NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Physics

Examination paper for TFY4345 - (Classical Mechanics)

Academic contact during examination: Paul Gunnar Dommersnes

Phone: 94 18 61 10

Examination date: December 12, 2016

Examination time (from-to): 9-13

Permitted examination support material:

- **Approved, simple calculator**
- **Fysiske størrelser og enheter: navn og symboler, Angell, Carl - Lian, Bjørn Ebbe**
- **K. Rottmann (matematisk formelsamling)**
- **Barnett & Cronin: Mathematical Formulae**
- **Notice: *Supplementary Information* on the last page of the exam contains useful formulas**

Other information:

Grading: Problem 1 (35%), Problem 2 (30%), Problem 3 (25%), Problem 4 (10%)

Language: English

Number of pages (front page excluded): 5

Number of pages enclosed: 5

Informasjon om trykking av eksamensoppgave

Originalen er:

1-sidig ☐ 2-sidig ☐

sort/hvit ☐ farger ☐

Checked by:

____

Problem 1.   Motion of a particle confined to a surface (35%)

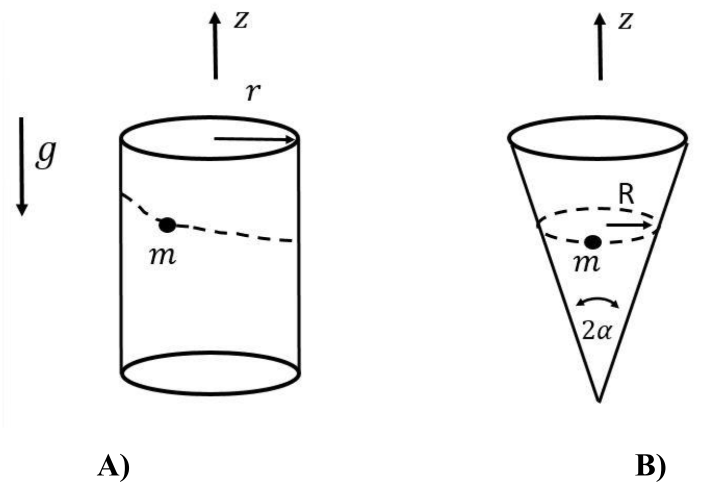


Fig 1. A) Particle confined on the surface of a cylinder   B) Particle confined on the surface of a cone of angle 2α. The particle is subject to a uniform gravitational field $\vec{g} = -g\vec{e}_z$. The particle is sliding on the surface without any friction, and remains on the surface at all times.

We shall first consider the motion of a point particle of mass m on the inside surface of a vertical cylinder with radius r, see Fig 1.A). The initial velocity of the particle is large enough to keep it at all times in contact with the cylindrical surface. The particle slides downwards due to the gravitational field.

1a) Use cylindrical coordinates to describe the motion of the particle on the cylindrical surface, and write down the Lagrangian for the particle.

1b) Find the equations of motion and identify conservation laws.

1c) At time $t = 0$ the velocity in the z-direction is zero. Find the solution for the trajectory of the particle as a function of time. Make a simple sketch of the motion of the particle on the cylinder surface.

We now consider a similar situation, only this time the particle is confined on the surface of a cone with half-angle α, see Fig 2.B).

1d) Write down the Lagrangian for the motion of the particle on the cone. You may use spherical coordinates, where $r = 0$ corresponds to the tip of the cone.

1e) Find the equations of motion and identify conservation laws.

1f) Show that there exist a solution of the equations of motion where the particle is moving in a circle without falling (horizontal movement). Determine the frequency ω of the circular motion as function of the radius R of this circle, the cone half-angle α, and the gravitational acceleration g.

1g) Find the equation of motion for a particle that is perturbed slightly from this circular motion (small deviations from the circular motion). Show that the perturbation has an oscillatory motion and find the expression for its oscillation frequency Ω . Show that $\omega = \Omega$ for a specific angle α.

Problem 2. Particle in a central force potential (30%)

A particle of mass m is subject to an attractive central force potential:

$$V = -\frac{k}{r^4}$$

2a) Show that the angular momentum of the particle is conserved and explain why this implies that the motion of the particle occurs in a plane.

2b) Use polar coordinates (r and ϕ) and find the Lagrangian for the particle. Determine all conservation laws.

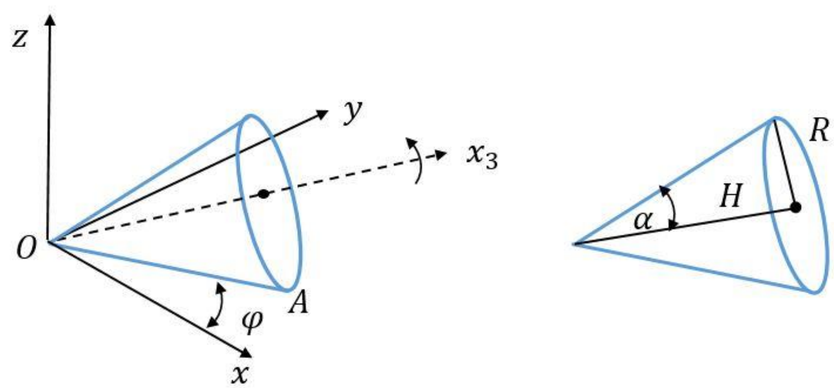
2c) Use conservation laws to transform the central force problem into a one-dimensional problem of a particle in an effective one-dimensional potential V_{eff}

2d) Draw a plot of the effective potential as a function of the distance r.

