Problem 1: X-ray mammography

1 a

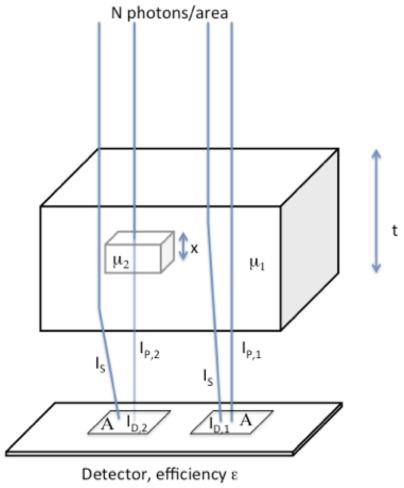

Figure 1: The parameters of the target visibility calculation.

In order to solve this problem correctly, the following issues must be answered:

- State/calculate the correct expressions for S_1 and S_2, including the assumption of equal scattering signal in both.
- Assume the noise (STD) is similar in both and can be estimated from S_1.
- Poisson noise given by the variance in arrival of photons.
- State correct interpretation of scattering ratio R and $\Delta\mu$.

Starting from the definition of the visibility k given on the sheet of expres-

sions:

$$\begin{aligned}
k &= \frac{S_1 - S_2}{STD_S} \\
S_1 &= \varepsilon A(I_{P,1} + I_S) = \varepsilon A(N_0 E e^{-\mu_1 t} + I_S) \\
S_2 &= \varepsilon A(I_{P,2} + I_S) = \varepsilon A(N_0 E e^{-\mu_1 (t-x)} e^{-\mu_2 x} + I_S) \\
STD_S &= E\sqrt{N_1} = E\sqrt{\varepsilon A(N_0 e^{-\mu_1 t} + N_S)} \\
k &= \frac{\varepsilon A N_0 E e^{-\mu_1 t}(1 - e^{\Delta\mu x})}{E\sqrt{\varepsilon A N_0 e^{-\mu_1 t}(1+R)}} = (1 - e^{\Delta\mu x})\sqrt{\frac{\varepsilon A N_0 e^{-\mu_1 t}}{1+R}} \\
\Delta\mu &= \mu_1 - \mu_2 \\
R &= \frac{I_S}{I_P} \approx \frac{N_S}{N_P}
\end{aligned}$$

1 b

First we solve the expression in 1a for N_0:

$$N_0 = \frac{k^2(1+R)e^{\mu_1 t}}{\varepsilon x^4 \Delta\mu^2}$$

where we have assumed the target to be a cube ($A = x^2$) and we have assumed $\Delta\mu x << 1$.

There is one complication in this problem: The detector efficiency is not given directly as part of the problem, it has to be calculated from the attenuation coefficient and thickness of the amorphous Selenium detector:

$$\varepsilon = 1 - e^{-\mu_{Se} t_{Se}} = 1 - e^{216 cm^{-1} \cdot 0.01 cm} = 0.88$$

The resulting value for N_0 becomes:

$$\begin{aligned}
N_0 &= \frac{5^2(1+0.5)e^{0.87 cm^{-1} \cdot 2cm}}{0.88 \cdot (0.01cm)^4 (0.87 cm^{-1} - 9.29 cm^{-1})^2} \\
&= 3.42 \cdot 10^8 cm^{-2}
\end{aligned}$$

From the expression sheet we have:

$$D = EN_0(\frac{\mu_{en}}{\rho})$$

The final part of the calculation is mostly about getting units correct:

$$\begin{aligned}
D &= EN_0(\frac{\mu_{en}}{\rho}) = 20keV \cdot 1.6 \cdot 10^{-16} J/keV \cdot 3.42 \cdot 10^8 cm^{-2} \cdot 0.56 cm^2/g \cdot 10^3 g/kg \\
&= 6.1 \cdot 10^{-4} J/kg = 0.61 mGy
\end{aligned}$$

Note that this is an approximate value, and number of significant digits used vary somewhat along the calculation. Also it makes sense to double check if the approximation $\Delta\mu x << 1$ holds: $(0.87 cm^{-1} - 9.29 cm^{-1}) \cdot 0.01 cm = -0.084$, which is borderline OK.

1 c

There are several possible ways to produce a suitable X-ray spectrum. The key point in this question is the term "suitable", where the student should identify that a suitable spectrum for the above situation is a spectrum with its peak intensity at around 20 keV.

