NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Physics

Examination paper for TFY4340/FY8909 Nanophysics

Academic contact during examination: Professor Arne Brataas

Phone: 90643170

Examination date: June 6, 2016

Examination time (from-to): 09.00am-1.00pm

Permitted examination support material: C, K. Rottmann: Mathematical formulae and approved calculator with empty memory.

Other information:

Language: English

Number of pages: 1+3

Number of pages enclosed: 3

Checked by:

Date Signature

Problem 1: Electron Concentration and Fermi Wavelength

(a) Calculate the Fermi wavevector k_F of a three dimensional electron gas (3DEG) expressed in terms of the electron concentration n at a temperature $T = 0$. Do the same for 2D and 1D systems. You should include the spin-degeneracy $g = 2$ in all these cases.

(b) The dimensional unit in which the electron concentration is expressed changes with the dimensionality of the system. To compare systems with different dimensionalities, the electron concentration can be expressed in terms of the effective distance between the electrons d_{e-e} by assuming that each electron resides in a box of size d_{e-e}.

Express the results for k_F in one, two, and three dimensions in terms of d_{e-e} and calculate the numerical value of the prefactors.

Problem 2: Weak Localization

(a) What is weak localization?

(b) Sketch the resistance versus the magnetic field in a device in which weak localization occurs.

Problem 3: Coherent Transport

A one dimensional conductor contains a single scatterer with transmission coefficient t and an electron with wavefunction $\psi(x) = \exp ikx$ is sent into the conductor. The incoming,

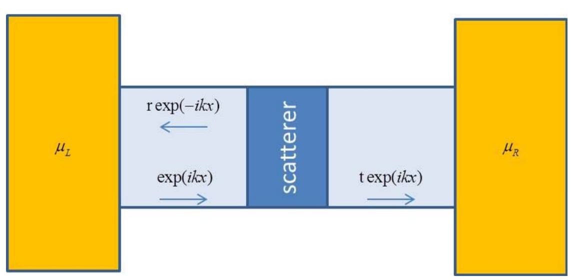

FIG. 1. A one dimensional conductor with a single scatterer connected to a left and a right reservoir. The wave function to the left of the scatterer is $\psi(x) = \exp ikx + r \exp -ikx$ and the wave function to the right of the scatterer is $\psi(x) = t \exp ikx$.

reflected, and transmitted waves on both sides of the scatterer as well as the left and right reservoirs are shown in Fig. 1.

(a) The current density J is given by

$$J = \frac{\hbar}{2mi}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{dx}\right) . \tag{1}$$

Express the current density in terms of t and the velocity of the electron.

(b) Assume that the scatterer is a rectangular potential of height $V_0 > E_F$ and width d within $-d/2 < x < d/2$. Calculate the (energy-dependent) transmission probability $T(E) = |t(E)|^2$.

Problem 4: Single-electron Tunneling and Coulomb Blockade

(a) An amount of charge Q_0 is placed on a capacitor with capacitance C (there is a charge $+Q_0$ on one side of the capacitor and $-Q_0$ on the other side of the capacitor). There is a resistance R to the ground, see Fig. 3. Show that the charge $Q(t)$ on the capacitor

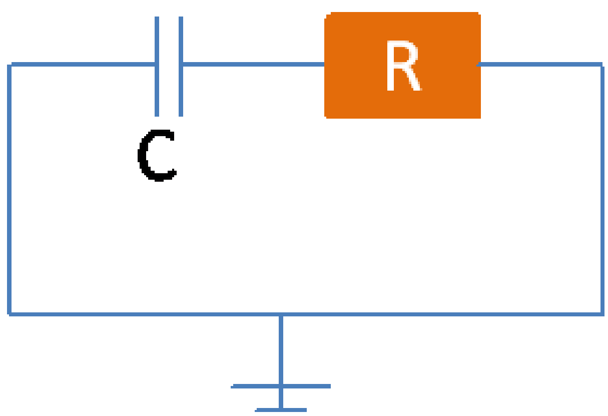

FIG. 2. An electric circuit with a capcitance in series with a resistor.

evolves as

$$Q(t) = Q_0 \exp -t/\tau \tag{2}$$

and determine the decay time τ.

(b) The Heisenberg uncertainty principle states that the energy of an electron is ill defined when the electron stays in a state only for a short time:

$$\Delta E \Delta t \geq \hbar/2 \, . \tag{3}$$

What is the uncertainty in energy for the system discussed in Problem 4a?

(c) We now assume that we have a nano-scale system where a junction exhibits both capacitive effects and tunneling, e.g. that a single junction consists both of a capacitance C and a (tunnel) resistance R as in Fig. 3.

Find a condition, expressed in terms of the tunnel conductance $G = 1/R$, for when quantum fluctuations can be avoided so that single-electron tunneling effects can be clearly seen.

Problem 5: Landau Levels

We consider electrons in a two-dimensional electron gas (2DEG) subject to a perpendicular magnetic field. We disregard the effects of the electron spin. The Schrödinger equation is then

$$\frac{(\mathbf{p} + e\mathbf{A})^2}{2m}\psi(x, y) = E\psi(x, y)\,, \tag{4}$$

where $-e$ is the electron charge and $\mathbf{A}$ is the electromagnetic vector potential.

(a) The perpendicular magnetic field is $\mathbf{B} = B\mathbf{z}$, where B is its magnitude and $\mathbf{z}$ is a unit vector in the z-direction. Let us assume that the electromagnetic vector potential only depends on the in-plane coordinates x and y and only has components along the x and y directions. Find all possibilities for the vector potential $\mathbf{A}$ to the first order in the coordinates x and y, e.g. $\mathbf{A}$ is of the form

$$\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_1 x + \mathbf{A}_2 y\,. \tag{5}$$

when we use the Coulomb gauge

$$\nabla \cdot \mathbf{A} = 0\,. \tag{6}$$

(b) It can be shown that the eigenvalues of the Schrödinger equation (19) in the Landau gauge $\mathbf{A} = (0, Bx, 0)$ are

$$E_n = \hbar\omega_c\left(n + \frac{1}{2}\right) \tag{7}$$

where n is an integral number $n = 0, 1, 2, \ldots$ and $\omega_c = eB/m$ is the cyclotron frequency. Find the eigenvalues in the other possible choices of the gauge for the electromagnetic vector potential.