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EXAM
TFY4340 MESOSCOPIC PHYSICS
Wednesday May 18 2011, 0900 - 1300
English

Remedies: C

- K. Rottmann: Mathematical formulae
- Approved calculator with empty memory (Citizen SR-270X, HP30S, or similar).

Pages 2 – 6: Questions 1 – 4. The four questions are relatively unrelated and may be answered in any order. Also, many of the subquestions within a given question may be answered independently from the others.

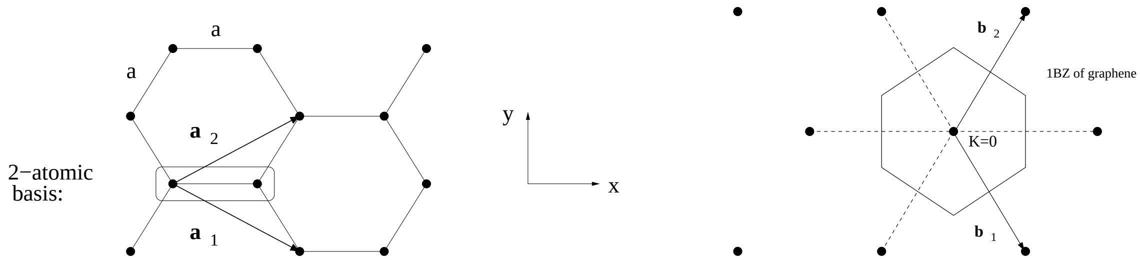
Notation: Vectors are given in bold italic. Unit vectors are given with a hat above the symbol.

Some constants:
Electron mass: $m_e = 9.1 \cdot 10^{-31}$ kg. Elementary charge: $e = 1.6 \cdot 10^{-19}$ C.
Boltzmann constant: $k_B = 1.38 \cdot 10^{-23}$ J/K. Planck constant: $\hbar = h/2\pi = 1.05 \cdot 10^{-34}$ Js.

The grades will be available around May 27.

QUESTION 1

Graphene is a single 2D layer of graphite, i.e., a honeycomb lattice of carbon atoms with nearest neighbour distance $a \simeq 1.4$ Å. The primitive cell consists of two C atoms. The reciprocal lattice is triangular, hence, the first Brillouin zone (1BZ) is hexagonal.



Primitive vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$, primitive vectors $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ in reciprocal space (with $|\boldsymbol{b}_1| = |\boldsymbol{b}_2| = 4\pi/3a$), as well as the hexagonal 1BZ are shown in the figure.

Within the LCAO approximation (”Linear Combination of Atomic Orbitals”), the $2s$, $2p_x$, and $2p_y$ orbitals of the two C atoms are responsible for the strong C-C bonds, in terms of hybridized sp^2 states and corresponding energy bands. For electronic transport at low temperatures, however, the energy bands of interest arise from the coupling between $2p_z$ orbitals on neighbouring C atoms. Within the nearest neighbour tight binding approximation, the resulting valence band (bonding π band) and conduction band (anti-bonding π^* band) are described by the dispersion relation

$$E^{\pm}(\boldsymbol{k}) = E_0 \pm |\gamma| \sqrt{1 + 4\cos\frac{3k_x a}{2}\cos\frac{\sqrt{3}k_y a}{2} + 4\cos^2\frac{\sqrt{3}k_y a}{2}}.$$

Here, γ is the nearest neighbour transfer integral,

$$\gamma = \langle 2p_z^A | \Delta V | 2p_z^B \rangle < 0,$$

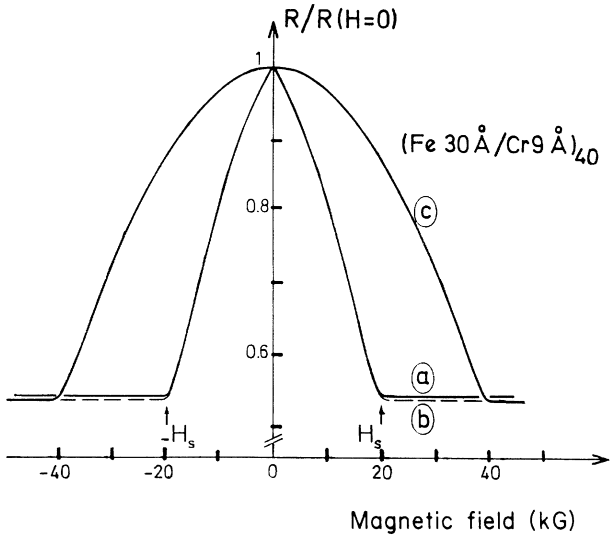
i.e., the matrix element of the perturbation $\Delta V = H - H_a$, and $|2p_z^A\rangle$ and $|2p_z^B\rangle$ are eigenstates of the atomic hamiltonian H_a, for nearest neighbour carbon atoms A and B. (E_0 is an ”uninteresting” constant.)