Example answer: Molybdenum anode, has characteristic X-rays at 17-20 keV. Suggested acceleration voltage should be 25-35 keV in order to have sufficient intensity at 20 keV. A Molybdenum filter will attenuate both the hard end (above 20 keV) and the soft end (below 15 keV), and with proper filter thickness the end result will look like the spectrum shown on the right in figure 2 (from lecture notes). Note that the shown specturm is not ideal, since it has its peak energy below 20 keV.

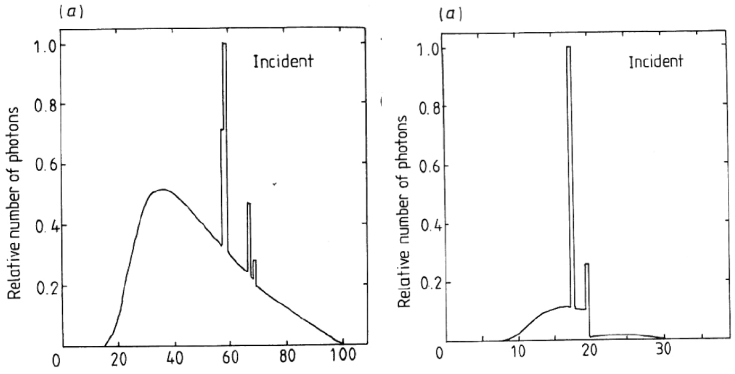

Figure 2: Example tube X-ray spectra in Chest radiography (left) and mammography (right). The final spectra is a function of anode material, acceleration voltage and filtering.

As an alternative a combination of Tungsten anode and Rhodium filter could also be used (not discussed in the lectures), or Rhodium anode and Rhodium filter.

2 a

The expression to discuss:

$$\#counts \propto \int_0^{t_S} A_0 e^{-\lambda t} dt \cdot Y_\gamma (1 - \bar{\phi_\gamma}) \cdot g \cdot \varepsilon_D(E_\gamma) \tag{1}$$

Explanation for each term:

- $A_0 e^{-\lambda t}$: A_0 is the injected activity at time t=0. $e^{-\lambda t}$ is to take into account the decay in the activity given by the decay rate λ.
- $\int_0^{t_S} dt$: Integration over the acquisition time t_S.

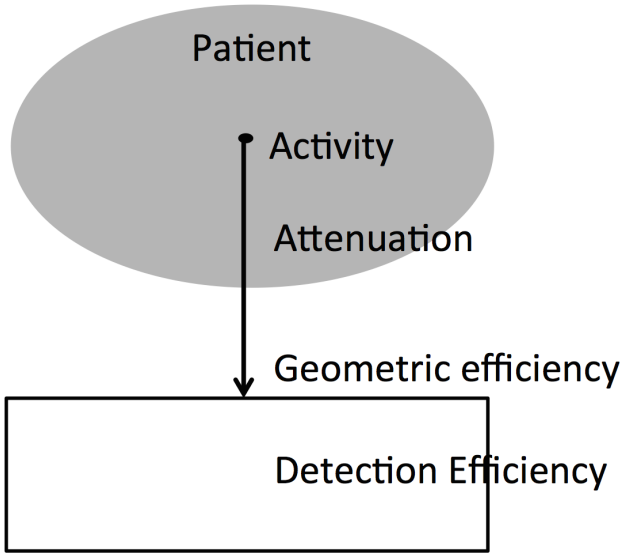

Figure 3: Very basic illustration of process in single gamma emission imaging.

- Y_γ: The yield of the decay branch that results in the emission of single gamma-photon, will be a number between 0 and 1.
- $\bar{\phi_\gamma}$: The average attenuated fraction of gamma-photons as result of propagation through the patient tissue. $(1 - \bar{\phi_\gamma})$ is then the average fraction of photons NOT attenuated due to patient tissue (number between 0 and 1).
- g: The geometrical efficiency of the detector, this is also a fraction (number between 0 and 1). For each emitted photon, what is the probability of reaching the detector surface (not taking into account attenuation through tissue). This factor will depend strongly on the choice of collimator and is usually in the order of 10^{-4} for gamma cameras.
- ε_D: The detector efficiency, or probability of detection. Will depend on the attenuation coefficient and thickness of the scintillator material. Usually we can approximate this by the expression $1 - e^{\mu_D \cdot x_D}$, by assuming 100% photo fraction (no scattering in detector). This approximation will be an upper bound on the actual detection efficiency.

