Design of one-dimensional random surfaces with specified scattering properties

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We propose a method for designing a one-dimensional random perfectly conducting surface which, when illuminated by a plane wave, scatters it with a prescribed angular distribution of intensity. The method is applied to the design of a surface that scatters light uniformly within a specified range of scattering angles, and produces no scattering outside this range. It is tested by computer simulations, and a procedure for fabricating such surfaces on photoresist is described. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1495900]

In a recent series of papers the present authors and their colleagues have presented a method for designing one-dimensional random surfaces that scatter light in a specified manner.[1,2] This method is based on expressing the random surface profile function as a superposition of equally spaced trapezoidal grooves, whose statistically independent random amplitudes are drawn from a probability density function (PDF) that is determined in such a way that the mean intensity of the scattered light has the specified angular distribution. It has been shown that such surfaces can be fabricated on photoresist,[1,2] to produce the specified angular distribution of the intensity of the scattered light.[2,3]

In this letter we present a method for designing a one-dimensional random surface that scatters light in a prescribed manner, that is simpler to implement, both theoretically and experimentally than the method of Refs. 1 and 2. Like the latter it is based on the geometrical optics limit of the Kirchhoff approximation for the scattering of s-polarized light incident normally on a perfectly conducting surface. This method is illustrated by applying it to the determination of a surface that scatters light uniformly within a specified range of scattering angles, and produces no scattering outside this range (a band-limited uniform diffuser). It is tested by rigorous computer simulation calculations. Finally, we indicate how the kinds of surfaces generated by our approach can be fabricated on photoresist.

The physical system we initially consider consists of vacuum in the region $x_3 > \zeta(x_1)$ and a perfect conductor in the region $x_3 < \zeta(x_1)$. The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1 that is differentiable, and constitutes a random process. The surface $x_3 = \zeta(x_1)$ is illuminated from the vacuum region by an s-polarized plane wave of frequency ω, whose plane of incidence is the x_1x_3-plane.

Our starting point is the geometrical optics limit of the Kirchhoff approximation for the mean differential reflection coefficient which, in the case of normal incidence, is given by[1]

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \exp(iqu) \times \langle \exp(iau\zeta'(x_1)) \rangle, \tag{1}$$

where L_1 is the length of the x_1-axis covered by the random surface, $q = (\omega/c)\sin\theta_s$, where θ_s is the angle of scattering measured clockwise from the x_3-axis, and $a = (\omega/c)(1 + \cos\theta_s)$. The angle brackets denote an average over the ensemble of realizations of $\zeta(x_1)$. We now define a set of equally-spaced points along the x_1-axis by $x_n = nb$, where b is a characteristic length and $n = 0, \pm1, \pm2, \ldots$. The surface profile function $\zeta(x_1)$ is then given by

$$\zeta(x_1) = a_n x_1 + b_n; \quad nb \leqslant x_1 \leqslant (n+1)b, \tag{2}$$

where the $\{a_n\}$ are independent random deviates. Therefore, the probability density function of a_n,

$$f(\gamma) = \langle \delta(\gamma - a_n) \rangle, \tag{3}$$

is independent of n. In order that the surface be continuous at $x_1 = (n+1)b$, the relation

$$b_{n+1} = b_n - (n+1)(a_{n+1} - a_n)b \tag{4}$$

must be satisfied. From this recurrence relation the $\{b_n\}$ can be determined from a knowledge of the $\{a_n\}$, provided that an initial value, for example, that of b_0, is specified. It is convenient to choose $b_0 = 0$, and we will do so in what follows.

The double integral in Eq. (1) can now be evaluated, with the result

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{1+\cos\theta_s} f\left(\frac{-\sin\theta_s}{1+\cos\theta_s} \right). \tag{5}$$

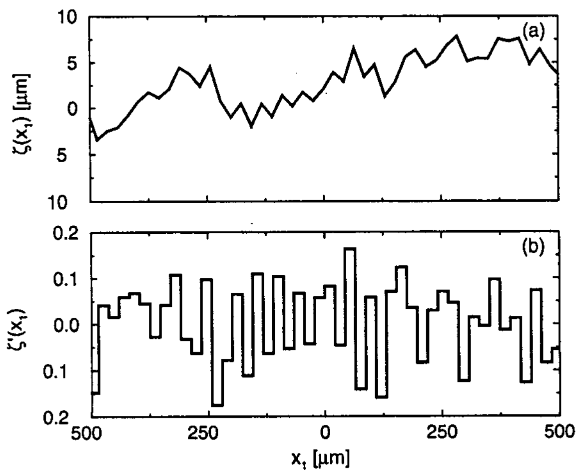

FIG. 1. (a) A segment of the surface profile function $\zeta(x_1)$ for the case of a band-limited uniform diffuser. (b) The derivative $\zeta'(x_1)$ of this surface profile function.