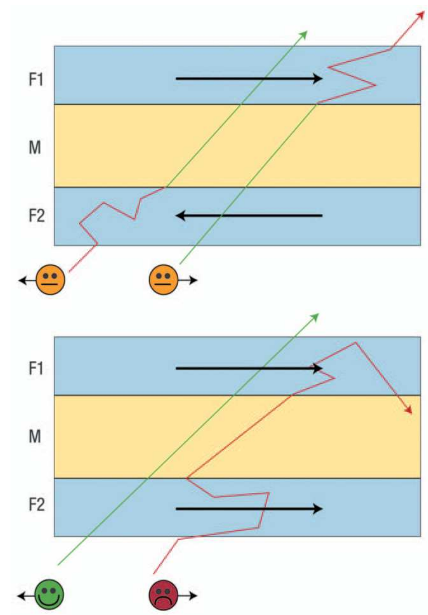
a) The top of the valence band, $E^-(\boldsymbol{k})$, and the bottom of the conduction band, $E^+(\boldsymbol{k})$, are both located at the corners of the 1BZ. Use this information to calculate the bandgap of graphene.

b) The low temperature electronic excitations in graphene are often refered to as ”massless relativistic particles” (or ”massless Dirac fermions”). Justify this expression and calculate the velocity of these particles, assuming a value of 2 eV for the transfer integral $|\gamma|$.
Hint: Feel free to choose a *particular* direction around a *particular* corner of the 1BZ.

Relativistic energy: $E^2 = p^2c^2 + m^2c^4$.

QUESTION 2



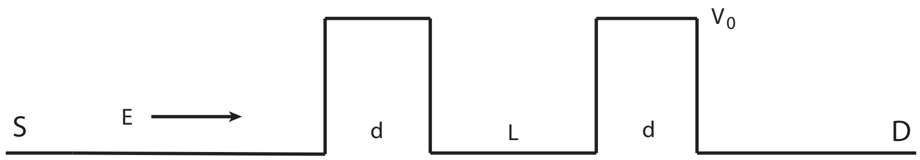


The figure to the left [Baibich et al, *Phys Rev Lett* **61**, 2472 (1988)] illustrates the so-called "giant magnetoresistance" effect (GMR), where the application of a magnetic field H to a structure of alternating layers of ferromagnetic iron and nonmagnetic chromium (actually, a superlattice of forty Fe/Cr layers) results in a reduction in the resistance R when compared with the zero field resistance $R(H = 0)$. Here, the reduction is about 45 % when the applied field is larger than the saturation field H_s. An explanation of the observed GMR effect - in terms of the so-called two-current model for the conduction in ferromagnetic metals - is suggested in the figure to the right [Chappert et al, *Nature Materials* **6**, 813 (2007)]. In the upper figure, $H = 0$, and an antiferromagnetic coupling between the magnetic iron layers F1 and F2 yields antiparallel magnetizations $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ (black arrows). In the lower figure, $H > H_s$, and $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ are both aligned with the external field $\boldsymbol{H}$.

Let R_+ and R_-, with $R_+ > R_-$, denote the ferromagnetic layer resistances experienced by electrons that have their spin aligned parallel and antiparallel, respectively, with the magnetization. Assume that the resistance of the nonmagnetic chromium layer (M) is negligible.

Derive expressions for $R(H = 0)$ and $R(H > H_s)$. Estimate a value for the ratio R_+/R_- in the experiment by Baibich et al.

QUESTION 3



a) Explain briefly how the symmetric double potential energy barrier structure (DBS) in the figure above is related to the electronic band structure of semiconductor materials, e.g., GaAs and $Al_xGa_{1-x}As$ (with $x \simeq 0.3$).

An electron coming in from the left (S = "source"; D = "drain") with energy $E = \hbar^2 k^2/2m$ can be described by a plane wave $\exp(ikx)$ ($k > 0$). The DBS energy barrier height is V_0, the barrier width is d, and the well width is L. For a *single* barrier structure (SBS) located at $0 < x < d$, the transmitted wave is $t\exp(ikx)$ and the reflected wave is $r\exp(-ikx)$, with transmission and reflection amplitude

$$
\begin{aligned}
t &= e^{-ikd}\left(\cosh\kappa d + i\frac{\delta}{2}\sinh\kappa d\right)^{-1}, \\
r &= -i\frac{\sigma}{2}e^{ikd}t\sinh\kappa d.
\end{aligned}
$$

Here, $\kappa = \sqrt{2m(V_0 - E)}/\hbar$, $\sigma = \kappa/k + k/\kappa$, and $\delta = \kappa/k - k/\kappa$.

b) For the SBS, use these expressions to show that the reflection probability $R = |r|^2$ and the transmission probability $T = |t|^2$ are, to leading order when $E \ll V_0$ and $\kappa d \gg 1$,

$$
\begin{aligned}
R &\simeq 1 \\
T &\simeq \frac{16k^2}{\kappa^2}e^{-2\kappa d}.
\end{aligned}
$$

We may call this the "opaque" barrier limit. (Of course, $R + T = 1$.)

