NTNU – Trondheim
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Department of Physics

Examination paper for TFY4340 Mesoscopic Physics

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Date Signature

Problem 1: Various

(a) What is a Shottky barrier between a metal and a semiconductor ?

(b) Derive the classical Drude formula for the electron conductivity σ based on Newton's second law:

$$\sigma = \frac{e^2 n\tau}{m}\,, \tag{1}$$

where τ is the typical scattering time between collisions, n is the electron density, e is (minus) the electron charge, and m is the electron mass. You should also state what the basic assumptions in deriving the Drude formula are.

(c) Consider a spin-degenerate ($g_s = 2$) free electron gas and compute the density of states as a function of energy ϵ, $D(\epsilon)$, when the system is three-dimensional with volume V.

Problem 2: Transmission and Landauer-Büttiker conductance

Consider a spin-degenerate one-dimensional system attached to a left and a right reservoir. The Landauer-Büttiker conductance G is

$$G = \frac{2e^2}{h} T\,, \tag{2}$$

where e is (minus) the electron charge, h is Planck's constant, and $T = |t|^2$ is the transmission probability in terms of the transmission amplitude t.

(a) We now assume a particle with mass m and energy ϵ moves in a one-dimensional channel in the presence of a single scatterer with a potential

$$V(x) = V_0 \delta(x)\,, \tag{3}$$

where $\delta(x)$ is the Dirac-delta function $[\int_{-\infty}^{\infty} dx f(x)\delta(x-y) = f(y)]$ and V_0 is the strength of the scatterer with dimension $[V_0] = Jm$. We define a scattering matrix consisting of reflection r (r') and transmission amplitudes t (t') for electrons coming from the left (right) lead as

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}. \tag{4}$$

Show that the reflection amplitudes are

$$r = r' = \frac{V_0}{i\hbar v - V_0} \tag{5}$$

and the transmission amplitudes are

$$t = t' = \frac{i\hbar v}{i\hbar v - V_0} \,, \tag{6}$$

where the velocity $v = \sqrt{2E/m}$ is the same in both leads since the potential vanishes there.

(b) Imagine that we have two scatterers in series with scattering matrices S_1 and S_2, respectively. These two scatterers together define a total scattering matrix S_{12}.

Show that the reflection and transmission amplitudes of the total scattering matrix are

$$t_{12} = t_1[1 - r_2 r_1']^{-1} t_2 \tag{7}$$

$$r_{12} = r_1 + t_1 r_2 [1 - r_2 r_1']^{-1} t_1' \tag{8}$$

$$t_{12}' = t_2'[1 - r_1' r_2]^{-1} t_1' \tag{9}$$

$$r_{12}' = r_2' + t_2' r_1' [1 - r_1' r_2]^{-1} t_2 \tag{10}$$

in terms of the reflection and transmission amplitudes of the scattering matrices S_1 and S_2. It may be useful to know that $(1 - x)^{-1} = \sum_{i=0}^{\infty} x^i$.

(c) We now consider two identical scatterers in the one-dimensional channel located at $x = -a/2$ and $x = a/2$ with a total potential

$$V(x) = V_0 \delta(x + a/2) + V_0 \delta(x - a/2) \,. \tag{11}$$

As a scattering problem, we can now view the system as consisting of three scattering matrices in series where the leftmost and rightmost scattering matrices are identical to the one in problem a), $S_1 = S_3 = S$, and in the middle of the system $(-a/2 < x < a/2)$, there is no reflection, but the transmission amplitude acquires a phase ka so that its scattering matrix is

$$S_2 = \begin{pmatrix} 0 & \exp ika \\ \exp ika & 0 \end{pmatrix} \,. \tag{12}$$

What is the total transmission *amplitude* for the whole system (consisting of scattering matrices S_1, S_2, and S_3) ? Show that the total transmission *probability* is

$$T_{123} = \frac{T^2}{1 - 2R\cos\theta + R^2} \tag{13}$$

where $\theta = 2\,[ka + \arctan \hbar v / V_0]$.

(d) Consider that the transmission probability T for each scatterer is very small $T \ll 1$. What is the maximum transmission probability T_{123} and the associated Landauer-Büttiker conductance G for an arbitrary energy ϵ ? Comment on the physical interpretation of this result.

Problem 3: Single-electron tunneling

Consider a junction that has a capacitance C and a large resistance R due to weak tunneling. Assume there is a charge Q to the left of the junction and a charge $-Q$ to the right of the junction. This charge may depend on time.

(a) Demonstrate that an energy $E_C = Q^2/2C$ is required to charge the capacitor from a zero charge to a charge Q.

(b) At what thermal energies with respect to E_C do we expect single-electron tunneling effects to become important and why ?

(c) Assume the junction (with capacitance C and tunnel resistance R) is an open circuit. Demonstrate that the typical decay time of the charge Q is $\tau = RC$.

(d) Find a condition using the tunnel conductance $G = 1/R$ when quantum flucutations can be avoided so that single-electron tunneling effects can be clearly seen.