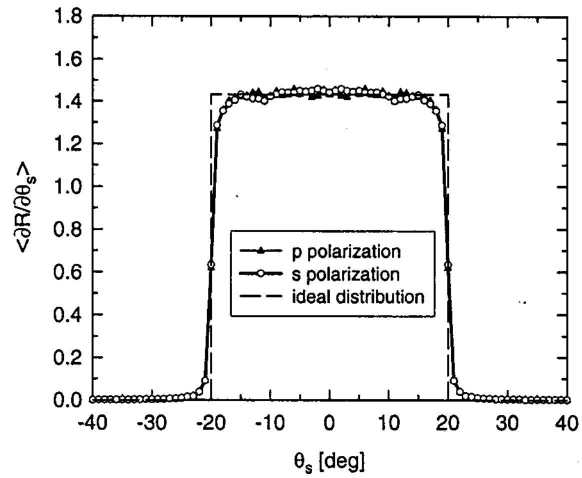

FIG. 2. The mean differential reflection coefficient $\langle \partial R/\partial\theta_s \rangle$ estimated from $N_p = 20\,000$ realizations of perfectly conducting surface profiles: $\theta_m = 20°$, $b = 22$ μm, $\lambda = 632.8$ nm.

Thus, we find that the mean differential reflection coefficient is expressed in terms of the PDF of the random deviate a_n. We now make the change of variable $\sin\theta_s/(1+\cos\theta_s) = \tan(\theta_s/2) = \gamma$ and obtain from Eq. (5) that $f(\gamma)$ is given by

$$f(\gamma) = \frac{2}{1+\gamma^2}\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle(-\gamma). \tag{6}$$

It should be noted that this result does not depend explicitly on the wavelength of the incident light. A long sequence of $\{a_n\}$ is then generated, for example, by the rejection method,[4] and the corresponding sequence of $\{b_n\}$ is obtained from Eq. (4). The surface profile function $\zeta(x_1)$ is then constructed on the basis of Eq. (2).

We illustrate this approach by applying it to the design of a random surface that gives rise to a mean differential reflection coefficient that is a constant in the angular interval $|\theta_s| < \theta_m < \pi/2$, and vanishes for $|\theta_s| > \theta_m$ (a band-limited uniform diffuser),

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{\theta(\theta_m - |\theta_s|)}{2\theta_m} = \frac{\theta(\gamma_m - |\gamma|)}{4\tan^{-1}\gamma_m}, \tag{7}$$

where $\theta(z)$ is the Heaviside unit step function and $\gamma_m = \tan(\theta_m/2)$. We find from Eq. (6) that the PDF of a_n is given by

$$f(\gamma) = \frac{1}{2\tan^{-1}\gamma_m}\,\frac{\theta(\gamma_m - |\gamma|)}{1+\gamma^2}. \tag{8}$$

A segment of the surface profile function $\zeta(x_1)$ and its derivative calculated by the approach proposed here are plotted in Figs. 1(a) and 1(b), respectively. The parameters used in generating these functions were $\theta_m = 20°$ and $b = 22$ μm.

Although the derivation of the PDF $f(\gamma)$ given by Eq. (6) was based on a single-scattering approximation and the assumption of s-polarization, the random surfaces generated by its use retain the scattering properties for which they were designed when multiple scattering is taken into account and the incident light is p-polarized. To demonstrate this, we present in Fig. 2 plots of $\langle \partial R/\partial\theta_s \rangle$ as functions of θ_s for the scattering of both p- and s-polarized light from a one-dimensional perfectly conducting random surface designed to act as a band-limited uniform diffuser with $\theta_m = 20°$, obtained by means of rigorous computer simulation calculations[5] that take into account multiple-scattering processes of all orders. We see that the surface displays the scattering property for which it was designed for both polarizations of the incident light.

In Fig. 3 we present the results of a rigorous computer simulation calculation of $\langle \partial R/\partial\theta_s \rangle$ as a function of θ_s for s-polarized light incident normally on a one-dimensional random silver surface designed to act as a band-limited uniform diffuser with $\theta_m = 20°$. Results are presented for three wavelengths of the incident light: (a) $\lambda = 632.8$ nm (He-Ne laser); (b) $\lambda = 532$ nm (the second harmonic of the YAG laser); (c) $\lambda = 442$ nm (He-Cd laser). These wavelengths cover the entire visible region of the optical spectrum. We see that even when the scattering medium is a finitely conducting metal, a surface ruled on it in accordance with Eqs. (2), (4),