2e) Give the general condition for a circular orbit (r = constant). Express the radius of the orbit as a function of the constant k, mass m, and conserved quantities.

2f) Are these circular orbits stable or unstable? Explain your answer.

Problem 3. Cone rolling on a plane (25%)



A) B)

Fig 2. A) A solid cone rolls without slipping on a flat horizontal plane (xy-plane). The momentary line of contact with the plane is OA. The angle between the x-axis and the line OA is φ. The rolling induces rotation of the cone around the principal symmetry axis of the cone (x_3-axis in the figure).

B) Dimensions of the cone: height (H), radius (R) and cone half-angle (α). Notice that these are related by: $\frac{R}{H} = \tan(\alpha)$

We shall consider the motion of a solid cone that is rolling on surface (xy plane) without slipping, see Fig. 2.

3a) The center of mass of the cone is situated on the symmetry axis x_3, at a distance ℓ from the origin O. Calculate the velocity of the center of mass, V_{cm}, as a function of ℓ, α and $\dot{\varphi}$. Here $\dot{\varphi}$ is the time derivative of φ.

3b) Explain why the angular velocity vector $\vec{\omega}$ of the rolling cone is directed along the line OA. Show that $|\vec{\omega}| = \frac{\cos(\alpha)}{\sin(\alpha)}\dot{\varphi}$

3c) Find the components of $\vec{\omega}$ along the principal axes (symmetry axes) of the cone.

3d) The principal moment of inertia of the cone (for rotation around the point O) is: $I_1 = I_2 = \frac{3}{20}m(R^2 + 4H^2)$ and $I_3 = \frac{3}{10}mR^2$, where m is the mass of the cone. Calculate the kinetic energy of the cone as a function of m, H, α and $\dot{\varphi}$.

Problem 4. Light from a fluorescent tube (10%)

A fluorescent tube lamp is stationary in a reference frame S. The tube is arranged such that it lights up simultaneously (in S) along its entire length L_0 at the time $t = 0$

Consider an observer in a reference system S' moving with a velocity *v* parallel to the orientation of the tube.

Suppose the tube lies along the z axis, at rest in the reference frame S, with one end at $z = 0$, and the other at $z = L_0$.

4a) We now consider two space time events in S: the lighting up of the tube in position z at time an t, and in position $z + \Delta z$ also at time t.

Use the Lorentz transformation to calculate the space-time coordinates of these two events in the S' frame (z' at time t') , ($z' + \Delta z'$ at time $t' + \Delta t'$).

4b) For the observer in S' the light does not appear to turn on simultaneously along the tube. Show that for the observer in S' the lighting up of the tube propagates with an apparent velocity:

$$|u| = \frac{c^2}{v}$$

Supplementary information

Cylindrical coordinates:

$$x = r\cos(\phi)$$
$$y = r\sin(\phi)$$
$$z = z$$

$$\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$$

$$\vec{r} = r\vec{e}_r + z\vec{e}_z$$

Spherical coordinates:

$$x = r\cos(\phi)\sin(\theta)$$
$$y = r\sin(\phi)\sin(\theta)$$
$$z = r\cos(\theta)$$

$$\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$$

$$\vec{r} = r\vec{e}_r$$

Series expansion valid for $|x| \ll 1$:

$$(1+x)^a = 1 + a\,x + O(x^2)$$

Angular momentum of a particle:

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \vec{N} = \vec{r} \times \vec{F}$$

Angular momentum of a rigidy body along principal axes of rotation:

$$L_1 = I_1\omega_1$$
$$L_2 = I_2\omega_2$$
$$L_3 = I_3\omega_3$$

Kinetic energy of a rigid body:

$$T = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 + \frac{1}{2}I_3\omega_3^2$$

Where I_1 , I_2 , I_3 are the principal moment of inertia, and ω_1, ω_2 , ω_3 are the angular velocity components around the principal axes.

Lorentz transforms between a reference system S' moving with a constant velocity v in the z-direction, with respect to a reference system S:

$$\boxed{\begin{aligned} x' &= x \\ y' &= y \\ z' &= \gamma(z - vt) \\ t' &= \gamma\left(t - \frac{vz}{c^2}\right) \end{aligned}}$$

$$\boxed{\begin{aligned} x &= x' \\ y &= y' \\ z &= \gamma(z' + vt') \\ t &= \gamma\left(t' + \frac{vz'}{c^2}\right) \end{aligned}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$