In order to go from counts to activity all the factors in the above expression must be correctly accounted for. Some are easily available, like the yield Y_γ and geometric efficiency g, while some are quite difficult to access. In particular the attenuation factor $\bar{\phi_\gamma}$ is unique for each patient and position inside the patient, and so it must be measured/calculated individually. In modern SPECT-CT

systems it can be estimated from CT-images using for example the Chang-method.

One additional effect not taken into account by the above model is scattering. Scattered photons will give a higher count than predicted by the model and this has to be subtracted. Several methods have been proposed in order to achieve this (not required to go into detail here).

2 b

This problem consists of four questions, equal weight on each. Estimation of the geometric efficiency of the PET ring: Cylinder with diameter $D = 80cm$ and height $h = 4mm \cdot 9 = 36mm$:

$$g \approx \frac{\pi Dh}{\pi D^2} = \frac{h}{D} = \frac{3.6cm}{80cm} = 0.045 = 4.5\%$$

The spatial resolution in PET is the combined effect of three main mechanisms: Crystal size, non-colinearity and positron range. The first two depends on the PET-system, the last on the type of decay process. The best possible spatial resolution is found by assuming the positron range is zero.

- Effect from crystal size: The spatial resolution is limited by the crystal size. As a rule of thumb the FWHM from this effect is given by $d/2 = 2.0mm$ in the center of the ring, while it increases towards d at the outer part towards the ring itself.
- Effect of non-colinearity: The non-colinearity effect is an angular mechanism, so the spatial resolution will depend on the ring diameter: $FWHM = 0.0022rad \cdot D = 1.8mm$.
- Maximum spatial resolution (or minimum FWHM) = $\sqrt{2.0^2 + 1.8^2}mm = 2.7mm$.

The coincidence detection probability is given by the product of the detection probabilities for each photon:

$$\varepsilon_D = (1 - e^{-\mu_D \cdot x_D})^2 = (1 - e^{-0.88cm^{-1} \cdot 2.5cm})^2 = 0.79 = 79\%$$

In the above calculation we have assumed that all attenuated photons give rise to a count, or in other words we ignore the effect of scattering in the detector. The true coincidence detection probability will be lower, since a significant fraction of the interactions between incoming photons and the scintillator will be partial absorption via the Compton scattering effect (not in the photo peak).

Finally, if the point source is located inside a water cylinder with $D_W = 20cm$:

$$e^{\mu_W \cdot D_W} \cdot \varepsilon_D = e^{0.097cm^{-1}20cm}(1 - e^{-0.88cm^{-1} \cdot 2.5cm})^2 = 0.11 = 11\%$$

Each photon will only experience $e^{\mu_W \cdot D_W/2}$, but the total attenuation effect will be the product of the two individual attenuation factors.

3a

The physical interaction process described by the equation $\vec{N} = \vec{\mu} \times \vec{B}$ is electromagnetic in origin: A magnetic dipole moment $\vec{\mu}$ placed in an external magnetic field $\vec{B}$ will experience a torque N around the axis defined by the cross-product of $\vec{\mu}$ and $\vec{B}$. Classically the equation can be derived from analyzing the Lorentz forces on a current loop immersed in a magnetic field.

The second equation comes from mechanics and states that a torque N drives a change in the angular momentum J. It is the angular analogue to Newtons 2.law where a force drives a change in the linear momentum: $\vec{F} = d\vec{p}/dt = m\vec{a}$.

A nucleus with non-zero quantized spin J has an associated magnetic dipole moment $\vec{\mu} = \gamma J$. The proportionality constant γ is called the gyromagnetic ratio and is unique for each nuclide.

By combining the above we can derive the classical equation of motion for a magnetic dipole moment in an external field:

$$\begin{aligned} \vec{N} &= \vec{\mu} \times \vec{B} \\ \frac{d\vec{J}}{dt} &= \vec{\mu} \times \vec{B} \\ \frac{d\vec{\mu}}{dt} &= \gamma \cdot \vec{\mu} \times \vec{B} \end{aligned}$$

The solution for this equation is a precessional motion of $\vec{\mu}$ around $\vec{B}$. This result from classical derivation also holds for the behaviour of the nuclear magnetic moment in MRI, since the expectation value for a large number of measurements of the spin state is given by the classical result.

