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Contact during the exam:
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EXAM
TFY4340 MESOSCOPIC PHYSICS
Wednesday May 12 2010, 0900 - 1300
English

Remedies: C

- K. Rottmann: Mathematical formulae
- Approved calculator with empty memory (Citizen SR-270X, HP30S, or similar).

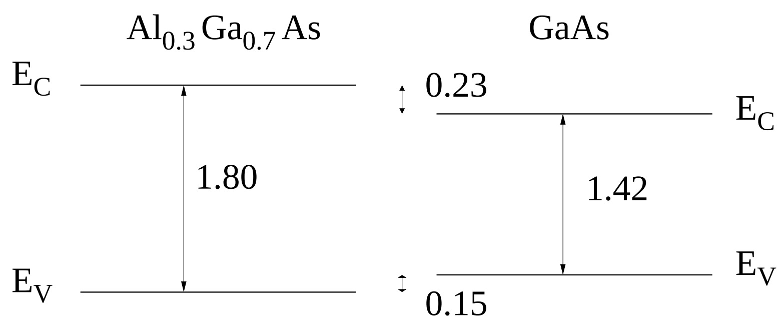
Pages 2 – 6: Questions 1 – 3. The three questions are relatively unrelated and may be answered in any order. Also, many of the subquestions within a given question may be answered independently from the others.

Notation: Vectors are given in ***bold italic***. Unit vectors are given with a hat above the symbol.

Some constants:
Electron mass: $m = 9.1 \cdot 10^{-31}$ kg. Elementary charge: $e = 1.6 \cdot 10^{-19}$ C.
Boltzmann constant: $k_B = 1.38 \cdot 10^{-23}$ J/K. Planck constant: $\hbar = h/2\pi = 1.05 \cdot 10^{-34}$ Js.

The grades will be available no later than May 28.

QUESTION 1



The figure above shows the energy band characteristics at the Γ–point ($\boldsymbol{k} = 0$) in the two semiconductors $Al_{0.3}Ga_{0.7}As$ and GaAs, with bandgaps and conduction and valence band offsets in the unit eV.

An interface is formed between (uniformly) n–doped $Al_{0.3}Ga_{0.7}As$ and undoped ("intrinsic") GaAs. We assume that the Fermi level (or: chemical potential) μ in bulk n–$Al_{0.3}Ga_{0.7}As$ lies 0.13 eV below the conduction band edge E_C.

a) Make a sketch of the resulting energy band diagram near the interface when equilibrium has been established. Explain briefly the difference between your figure and the figure given above.

b) Concerning the properties of the two–dimensional electron gas (2DEG) that is now formed in GaAs next to the interface, what would be the benefit of introducing an undoped layer of $Al_{0.3}Ga_{0.7}As$ between n–$Al_{0.3}Ga_{0.7}As$ and GaAs?

Let $z = 0$ denote the location of the interface. To a first approximation, the conduction band edge may, in the vicinity of the interface, be described by the potential $V(z) = Fz$ for $z > 0$ and $V(z) = \infty$ for $z < 0$. Here, F is a constant. In the xy–plane, the electrons move essentially as free particles, with kinetic energy $\hbar^2 k^2/2m^*$, and effective mass $m^* = m^*(\text{GaAs}) = 0.067m$ due to the periodic potential felt by the electrons. The total energy of the two–dimensional (2D) subbands is therefore

$$E_n(\boldsymbol{k}) = E_n + \frac{\hbar^2 k^2}{2m^*} \quad , \quad n = 1, 2, 3, \ldots,$$

with

$$\boldsymbol{k} = k_x \hat{x} + k_y \hat{y}.$$

Here, zero energy is chosen at the conduction band edge in GaAs when $z \to 0$, i.e., at the bottom of the triangular potential well. The corresponding wave functions are

$$\Psi_{n\boldsymbol{k}}(\boldsymbol{r}) = \Phi_n(z) u_{\boldsymbol{k}}(x, y) e^{i(k_x x + k_y y)},$$

with the (x, y)–dependent part on Bloch form. The Schrödinger equation is now separable.

