NTNU

Department of (department)

Examination paper for TFY4345 Classical Mechanics

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Examination date: 12.12.2017

Examination time (from-to): 15:00-19:00

Permitted examination support material:

- Approved, simple calculator
- Fysiske størrelser og enheter: nav og symboler, Angell, Carl - Lian, Bjørn Ebbe
- K. Rottmann (matematisk formelsamling)
- Barnett & Cronin: Mathematical Formulae
- Notice: Supplementary Information on the last page of the exam contains useful formulas

Other information:

Grading: Problem 1 (4p), Problem 2 (6p), Problem 3 (6p), Problem 4 (6p), Problem 5 (6p) – Total 28 p

Language: English

Number of pages (front page excluded): 3

Number of pages enclosed: 3

Informasjon om trykking av eksamensoppgave

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Checked by:

5.12.2017 Jaakko Akola

Date Signature

Problem 1. Special theory of relativity (4p ~ 14%)

(a) Mary is travelling in a space ship with velocity $0.8c$ to a nearby star system and back. Each leg takes 10 years according to Frank's clock whereas Mary's clock shows only 6 years. Mary sends a signal to Frank every year. How often does Frank receive signals from Mary during each leg?

(b) Let us denote the four-vector for position as $\mathbf{x} = (x_1, x_2, x_3, ict)$ following the covariant 3+1 formulation where time is the 4th component. The standard 3D velocity of the particle is $\mathbf{v}$. Derive the velocity four-vector $\mathbf{u}$ of the particle by using the invariance of the displacement $ds^2 = dx_\mu dx_\mu$ in the Minkowski-space and proper time.

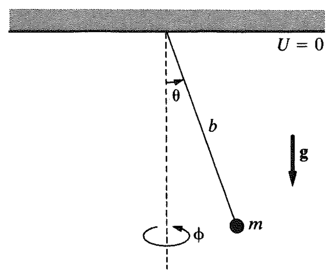
Problem 2. Bead in a rotating string (6p ~ 21%)

A bead slides along a smooth wire bent in the shape of a parabola $z = cr^2$. The wire is rotating around its vertical axis with constant angular velocity ω.

(a) Determine the Lagrangian function for the rotating bead. Is the system holonomic?
(b) Derive the equations of motion from Lagrange's equations. What can you say about cyclic coordinates?
(c) Let us assume next that the bead follows a circular trajectory with $r = R$ (set by initial conditions). Find the value of c.

Problem 3. Spherical pendulum (6p ~ 21%)

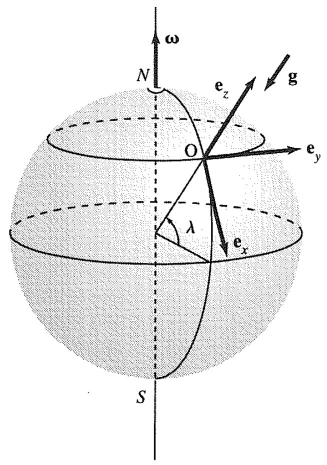
Use the Hamiltonian method for a spherical pendulum of mass m and length b.



(a) Find the Hamiltonian equations of motion.
(b) What can you say about conserved quantities?
(c) Using the Poisson's bracket, evaluate the total time-derivative of the kinetic energy T. When is T a constant of motion?

Problem 4. Coriolis effect (6p ~ 21%)

A particle is projected vertically upward to a height h above Earth's surface at a northern latitude λ. In the following, neglect air resistance and consider only small vertical heights.



(a) Write the expression for the Coriolis effect ("force") and determine the effective acceleration vector.
(b) Where does the particle strike the ground? In which direction?

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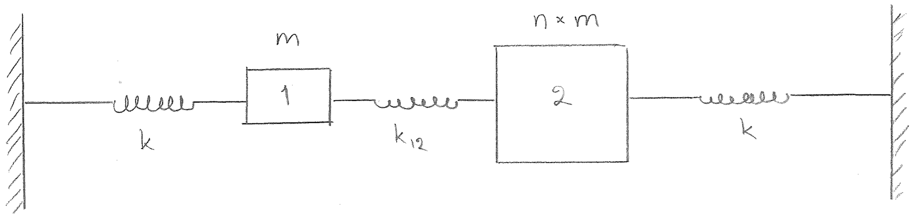
Problem 5. Coupled oscillations (6p ~ 21%)

Two bodies are coupled to a wall, each, and to each other by springs according to the image below. The mass of the second body is n times the first one.



(a) Derive the secular determinant of coupled oscillations from the kinetic and potential energies.
(b) Calculate a general expression for the eigenfrequencies squared (ω^2).
(c) Consider the limit where n approaches infinity. What is the physical interpretation of this result for frequencies? Compare here with n = 1.

Some useful formula and equations:

Cylindrical coordinates:

$$ds^2 = dr^2 + r^2 d\theta^2 + dz^2$$

Spherical coordinates:

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Displacement in Minkowski-space:

$$ds^2 = x_\mu x_\mu = dx_1^2 + dx_2^2 + dx_3^2 - c^2 t^2$$

Lorentz transformation:

$$\begin{aligned} x' &= x \\ y' &= y \\ z' &= \gamma(z - vt) \\ t' &= \gamma(t - \frac{vz}{c^2}; \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

Doppler effect:

$$\nu = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}\nu_0; \quad \beta = \pm v/c$$

Hamilton's equations:

$$\dot{q}_i = \frac{\partial H}{\partial p_i}; \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Poisson brackets:

$$[u, v]_{q,p} = \sum_{i=1}^{n} \left(\frac{\partial u}{\partial q_i}\frac{\partial v}{\partial p_i} - \frac{\partial v}{\partial q_i}\frac{\partial u}{\partial p_i} \right)$$

Observer in rotating coordinate system:

$$\vec{F}_{eff} = \vec{F} - 2m(\vec{\omega} \times \vec{v}_r) - m\vec{\omega} \times \vec{\omega} \times \vec{r}$$

Coupled oscillations:

$$V = \frac{1}{2}\sum_{j,k} A_{jk} q_j q_k; \quad T = \frac{1}{2}\sum_{j,k} m_{ij}\dot{q}_j \dot{q}_k$$