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Department of Physics

Examination paper for TFY4340 Mesoscopic Physics

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Problem 1: Weak Localization

(a) What is the origin of the weak localization effect ?

(b) Why is the effect called "weak" localization ?

(c) Why will a magnetic field affect the weak localization effect ?

Problem 2: The Quantum Hall Effect

Consider a two-dimensional electron gas in the x-y plane, where there is a transverse harmonic potential of the form $V(y) = m\omega_0^2 y^2/2$ and a magnetic field $\mathbf{B} = B\mathbf{z}$ applied along the z-direction. The Schrödinger equation is

$$\left[-\frac{\hbar^2}{2m}\left(\nabla + \frac{ie}{\hbar}\mathbf{A}\right)^2 + \frac{1}{2}m\omega_0^2 y^2\right]\psi(x,y) = E\psi(x,y)\,. \tag{1}$$

Choose the Landau gauge where the electromagnetic vector potential is $\mathbf{A} = (-yB, 0, 0)$.

(a) Use the Schrödinger equation (1) and the ansatz $\psi_{k,n}(x,y) = \phi_n(y)\exp ikx$ to show that the equation for ϕ_n is

$$\left[-\frac{d^2}{du^2} + (u-K)^2 + R^2u^2\right]\phi_n(u) = \epsilon_{k,n}\phi_n(u)\,, \tag{2}$$

where we have introduced the dimensionless variables $u = y/l_b$, $R = \omega_0/\omega_c$, $\epsilon = E/(\hbar\omega_c/2)$, $K = kl_B$. Additionally, $l_B = (\hbar/(eB))^{1/2}$ is the magnetic length, and $\omega_c = eB/m$ is the cyclotron frequency.

(b) Demonstrate that Eq. 2 can be re-written into the form of an equation for a particle in a harmonic potential and that the solution in terms of the original variables is

$$\psi_{k,n}(x,y) = e^{ikx}\phi_n(y - L_B^2 k)\,, \tag{3}$$

where $\phi_n(y)$ is a harmonic oscillator function that satisfies

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dy^2} + \frac{1}{2}m\Omega^2y^2\right)\phi_n(y) = \hbar\Omega\left(n+\frac{1}{2}\right)\phi_n(y)\,, \tag{4}$$

$$L_B^2 = \frac{\omega_c^2}{\omega_c^2+\omega_0^2}l_B^2 \tag{5}$$

and the eigenenergy for the eigenstate $\psi_{k,n}(x,y)$ is

$$E_{k,n} = \hbar\Omega(n+\frac{1}{2}) + \frac{\hbar^2k^2}{2M_B}\,, \tag{6}$$

where $\Omega = (\omega_c^2+\omega_0^2)^{1/2}$, and $M_B = m\Omega^2/\omega_0^2$.

(c) Compute the particle current density

$$\mathbf{j} = \frac{\hbar}{m}\mathrm{Im}\left(\psi^\dagger \nabla \psi\right) + \frac{e}{m}\mathbf{A}|\psi|^2 \tag{7}$$

for the eigenstate $\psi_{k,n}$ that we found above. What is the sign of the current density along the x-direction, j_x ?

Problem 3: The Landauer-Büttiker formalism

The Landauer-Büttiker formula for the conductance is

$$G = \frac{e^2}{h}\sum_n T_n \,, \tag{8}$$

where T_n is the transmission probability for transverse wave guide mode n and the sum is over transverse wave guide modes.

(a) Consider a one-dimensional system (only one wave guide mode) with a left and right reservoir with chemical potentials μ_L and μ_R, respectively, such that $eV = \mu_L - \mu_R$. Find arguments for how the current should be expressed in terms of the velocity $v(\epsilon)$ of an electron at energy ϵ, the density of states in one dimension $N(\epsilon) = 2/[hv(\epsilon)]$, the transmission probability $T(\epsilon)$, and the distribution functions in the left and the right reservoirs $f(\epsilon - \mu_L)$ and $f(\epsilon - \mu_R)$. Derive from these arguments the Landauer-Büttiker conductance in the linear response regime (the bias voltage is much smaller than the Fermi energy) at zero temperature, Eq. 8 with only one transverse wave guide mode, $G = (e^2/h)T$.

(b) We consider a narrow quantum wire that is created in a two-dimensional electron gas such that the system is infinite in the x-direction, but has a finite width L in the y-direction. The potential is assumed to be infinite outside the wire where the wave function vanishes. We assume there is no scattering in the wire and transport is ballistic. Show that the conductance is

$$G = \frac{2e^2}{h}\left[\frac{Lk_F}{\pi}\right] \,, \tag{9}$$

where $[\cdots]$ represents the integral part of the number, e.g. $[2.1] = 2$.

Problem 4: Magnetoresistance

(a) What do the abbrevations AMR, GMR, and TMR mean ? What is the cause of AMR, GMR, and TMR ?