Problem 1: X-ray and CT Imaging

1a

Which detector material: For a fixed detector thickness the material with the highest linear attenuation coefficient will have the best detector efficiency and should therefore be chosen. Figure 1 shows the mass attenuation coefficients for CsI and Se, and since the density for both materials are the same, the values can be directly compared. At 50 keV the values are 12.9 cm^2/g and 3.9 cm^2/g respectively, so CsI should be chosen as material. (Some variation in the extracted values is accepted, but the student should be able to identify CsI as the best material).

For CsI there are two possibilities: structured and unstructured. Structured scintillators are engineers so that the visible light produced in the scintillator does not spread isotropically inside the material, but is guided towards the photodetector. This will increase the spatial resolution for a given scintillator thickness. Therefore we should choose the structured CsI.

Required thickness x to achieve 99 % detector efficiency:

$$\begin{aligned} 1 - e^{-\mu x} &= 0.99 \\ x &= -\frac{ln(0.01)}{\mu} = \frac{4.61}{4.5g/cm^2 12.9cm^2/g} = 0.8mm \end{aligned}$$

Image SNR: Number of incoming X-ray photons n per pixel:

$$n = \frac{8000MeV/cm^2}{50keV 0.02^2 cm^2} = 64$$

With 99 % detector efficiency, the expected value of detected X-ray photons will be $n_{det} = 63.36$. With this low number of photons per pixel, the noise will be dominated by the stochastic arrival of X-ray photons, which obeys the Poisson distribution which in this case can be approximated by a normal distribution with identical mean and variance:

$$SNR = \frac{Mean}{STD} = \frac{n_{det}}{\sqrt{n_{det}}} = \sqrt{n_{det}} = 7.96 \approx 8$$

1b

The starting point for this task should be the collected set of projections in a sinogram.

1D filtering in image domain:

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Create ramp filter:
1. Define a linear function s = |k|.
2. Calculate the inverse Fourier transform

For each measured projection:
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   calculate the convolution between projection and ramp filter
   perform backprojection of the resulting filtered projection
end
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2D filtering in Fourier domain:

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For each measured projection:
   perform backprojection
end

Calculate the 2D Fourier transform of resulting image.

Create ramp filter:
-Define a 2D cone function.

Multiply the ramp filter and the 2D Fourier transform of the image.

Calculate inverse Fourier transform.
end
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Problem 2: Nuclear Medicine Imaging

2a

See figure for qualitative shape of spectra.

Starting from mono energetic gamma photons at E_γ:

- Propagation through soft tissue will give rise to some full absorption and some Compton scattering. The energy of the scattered photon depends on the scattering angle starting from E_γ for forward scattered photons and continually decreasing down to a minimum for backward scattering (180 degrees).
- Energy absorption in scintillator will consist of the same two processes; some full absorption and some Compton scattering. Only difference is that now the relevant energy is E_γ- E'_γ which starts from 0 for forward scattered photons and continually increasing up to a maximum for backward scattering (180 degrees).
- The measured spectrum will be a smoothed version of the absorbed spectrum due to the Poisson noise caused by the finite number of visible light photons. Typical smoothing is up to 10 %.

2b

The spatial resolution in a gamma camera with a parallel hole collimator is given by (can be found on equation sheet):

$$R_c = \frac{d(L+z)}{L}$$

where L is the hole length and d is the hole diameter, z is the distance to the object. Setting z to 10 cm, we get for the two available collimators:

$$R_{c,1} = \frac{3.4mm(36mm + 100mm}{36mm} = 12.8mm$$
$$R_{c,2} = \frac{1.8mm(40mm + 100mm}{40mm} = 6.3mm$$

Collimator 2 has the best spatial resolution.

The disadvantage of the chosen collimator is the lower geometric efficiency. This can be shown by calculating g (not shown).

2c

The original intention of this question was to explain the design of the block-detectors used in PET, where an array of for example 9x9 pencil shaped crystals are put together in a matrix and varying degree of reflective material between each crystal helps to guide the visible light so that a flood field map is created to facilitate localization of each detected high energy photon to a single crystal. However, all students understood the question to be about general PET detection, which is fine as long as there is also a description of the geometry of crystals and PMTs.

Problem 3: Magnetic Resonance Imaging

Anders: Guess you do not need solution set here.

Problem 4: Ultrasound Imaging

4a

Example derivation can be found in Webbs, p 355-356.

4b

Speckle is caused by varying degree of constructive and destructive interference between the reflected waves from different scattering objects closer together than the resolution volume of the ultrasound beam. This gives rise to a texture pattern that follows the tissue.