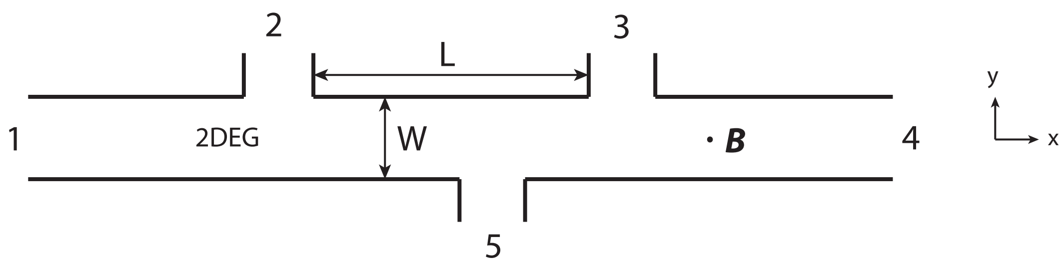
c) For the DBS, use the notion of Feynman paths (or, if you like, the analogy to interference in optics; see figure below) to show that the transmission amplitude t_{SD} (from "source" S to "drain" D) is

$$
t_{SD} = \frac{t^2 e^{ikL}}{1 - r^2 e^{2ikL}}.
$$

d) For opaque barriers (see **b**), the phase of the reflection amplitude is approximately $-\pi$, i.e., $r \simeq -|r|$. Then, show that *resonant tunneling* (i.e., $T_{SD} = |t_{SD}|^2 = 1$) takes place for electron energies corresponding to *standing waves* in the well between the two barriers.

e) Show - with qualitative arguments - how the current-voltage curve $I(V)$ of such a DBS may possess a region of negative differential resistance, $(dI/dV)^{-1} < 0$. Use figures.

QUESTION 4



The figure above shows a 5-terminal device with ideal contacts, for measurement of the Hall resistance R_H and the longitudinal resistance R_L. A relatively strong uniform magnetic field $\boldsymbol{B}$ is applied perpendicular to the 2DEG and points out of the plane.

The Büttiker–Landauer equations,

$$I_\alpha = \sum_{\beta \neq \alpha} G_{\alpha\beta} \left(V_\alpha - V_\beta \right),$$

with conductances

$$G_{\alpha\beta} = \frac{2e^2}{h} T_{\alpha\beta},$$

relate the current in terminal α to the potentials at the various terminals. Here, $T_{\alpha\beta}$ denotes the "direct transmission sum" from terminal β to terminal α. A small voltage $V = V_1 - V_4$ is applied between terminals 1 and 4, resulting in a net current I flowing from terminal 1 to terminal 4 (i.e., $I_1 = -I_4 = I > 0$). Terminals 2, 3, and 5 are used as voltage probes.

a) Use classical physics to argue that the net current through the system is carried by electrons located near the two edges (upper and lower) of the 2DEG. Draw electron orbits, both near and far from the edges of the 2DEG.

b) Assume N states are present near each edge of the 2DEG at the Fermi level. Use your ideas from **a)** to argue why e.g. $T_{43} = N$ whereas e.g. $T_{52} = 0$. Write down all the non-zero elements $T_{\alpha\beta}$ of the transmission matrix.

c) Find the Hall resistance $R_H = R_{14,25}$ and the longitudinal resistance $R_L = R_{14,23}$. (Notation: $R_{\alpha\beta,\kappa\eta} = (V_\kappa - V_\eta)/I$, $I = I_\alpha = -I_\beta$, $I_\kappa = I_\eta = 0$.) For convenience, you may choose $V_4 = 0$.

d) The figure below is taken from D. C. Tsui and A. C. Gossard, Appl Phys Lett **38**, 550 (1981), and shows an example of experimental curves for the longitudinal resistivity

$$\rho_{xx}(B) = \frac{E_x}{j_x}$$

and the Hall resistivity

$$\rho_{xy}(B) = \frac{E_y}{j_x}$$

for a similar device. Discuss your results in **c)** - both similarities and discrepancies - in view of these experiments.
(As a hint, some keywords: Landau levels, Hall plateaus, Shubnikov - de Haas oscillations, disorder, back-scattering.)