Department of Physics

Examination paper for TFY4345 Classical Mechanics

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Examination date: 11.08.2018

Examination time (from-to): 9:00-13:00

Permitted examination support material:

- Approved, simple calculator
- Fysiske størrelser og enheter: nav og symboler, Angell, Carl - Lian, Bjørn Ebbe
- K. Rottmann (matematisk formelsamling)
- Barnett & Cronin: Mathematical Formulae
- Notice: Supplementary Information on the last page of the exam contains useful formulas

Other information:

Grading: Problem 1 (6p), Problem 2 (6p), Problem 3 (6p), Problem 4 (6p), Problem 5 (6p) – Total 30 p

Language: English

Number of pages (front page excluded): 4

Number of pages enclosed: 4

Informasjon om trykking av eksamensoppgave

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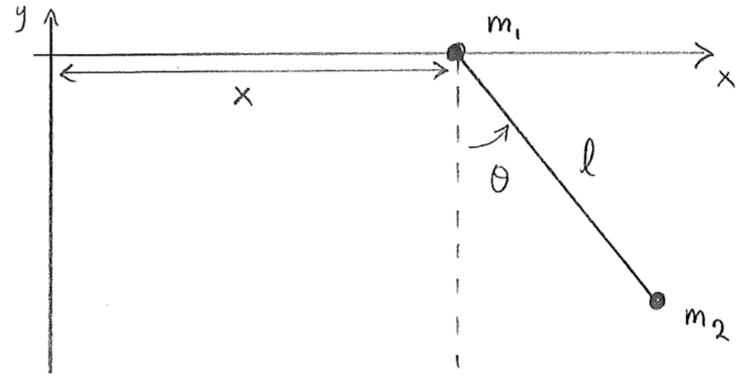
Date Signature

Problem 1. Special theory of relativity (6p = 20%)

(a) Derive the relativistic length contraction based on Lorentz transformation. Assume that you travel from Trondheim to Oslo (494 km) on a hyperloop with a constant velocity of $0.8c$. How long does the distance appear to you as a moving observer?

(b) Let us denote the four-vector for position as $\mathbf{x} = (x_1, x_2, x_3, ict)$ following the covariant 3+1 formulation where time is the 4th component. The standard 3D velocity of the particle is $\mathbf{v}$. Derive the momentum four-vector $\mathbf{p}$ of the particle by using the invariance of the displacement $ds^2 = dx_\mu dx_\mu$ in the Minkowski-space and proper time. Show here the explicit connection between the momentum four-vector and relativistic total energy.

Problem 2. Sliding pendulum (6p = 20%)



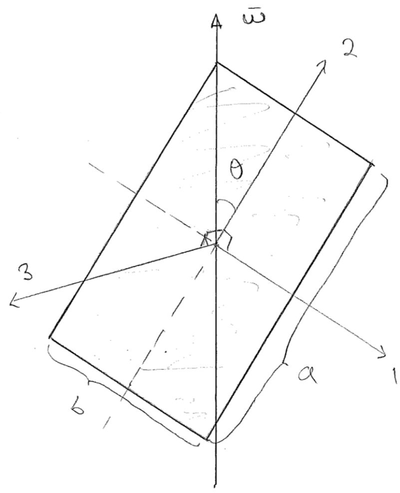
A pendulum system (see above) comprises two masses m_1 and m_2 where the first one acts as a frictionless sliding pivot along the x-axis for the latter mass.

(a) Determine the Lagrangian function. Is the system holonomic?
(b) Derive the equations of motion from Lagrange's equations.
(c) Let us assume next that the pivot (first mass, m_1) stops moving (*i.e.*, it jams). What is the effect on the equations of motion?

Problem 3. Rotating tilted slab (6p = 20%)

A very thin rectangular slab (see figure) rotates with an angular velocity $\boldsymbol{\omega}$ around its diagonal. The side lengths of the slab are a and b and the mass is m. The principal moments of inertia are $ma^2/12$, $mb^2/12$ and $m(a^2+b^2)/12$; the principal axes 1 and 2 go along the same directions as slab edges and 3 is perpendicular to the slab plane and goes through the slab center.