c) Show that the resulting equation for Φ_n may be written in dimensionless form,

$$\frac{d^2\Phi_n}{d\xi^2} - \xi\Phi_n = -\tilde{E}\Phi_n,$$

with $\xi = \kappa z$, $\tilde{E} = E\kappa/F$, and $\kappa = (2m^* F/\hbar^2)^{1/3}$.

d) The (dimensionless) eigenvalues of this equation are approximately $\tilde{E}_1 = 2.34$, $\tilde{E}_2 = 4.09$, $\tilde{E}_3 = 5.52, \ldots$. Show that with $F = 10$ meV/nm, and a Fermi level $\mu = 0.10$ eV, only the lowest 2D subband, $E_1(\boldsymbol{k})$, will be occupied by electrons.

The density of states (DOS), i.e., the number of available quantum states pr unit energy, $D_2(E)$, is independent of the energy E in a 2DEG. Let us prove this statement.

e) Assume that your sample is of size $L \times L$ in the xy–plane. Then, taking into account the spin degeneracy of two and the absence of valley degeneracy (i.e., $g_S = 2$, $g_V = 1$) near the Γ–point, argue that the density of states in (the 2D) $\boldsymbol{k}$–space is

$$D_2(\boldsymbol{k}) = \frac{L^2}{2\pi^2}.$$

f) Further, argue that the number of states $N_2(k)$ with wave number less than k (in absolute value) is then

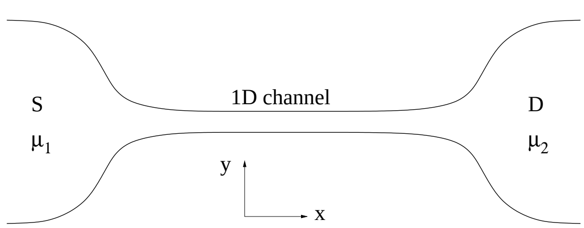
$$N_2(k) = \frac{(kL)^2}{2\pi}.$$

g) Use this result to write down $N_2(E)$ in "energy space", assuming $E = 0$ at the bottom of the 2D subband. Finally, show that

$$D_2(E) = \frac{m^* L^2}{\pi\hbar^2},$$

a constant, as advertised.

Next, we will consider electron transport through a 1D channel connecting a 2D source (S) at chemical potential (Fermi level) μ_1 and a 2D drain (D) at chemical potential μ_2 ($\mu_2 < \mu_1$):



The electric current from S to D, due to transverse subband j in the 1D channel, is

$$I_j^+ = (-e)\int_{E_j^t}^{\mu_1} dE\, \rho_j^+(E)\, v_j(E)\, T_j(E).$$

Here, $\rho_j^+(E)$ is the 1D DOS pr unit length for states with positive group velocity $v_j(E)$ along the 1D channel, and $T_j(E)$ is the probability of being transmitted through the 1D channel in subband j with energy E. We assume low temperatures.

h) Verify that this expression for I_j^+ has the correct unit (A).

i) From this expression for I_j^+, and an analogous expression for the current from D to S, I_j^-, derive the Landauer formula for the total conductance $G = I/V$ of the 1D channel,

$$G = \frac{2e^2}{h} \sum_j T_j(E_F).$$

Here, I is the total net current when a voltage V is applied between S and D, and we are assuming linear response, i.e., $\mu_1 \simeq \mu_2 \simeq E_F$. The 1D DOS is $D_1(E) = \sqrt{2m^*}L/\pi\hbar\sqrt{E}$ for a system of length L.

