate towards the anode due to the potential difference, and X-rays are generated as the electrons hit the anode. The heating current through the cathode filament I_F controls the flux of electrons, defined as the anode current (I_A). The high voltage potential controls the kinetic energy of the electrons hitting the anode, and thereby the maximum possible energy of generated X-rays.

Filtering of the spectrum means to remove/reduce either the low-energy part (soft-end filtering) or the high-energy part (hard-end filtering). The motivation for applying a filter is to obtain lower dose to the patient while maintaining the image quality.

Example spectrum should include the bremsstrahlung part with correct maximum energy according to acceleration voltage, and (if applicable) the characteristic lines correctly placed according to choice of anode material. Effect of filtering should be shown qualitatively correct.

1.2 Problem 1b

Two voxels of different content:

- Voxel A: Pure bone.
- Voxel B: Mixture of aluminum and soft tissue.

The first task is to estimate the volume fraction of aluminum. We know that at 60 keV the measured HU = 1857 for both voxels. Since the linear attenuation coefficients of aluminum, soft tissue and water are all given on page 2 in the exam, we have sufficient information to calculate the Aluminum volume fraction:

$$HU = 1000 \cdot \frac{\mu_{eff} - \mu_w}{\mu_w} \tag{1}$$

At 60 keV, HU = 1857 means that $\mu_{eff} = 0.60$. You can reach this result either by solving the HU-equation for HU = 1857, or by looking up μ_{bone} directly from the table on page 2. Next step is to apply the volume fraction equation and solve for f:

$$\mu_{eff} = f \cdot \mu_{Al} + (1 - f) \cdot \mu_{Soft} \tag{2}$$

$$f = \frac{\mu_{eff} - \mu_{Soft}}{\mu_{Al} - \mu_{Soft}} \tag{3}$$

$$f = = 0.72 \tag{4}$$

Next task is to calculate the HU values at higher keV. There was a typo in the exam-text resulting in an ambiguity between 80 keV or 100 keV as the higher energy case. Answers at both energies will be accepted. Here only the 80 keV case is shown. Answers are found by simply plugging into the HU-equation for the correct μ-values. For voxel A μ_{bone} is directly applied, for voxel B the effective μ must first be calculated:

$$HU_A = 1000 \cdot \frac{0.43 - 0.18}{0.18} = 1389 \tag{5}$$

$$\mu_{eff,B} = 0.72 * 0.54 + (1 - 0.72) * 0.19 = 0.44 \tag{6}$$

$$HU_B = 1000 \cdot \frac{0.44 - 0.18}{0.18} = 1444 \tag{7}$$

The two voxels will now have different HU values and can be distinguished. The brighter of the two voxels will be the one containing a mixture of Aluminum and soft tissue.

2 Problem 2

2.1 Problem 2a

Geometric efficiency calculated from formula given on page 2:

$$\begin{aligned} g &= \frac{d^4}{4\pi L^2(d+t)^2} && (8) \\ &= \frac{(2.5\cdot 10^{-3})^4}{4\pi(40\cdot 10^{-3})^2(2.5\cdot 10^{-3}+0.3\cdot 10^{-3})^2} && (9) \\ &= 2.48\cdot 10^{-4} && (10) \end{aligned}$$

Number of counts per second (n) in the gamma camera will be given by the multiplication of the activity of the source with the branching ratio (not given in the text, assumed 100 %), with the probability of escaping the body, with the geometric efficiency of the gamma camera and finally with the detection probability of the detector (photofraction should be set to 100 % according to text). Tc-99m de-excites to Tc-99 by the emission of a 141 keV photon. Table 2 includes linear att. values at this energy.

$$\begin{aligned} n &= A\cdot e^{-\mu_{soft}D}\cdot g\cdot(1-e^{-\mu_{NaI}l}) && (11) \\ &= 300\cdot 10^6 s^{-1} e^{-0.16\cdot 10}\cdot 2.48\cdot 10^{-4}\cdot(1-e^{-2.5\cdot 4}) && (12) \\ &= 1.5\cdot 10^4 s^{-1} && (13) \end{aligned}$$

where D is the thickness of soft tissue and l is the thickness of the NaI scintillator.

2.2 Problem 2b

The scatter contribution can be estimated by adding a narrow energy window of width W_s at each side of the photo-peak window (width W_m. The scatter count C_{scat} is then the trapezoid area given by the expression:

$$C_{scat} = (\frac{C_{left}}{W_s} + \frac{C_{right}}{W_s}) * \frac{W_m}{2} \quad (14)$$

2.3 Problem 2c

Attenuation correction in PET is based on the fact that for each detected coincidence event the whole projection line (or Line of Response LOR) between two detector elements are traversed by one of the two emitted 511 keV photons. This means that the count N for a given LOR is reduced by the total attenuation along the LOR:

$$N_{LOR} = N_{0,LOR}\cdot exp(-\int \mu dl) \quad (15)$$

where N_0 is the nominal count without attenuation. Given $im(x,y)$ and $mu(x,y)$ we can write the following expression for attenuation correction:

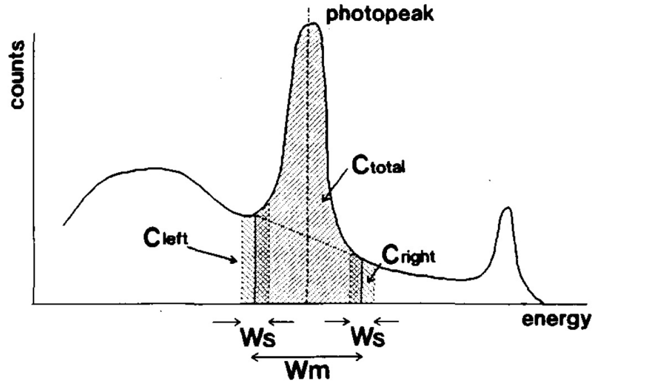

Fig. 2. Location and width of the windows. The main window is located at the photopeak and the subwindows are located on both ends of the main window. The counts of primary photons in the main window are estimated as a trapezoid using photons of the two subwindows.