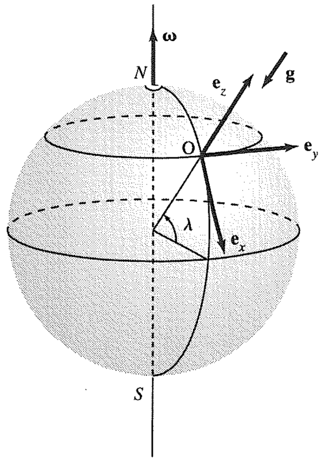
(a) Derive the angular momentum vector $\boldsymbol{L}$ of the slab.
(b) What is the angle between $\boldsymbol{L}$ and $\boldsymbol{\omega}$?
(c) What is the rotational kinetic energy T_{rot}?



[Turn page to see the rest of assignments!]

Problem 4. Coriolis effect (6p = 20%)

A projectile is fired due east from a point on the surface of the Earth at a northern latitude λ with a starting velocity V_0 and at an angle of inclination of α to the horizontal. In the following, neglect air resistance and consider only small vertical heights. Set your local coordinate system as shown in the figure.

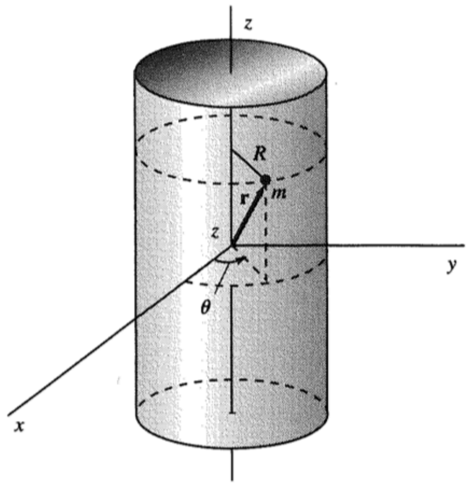


(a) Write the expression for the Coriolis effect ("force") and determine the effective acceleration vector.

(b) Next make an approximation that the Coriolis effect in the $\mathbf{e}_z$ direction is negligible. Derive the lateral deflection in the $\mathbf{e}_x$ direction as a function of V_0 and α. Is it towards South or North?

Problem 5. Hamiltonian dynamics on a cylinder surface (6p = 20%)

Use the Hamiltonian method to find the equations of motion of a particle of mass m constrained to move on the surface of a cylinder defined by $x^2 + y^2 = R^2$. The particle is subject to a force directed towards the origin and inverse-squarely proportional to the distance from the origin: $F = -k/r^2$ (for example, as for the Coulomb force).



(a) Find the Hamiltonian equations of motion.

(b) What can you say about conserved quantities?

(c) Using the Poisson's bracket, evaluate the total time-derivative of the kinetic energy T. When is T a constant of motion?

Some useful formula and equations in the next page:

Cylindrical coordinates:

$$ds^2 = dr^2 + r^2 d\theta^2 + dz^2$$

Spherical coordinates:

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Displacement in Minkowski-space:

$$ds^2 = dx_\mu dx_\mu = dx_1^2 + dx_2^2 + dx_3^2 - c^2 t^2$$

Lorentz transformation:

$$\begin{aligned} x' &= x \\ y' &= y \\ z' &= \gamma(z - vt) \\ t' &= \gamma(t - \frac{vz}{c^2}; \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

Relativistic kinetic energy:

$$T = \gamma mc^2 - mc^2$$

Hamilton's equations:

$$\dot{q}_i = \frac{\partial H}{\partial p_i}; \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Poisson brackets:

$$[u, v]_{q,p} = \sum_{i=1}^{n} \left(\frac{\partial u}{\partial q_i} \frac{\partial v}{\partial p_i} - \frac{\partial v}{\partial q_i} \frac{\partial u}{\partial p_i} \right)$$

Observer in rotating coordinate system:

$$\vec{F}_{eff} = \vec{F} - 2m(\vec{\omega} \times \vec{v}_r) - m\vec{\omega} \times \vec{\omega} \times \vec{r}$$

Coupled oscillations:

$$V = \frac{1}{2} \sum_{j,k} A_{jk} q_j q_k; \quad T = \frac{1}{2} \sum_{j,k} m_{ij} \dot{q}_j \dot{q}_k$$