NTNU – Trondheim
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Department of Physics

Examination paper for TFY4340/FY8909 Nanophysics

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Problem 1: Various

(a) Define the following length scales that are relevant in discussing transport in nano-scale systems: 1) Fermi wavelength λ_F, 2) elastic mean free path l_e, and 3) phase coherence length l_ϕ.

(b) We consider a 1D channel and define a scattering matrix consisting of reflection r (r') and transmission amplitudes t (t') for electrons coming from the left (right) lead as

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} . \tag{1}$$

Imagine that we have two scatterers in series with scattering matrices S_1 and S_2, respectively.

Make the assumption that there is incoherent scattering in the 1D channel so scattering by the two potentials is incoherent. What is the total transmission probability through the system in terms of the elements of the scattering matrices S_1 and S_2 in this case?

Problem 2: The Landauer-Büttiker formula

The Landauer-Büttiker formula for the relation between currents I_i and voltages V_j in a many-terminal system reads

$$I_i = \sum_j \left[G_{ji} V_i - G_{ij} V_j \right] , \tag{2}$$

where the sum is over the terminal indices $j = 1, 2, 3, \ldots N$ and $i = 1, 2, 3$ etc., N is the number of terminals, and G_{ij} are elements of a conductance matrix.

(a) Prove that $\sum_j G_{ji} = \sum_j G_{ij}$.

(b) In the remainder of the problems, we will assume that $G_{ij} = G_{ji}$. Which physical condition must be fulfilled to justify this assumption?

(c) Consider a three-terminal device, where a current $I = I_1 = -I_2$ passes from terminal 1 at voltage V_1 to terminal 2 at voltage V_2. In response to the current I, there is a voltage V_3 at the terminal 3 where there is no current, $I_3 = 0$.

Consider first (and here only) that the conductance element G_{23} is much bigger than all the other elements of the conductance matrix. In this limit, what is the voltage difference $V_3 - V_2$?

(d) Compute the potential difference $V_3 - V_2$ as a function of the current I and the conductance matrix G_{ij} of the system.

Problem 3: The Quantum Hall effect

(a) Give a physical explanation of what Landau levels are.

(b) Give a physical explanation of what semi-classical skipping orbits is as well as what their quantum mechanical analogue are.

Problem 4: Spintronics

(a) Explain what the following spintronics phenomena are: i) giant magnetoresistance (GMR) and 2) spin-transfer torques.

(b) We consider the Pauli equation for spin 1/2 electrons in a one-dimensional channel along the x-direction and disregard the spin-orbit interaction. We assume that there is a weak magnetic field B applied along the z-direction confined to a length $-L/2 < x < L/2$ and include its effect on the spin degrees of freedom only. The Hamiltonian is then

$$H = \begin{cases} -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}I + \frac{ge\hbar}{2m}B\sigma_z & ; -L/2 < x < L/2 \\ -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}I & ; |x| \geq L/2 \end{cases} , \tag{3}$$

where the unit matrix I and the Pauli-matrix σ_z are defined as

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{4}$$

and

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{5}$$

We may define a Zeeman energy

$$E_Z = \frac{ge\hbar}{2m}B \tag{6}$$

and assume that $E_Z > 0$ throughout this exam.

Consider an electron with energy $0 < E < E_Z$. Assume (in this question only) that the system is much longer than any other possible length scale, $L \to \infty$. Argue, without explicit calculations, what the reflection probability $R_\uparrow$ is for an incoming spin-up electron. For your information, the spin-up electron sees a larger potential barrier than the spin-down electron in the barrier region $-L/2 < x < L/2$.

(c) Consider still an electron with energy $0 < E < E_Z$, but the barrier region has now a finite length L. Compute the transmission probability for the passage of an electron through the one-dimensional channel for both a spin-up state and a spin-down state.