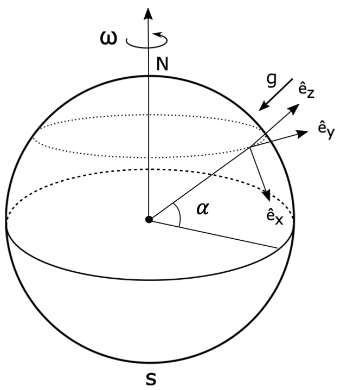
Problem 1. Rotation of Earth (7 points)

A. Felix Baumgartner made his record base jump[1] in 2012 with a starting altitude of 39 km (reached by using a giant helium balloon) and a supersonic top velocity of 1358 km/h in free fall. Some people[2] claim (?!) that if Earth was a spinning sphere, the starting point on the surface should have escaped him *visibly* while ascend (balloon trip). Make solid arguments why this should not be the case. Small measurable deflections are of course present (one effect mentioned below). The radius of Earth is 6371 km. [2p]



B. Derive the lateral displacement due to the Coriolis effect for the first part of the free fall by assuming that Felix reaches the maximum velocity (1358 km/h) in the end, but not reaching the ground yet. To be clear: Felix starts from 39 km at rest (checkpoint 1) and falls freely until maximum velocity (checkpoint 2). How much has it taken time and what is the altitude then? How much is the lateral deflection of the fall at that point with respect to Earth's surface and in which direction? You can neglect air resistance. The latitude of Roswell (New Mexico, USA) is 34 degrees (north), Earth's angular velocity is $\omega = 7.29 \times 10^{-5}$ s^{-1} and the acceleration of gravity is g = 9.81 m/s^2. [5p]

Hint: You can safely ignore the x- and y-components of velocity in the cross-product.

Problem 2. (10 points)

True (T) or false (F). Please explain as half of the points depend on it:

i. We can use the Lagrangian/Hamiltonian formalism for solving some mechanical problems with non-holonomic constraints.

ii. The Lagrangian function is uniquely defined.

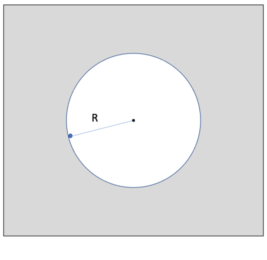
iii. The only central forces that can lead to bound orbits are the Hooke's law and the inverse-square law.

iv. We can solve the Hamilton-Jacobi equation by using Hamilton's characteristic function for the following Hamiltonian (A is a constant)

$$H = \frac{p^2}{2m} - mAtx$$

v. CO_2 is the most severe green-house gas because its concentration is the largest of all chemical species in the atmosphere.

Problem 3. Sliding particle inside a cylinder cavity with water (6 points)



Let us consider a particle (mass m) sliding down a cylinder cavity surface without any friction from the support. The cavity is filled with water. The particle experiences a drag force due to solvent that is directly proportional to velocity and has the form $F = -kmv$, where k is a constant. The radius of the cavity is R. The particle (point) experiences gravity but it has no volume, and you can neglect the buoyancy effect in water.

(a) Choose a convenient generalized coordinate and write out the Lagrangian. Derive the corresponding equation of motion. [3p]

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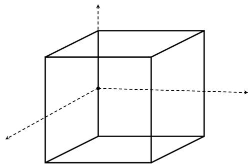
[1] Jumping from fixed objects using a parachute to descent safely to the ground.

[2] Flat Earth Society

(b) Let us assume small displacements around the equilibrium point in the bottom of the cylinder. By using new constants $\delta = k/(2R)$ and $\omega_0 = \sqrt{g/R}$, solve the resulting equation of motion (differential equation). What kind of solutions will you get and what kind of physical behavior they correspond to? [3p]

Problem 4. Inertia tensor of a cube (6 points)

Let us consider a homogeneous cube of density ρ and mass M. Its side length is a.



(a) Calculate the inertia tensor for a case where the fixed point is in the middle of one of its edges and the cube sides are aligned with coordinate axes. [3p]

(b) Solve the principal moments of inertia (eigenvalues). [3p]

Note: We solved the case with corner fixed in the lectures. Use a shorthand notation $b = Ma^2$ once you have solved the tensor elements.

Problem 5. Special theory of relativity – meson decay (5 points)

A meson of mass m_π at rest disintegrates to a meson of mass m_μ and a neutrino ν of effectively zero mass. Show that the kinetic energy of motion of the μ meson is

$$T_\mu = \frac{(m_\pi - m_\mu)^2}{2m_\pi} c^2 \qquad [5p]$$

Hint: We have conservation of both total energy and linear momentum. You can avoid 4-vectors. Note that we assign no mass for neutrino.

GOOD LUCK!