The figure below is copied from the paper *Quantized Conductance of Point Contacts in a Two–Dimensional Electron Gas*, by B. J. van Wees et al (*Phys Rev Lett* **60**, 848 (1988)):

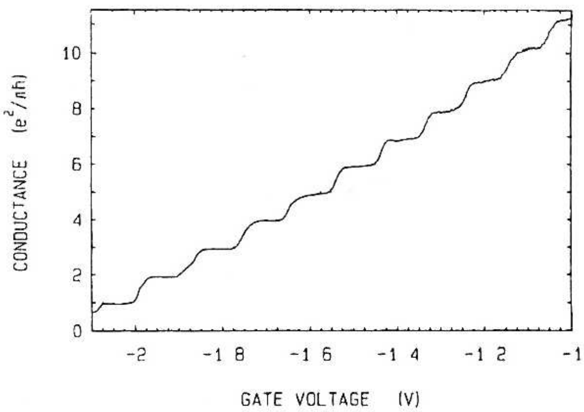


FIG. 2. Point-contact conductance as a function of gate voltage, obtained from the data of Fig. 1 after subtraction of the lead resistance. The conductance shows plateaus at multiples of $e^2/\pi\hbar$.

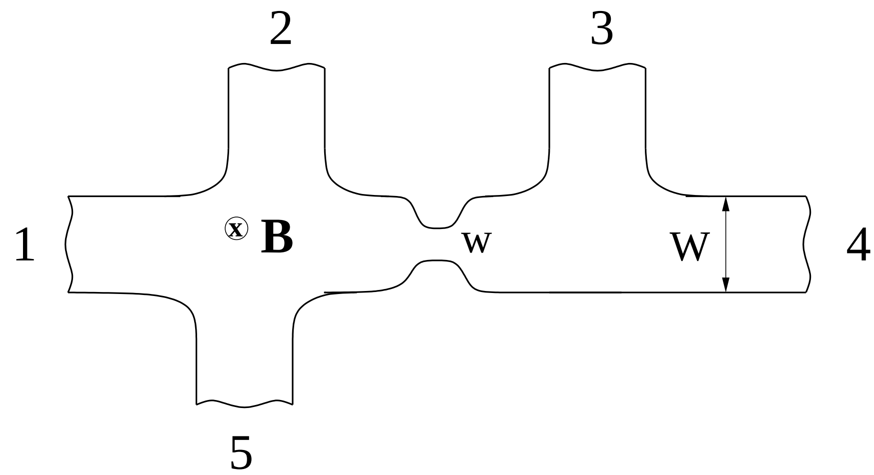
The figure shows the measured conductance of a narrow 1D channel (actually, a so–called "point contact"), where the width W of the channel is controlled by the voltage V_G on a split–gate electrode. The 2DEG "lives" in GaAs, so the effective electron mass is $m^* = 0.067m$, and the 2D density of electrons is $n_2 = 3.56\cdot 10^{15}\ \text{m}^{-2}$. Only the lowest 2D subband is occupied by electrons.

j) Use this information to calculate the Fermi level E_F in the 2DEG.
(Hint: Use the result of 1**g)**.)

k) Assume that the confining potential that defines the 1D channel is a potential box of width W and with hard walls (i.e. $V = \infty$ outside the 1D channel). With these assumptions, what is the width W of the 1D channel when the gate voltage is $V_G = -1.5$ V?

QUESTION 2

Consider the 5–terminal device shown in the figure below, with ideal contacts, and with a geometrical constriction of width w defined by a split–gate technique. We ignore tunneling and impurity scattering.