3b

- Duration of scan: $T_{acq} = TR \cdot N_{PE} = 1200ms \cdot 128 = 153.6sec = 2min33.6sec$
- Spatial Resolution, in-plane: Pixel size: $FOV/N_{RO/PE} = 156mm/128 = 2mm$, nominal spatial resolution according to Nyquist criteria is then 4 mm.
- Spatial Resolution, through-plane: Slice thickness is set to 5 mm, nominal spatial resolution according to Nyquist criteria is then 10 mm.
- Amplitude of slice-select gradient: We first need to find/derive an expression. From basic Larmor-frequency relationship we know that $f = \gamma B/2\pi$, where $\gamma/2\pi = 42.58MHz/T$. It is straight forward to apply the same re-

lation on a frequency bandwidth and a field gradient:

$$\begin{aligned} \Delta f &= \frac{\gamma}{2\pi}\Delta B = \frac{\gamma}{2\pi}\Delta x \cdot G_x \\ G_x &= \frac{2\pi\Delta f}{\gamma\Delta x} \\ &= \frac{2kHz}{42.58MHz/T \cdot 5mm} = 9.4mT/m \end{aligned}$$

where G_x is the magnetic field gradient, Δx is the slice thickness and Δf is the rf-pulse bandwidth.

- The echo-time TE is set to 20 ms. From this we can estimate the total time required to excite and sample one slice, and from that the total number of slices that fit within TR = 1200 ms. The highest number of slices to fit is if the read-out ends at TE, this gives 20 ms per slice and 60 slices within 1200 ms. A more realistic estimate would be a slice-time of around 30 ms, which gives 40 slices within TR.

3c

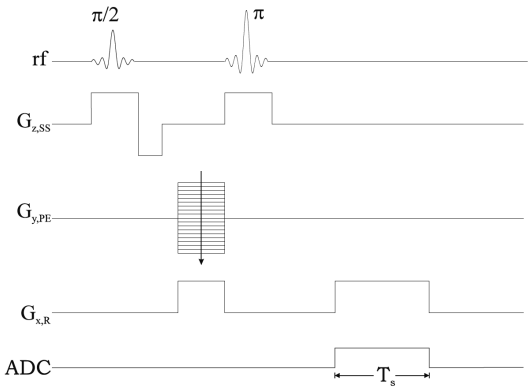

Fig. 10.17: Sequence diagram for a 2D spin echo imaging sequence. The arrow pointing downwards in the gradient table, indicating that the phase encoding gradient is stepped sequentially from its most positive value to its most negative value, has the same effect as an upward arrow in a sequence without a π-pulse. There is no rephasing gradient required for the π-pulse as a symmetric slice select gradient is self refocusing.

Figure 4: Partial solution for problem 3c. In addition the temporal development of longitudinal and transverse magnetization M should be added.

4a

Choice of frequency in normal pulse-echo ultrasound is determined by the compromise between the following two effects:

- Attenuation. The attenuation is strongly frequency dependent, meaning that for higher frequencies the pulse amplitude will decrease more strongly as function of tissue depth.
- Spatial resolution. Higher frequency provides better spatial resolution.

In effect, the frequency is chosen based on how deep into the tissue one needs to see. For shallow depths, high frequency is chosen, for larger depths the frequency must be lower.

The key point in terms of image contrast is "echo", which again requires reflection or scattering of the ultrasound wave. This happens whenever there are changes in the acoustic impedance along the path of the ultrasound pulse. Depending on the length-scale of the objects, we can define different categories of echoes:

- Specular/geometric reflection: On the interface between tissue types. Depending on the smoothness of the interface, the distribution of reflection/scattering angles varies.
- Stochastic scattering: When objects are of similar size as the wavelength. Typically mesoscopic scattering centers inside tissue, give rise to structure/texture in different types of tissue.
- Rayleigh scattering: For small scattering objects, like individual cells. Dominating scattering effect in blood.

4b

- The origin of the noise as seen on the left image is multiple reflections on various interfaces close to the transducer. These echoes will be imaged further down in the tissue based on the total path travelled.
- It is possible to reduce this noise due to non-linear wave propagation, which over time will modify the wave so that it will contain multiples of the base frequency. This will happen at some depth into the tissue, so that the high-frequency component of the wave does not include the multiple reflections close to the transducer, where only the base-frequency existed.
- The post-processing is a high-pass filter to remove the base-frequency part of the signal.