Figure 2: Illustration of one common method for scatter correction.

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mu_int = radon(mu,pixelsize); %Calculates the value of the integral of mu along each LOR.
% Pixelsize is included to get correct integration step length (units of cm).
N = radon(im,pixelsize); % Calculates N along each LOR.
N_0 = N.*exp(mu_int); % Multiplication of N and exp(mu_int) for each LOR.
im_ac = iradon(N_0); % Inverse radon transform to go from LOR-space back to image space.
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3 Problem 3

3.1 Problem 3a

The signal in the spin-echo sequence is given by:

$$S = S_0(1 - e^{-TR/T1})e^{-TE/T2} \tag{16}$$

This expression can be found on page 2, and does not need to be derived. The tast is to calculate the percentage signal change $\Delta S/S$ when T2 changes from 60 to 70 ms:

$$\begin{aligned}
\Delta S/S[\%] &= 100 \cdot \frac{S2 - S1}{S1} && (17)\\
&= 100 \cdot \frac{S_0(1 - e^{-TR/T1})e^{-70/70} - S_0(1 - e^{-TR/T1})e^{-70/60}}{S_0(1 - e^{-TR/T1})e^{-70/60}} && (18)\\
&= 100 \cdot \frac{e^{-70/70} - e^{-70/60}}{e^{-70/60}} && (19)\\
&= 18.1\% && (20)
\end{aligned}$$

In order to obtain a T1-weighted image, the effect on the signal from any variation in T2 should be minimized. This is obtained by minimizing the echo-time TE.

3.2 Problem 3b

Time-of-flight MRI uses the spoiled gradient-echo sequence with short repetition time TR and a flip angle θ below 90 degrees. In this situation the longitudinal magnetization M_z will decrease gradually for each applied rf-pulse towards its steady-state value, because there is not much time between each pulse for T1-relaxation to occur and regrowth of M_z. This means that the steady-state signal, given by $M_{z,ss}sin(\theta)$ is low compared to the signal during the first few rf-pulses. In TOF MRI the tissue is in the steady-state, while the moving spins in the blood have only experiences a few rf-pulses and therefore exhibit a higher signal and appear bright in the images.

3.3 Problem 3c

The extended phase graph method is used to analyse MR-sequences in which the assumption $TR >> T2$ no longer holds. An illustrative example is the three pulse experiment which gives five different echoes in total. Here only the twice-refocused spin-echo pathway and the stimulated echo pathway is shown.

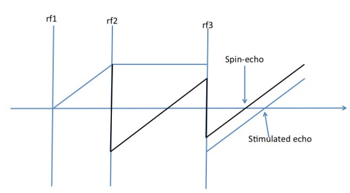

Figure 3: Phasegraph showing the twice refocused spin-echo and the stimulated echo.

4 Problem 4

4.1 Problem 4a

To solve this problem one needs to consider the difference in wave velocity c between the maximum and minimum particle velocity $v(x,t)$. Starting from the the expression for a harmonic displacement with amplitude a: $u(x,t) =$

$a \sin(kx - \omega t)$, the particle velocity is given by the time derivative of $u(x,t)$:

$$v(x,t) = \frac{\partial u}{\partial t} = -a\omega \sin(kx - \omega t) \tag{21}$$

The wave-velocity depends linearly on the particle velocity: $c = c_0 + \beta v$ where $\beta = 1 + 2B/A$ is a constant larger than 1. The net difference between the maximum and minimum wave velocity is then:

$$\Delta c_{max} = 2a\omega\beta \tag{22}$$

The time it takes for the fastest part of the wave to catch up with the slowest part determines the time required to reach the saw-tooth shape τ, and is equal to the time required for Δc_{max} to travel a distance equal to $\lambda/2$:

$$\tau = \frac{\lambda}{2\Delta c_{max}} = \frac{\lambda}{4a\omega\beta} \tag{23}$$

Finally, the distance l travelled by the wave during τ is our answer:

$$l = c_0\tau = \frac{c_0\lambda}{4a\omega\beta} \tag{24}$$

4.2 Problem 4b

The Doppler-shift f_d is given by the equation (given on page 2):

$$f_d = \frac{\pm 2 f_0 v \cos\theta}{c} \tag{25}$$

Using the information given in the text we find:

$$f_d = \frac{2 \cdot 2 \cdot 10^6 s^{-1} 2m/s \cos 30^\circ}{1540 m/s} = 4.50 kHz \tag{26}$$

Transit time broadening is the uncertainty (or bandwidth) in f_d due to the finite time a given scatterer is in the ultrasound beam. The lateral width of the beam is given by $1.2\lambda n_f$. The time τ required to pass through the beam is then given by:

$$\tau = \frac{1.2\lambda n_f}{v \sin\theta} \tag{27}$$

The Doppler-shift bandwidth is the reciprocal of this:

$$\Delta f_d = \frac{v \sin\theta}{1.2\lambda n_f} = \frac{2m/s \sin 30}{1.2 \cdot 0.77 \cdot 10^{-3} m \cdot 10} = 108 Hz \tag{28}$$

where $\lambda = c/f = 0.77mm$. Typo in the exam, it says "axial" instead of "lateral". This could be confusing.