A uniform perpendicular magnetic field $\boldsymbol{B}$ is pointing into the plane and causes the formation of N edge states (at each edge) in the wide regions of the device (width W). Inside the constriction, only n edge states (at each edge) are present at the Fermi level. Hence, we assume that n edge states are transmitted through the constriction with probability equal to one, whereas the remaining $N - n$ edge states do not enter the constriction. The Büttiker–Landauer equations,

$$I_\alpha = \sum_{\beta \neq \alpha} G_{\alpha\beta} \left(V_\alpha - V_\beta\right),$$

with conductances

$$G_{\alpha\beta} = \frac{2e^2}{h} T_{\alpha\beta},$$

relate the current in terminal α to the potentials at the various terminals. Here, $T_{\alpha\beta}$ denotes the "transmission sum" from terminal β to terminal α. A small voltage is applied between terminals 1 and 4, resulting in a net current I flowing from terminal 1 to terminal 4 (i.e., $I_1 = -I_4 = I$). Terminals 2, 3, and 5 are used as voltage probes.

a) Write down the dimensionless 5×5 conductance matrix (or "transmission matrix") with matrix elements $T_{\alpha\beta} = hG_{\alpha\beta}/2e^2$.

b) Find the resistances $R_{14,23}$, $R_{14,25}$, $R_{14,35}$, and $R_{14,14}$. (Notation: $R_{\alpha\beta,\kappa\eta} = (V_\kappa - V_\eta)/I$, $I = I_\alpha = -I_\beta$, $I_\kappa = I_\eta = 0$.) For convenience, choose $V_4 = 0$.

QUESTION 3

The figure below is copied from the paper *Weak Localization in Bilayer Graphene*, by R. B. Gorbachev et al (*Phys Rev Lett* **98**, 176805 (2007)):

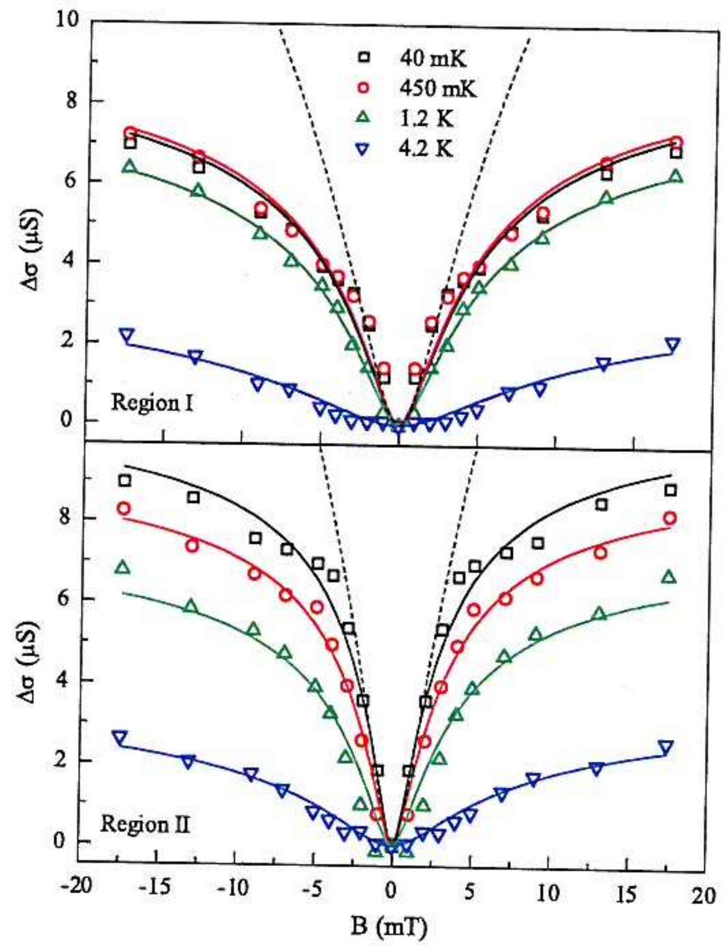


FIG. 3 (color online). Averaged magnetoconductivity for regions I and II. Dashed curves are the fits using only the first term in Eq. (1), and solid lines are the fits with the first two terms.

a) Describe briefly, in a sentence or two, the physics behind "weak localization".

b) Why does the conductivity increase when a weak magnetic field is applied to the sample?

c) Why does the effect vanish with increasing temperature (as shown in the figure)?