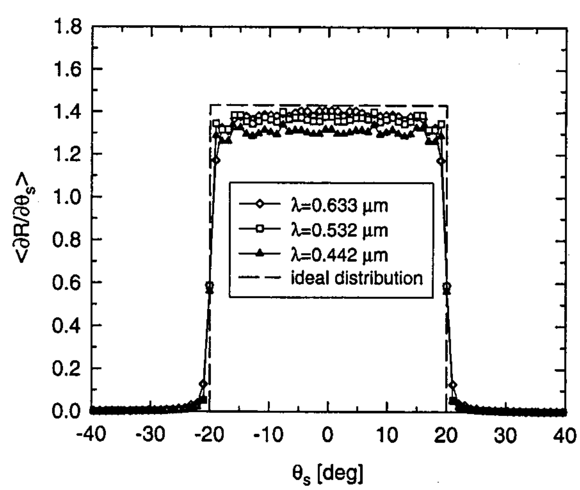

FIG. 3. The mean differential coefficient $\langle \partial R/\partial\theta_s \rangle$ estimated from $N_p = 40\,000$ realizations of metallic surface profiles in s-polarization: $\theta_m = 20°$, $b = 22$ μm.

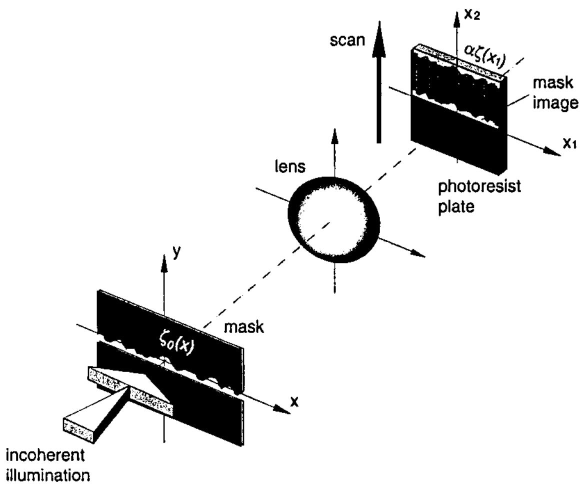

FIG. 4. Schematic diagram of the proposed experimental arrangement for the fabrication of surfaces with $f(\gamma)$ given by Eq. (6). The mask is imaged on the photoresist plate, which is then scanned along x_2.

and (8) still acts as a band-limited uniform diffuser, although with a weak dependence of the scattering pattern on the wavelength of the incident light. We attribute this dependence to the strong wavelength dependence of the dielectric function of silver in the range of wavelengths considered.

We conclude this paper by describing the manner in which one-dimensional surfaces of this kind can be fabricated on photoresist. First, a single realization of a profile function $\zeta_0(x)$, generated in accordance with Eqs. (2), (4), and (6), is used to fabricate a slit of variable width in the manner shown in Fig. 4. A good quality optical system is used to form an incoherent, demagnified image of the slit on the photoresist plate. Assuming that the object is resolvable, we express the intensity image on the photoresist plate as

$$I(x_1,x_2)=I_0\,\theta(x_2+d)\,\theta[\zeta(x_1)-x_2], \tag{9}$$

where I_0 is a constant, the coordinates x_1 and x_2 are fixed on the plate, and $\zeta(x_1)$ is a scaled version of the mask profile $\zeta_0(x)$. The plate is then scanned in the x_2 direction at speed v, producing a total exposure of the form

$$E(x_1)=K\int_{-T/2}^{T/2} I(x_1,x_2+vt)\,dt, \tag{10}$$

where K is a constant related to the sensitivity of the photoresist, $T=L_2/v$ is the time it takes to execute the scan, and the total scan length L_2 is assumed to be greater than the physical size of the plate. Then, the limits in Eq. (10) can be extended to infinity. Substitution of Eq. (9) into Eq. (10) gives

$$E(x_1)=E_0+\alpha\zeta(x_1), \tag{11}$$

where $E_0=KI_0d/v$ and $\alpha=KI_0/v$. This expression shows that the exposure has a linear dependence on the heights of the numerically generated realization of the surface profile function. Assuming that the relation between exposure and height is linear, the surface on the developed plate will have the desired properties. Note also that the vertical scale of the resulting profile can be adjusted through I_0 and the speed of the scan.

Thus, in this letter we have presented a method for generating a one-dimensional random surface that scatters light incident normally on it in such a way that the angular dependence of the intensity of the scattered light has a prescribed form. This method is simpler to implement than the one used in our earlier studies of the same problem.[1,2] Although it is based on a single-scattering approximation, for scattering from a perfectly conducting surface, we have shown by rigorous numerical simulation calculations that the surfaces generated by this approach yield the desired scattering pattern for both p and s polarizations of the incident light, when multiple scattering is taken into account, and when the perfect conductor is replaced by a finitely conducting metal as well. In addition, we have shown that the resulting scattering patterns are virtually independent of the wavelength of the incident light over a broad spectral range. Finally, we have described a procedure for fabricating such surfaces on photoresist.

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