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LETTER

Synchronization model for stock market asymmetry

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Abstract. The waiting time needed for a stock market index to undergo a given percentage change in its value is found to have an up–down asymmetry, which, surprisingly, is not observed for the individual stocks composing that index. To explain this, we introduce a market model consisting of randomly fluctuating stocks that occasionally synchronize their short term draw-downs. These synchronous events are parametrized by a 'fear factor', that reflects the occurrence of dramatic external events which affect the financial market.

Keywords: models of financial markets, stochastic processes

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The value of stocks varies from day to day, both relative to each other but also due to collective movements of the overall market. These variations of the market presumably reflect the psychological state of the surrounding society as affected by current events. An analysis technique based on inverse statistics has recently been applied to study the variation of stock indices, single stocks and exchange rates [1]–[3]. In the time dependent inverse statistics approach, one fixes a predetermined level of return (ρ), and, as explained in figure 1(a), asks for the waiting time needed to reach this level for the *first* time. Averaging over many investment events results in a histogram of waiting times.

The DJIA is an average over 30 of the most important stocks of the US market chosen from different sectors of the industry. When the gain (+)/loss (−) return levels are set to $\rho = \pm 5\%$, figure 1(b) shows the histograms obtained for the DJIA daily closing values over its entire 110 years of history. The histograms possess well defined and pronounced maxima, the so-called *optimal investment horizons* [1], followed by long power law tails. These $1/t^\alpha$ tails, with $\alpha \approx 3/2$, are well understood, and are a consequence of the uncorrelated increments of the underlying asset price process [4]–[7]. However, the interesting observation to be made from figure 1(b) is that the optimal investment horizons corresponding to the same return level magnitudes but opposite signs are *different*. More specifically, for $\rho = 5\%$ the maximum occurs after around 15 days, while for the mirrored (loss) case of $\rho = -5\%$ it occurs after about 8 days. Thus the market as a whole, as monitored by the DJIA, exhibits a fundamental *gain–loss asymmetry*. Other indices, such as SP500 and NASDAQ, also show this asymmetry [8], while, for instance, foreign exchange data do not [3].

The striking paradox is that a similar well pronounced asymmetry is not found for any of the individual stocks that constitute the DJIA [8]. This can be observed from the insert of figure 1(b), which shows the results of applying the same procedure, individually, to these stocks, and subsequently averaging over them to improve the statistics. The figure illustrates that single stocks show inverse statistic histograms that are similar to the DJIA index, but with the important difference that there is no asymmetry between gains and losses. How is it possible that the index exhibits a pronounced asymmetry while the individual stocks do not?

Motivated by numerous empirical studies, the classic assumption in theoretical finance is that stock and market prices are approximated by a *geometrical Brownian motion* [4, 5, 9, 10], i.e. the logarithm of the stock price is consistent with a standard, unbiased, random walk. We will adopt this assumption in the following. Moreover, it will be assumed, as is consistent with empirical findings [4, 5], that the stock price increments are small compared to the stock price level ensuring that the logarithmic return distribution is symmetric. Under these assumptions a single stock will not show any gain–loss asymmetry. Thus, we will have to understand how the average of many individual (log-normal distributed) stocks can collectively add up to exhibit a gain–loss asymmetry for the resulting index that was not present for the constituting single stocks. The prime idea is to introduce occasional *synchronous events* among the individual stocks, i.e. a collective phenomenon. To this end, we introduce a model consisting of N log-normally distributed stocks of price $S_i(t)$ ($i = 1, \ldots, N$) that, at each (discrete) time step, t, can adjust their logarithmic prices $s_i(t) = \ln S_i(t)$ up or down by a certain amount $\delta > 0$, which for simplicity is assumed to be constant, but with the direction $\varepsilon_i(t) = \pm 1$ chosen

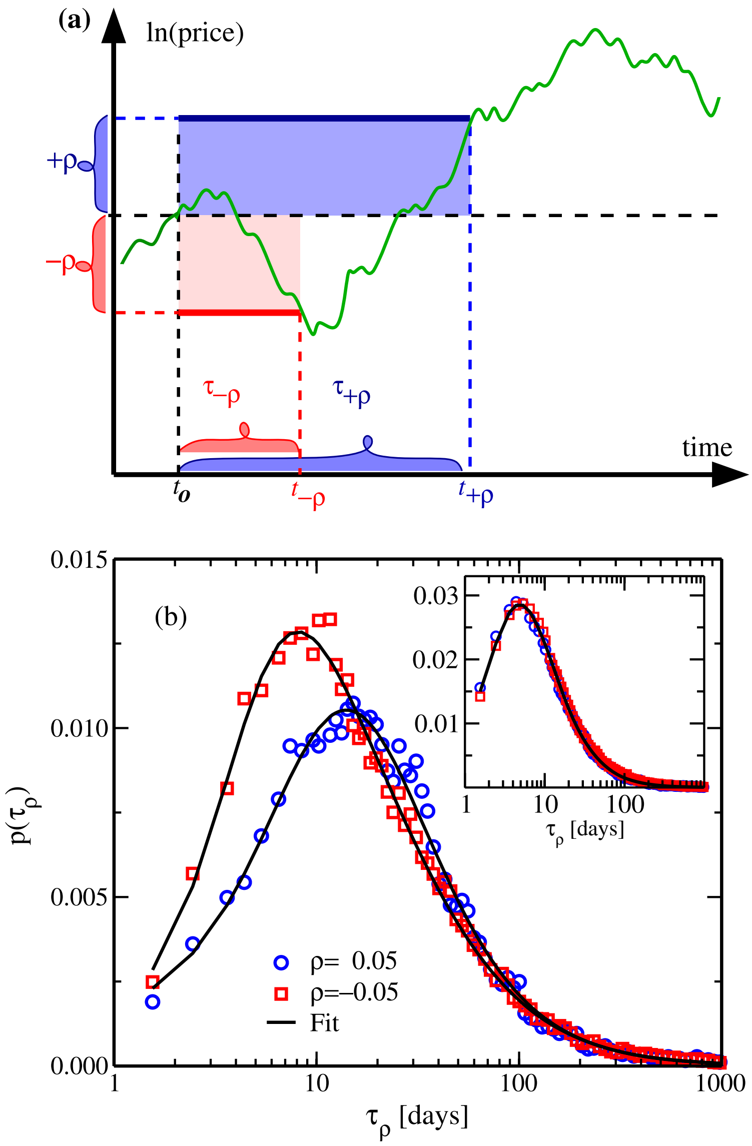

Figure 1. (a) Schematic picture of typical stock or index log-price variations with time. The predetermined return levels for gain (blue area)/loss (red area) are set to $+\rho > 0/-\rho < 0$. The corresponding investment waiting times (τ_ρ) are found by estimating where the horizontal $\pm\rho$ lines cross the logarithmic price curve for the *first* time (t_ρ), resulting in $\tau_\rho = t_\rho - t_0$. (b) The panel shows histograms of the inverse statistics for the DJIA obtained on the basis of the empirical daily close data covering its entire history of 110 years. The red data points are obtained using a loss level of $\rho = -0.05$ and the blue points are obtained using a gain level of $\rho = +0.05$ and both distributions are normalized. Note the clear asymmetry between the loss and the gain statistics. The full curves are fits using generalized inverse gamma distributions [1, 2]. The inset is the distribution obtained from using the same procedure on the individual stocks of the DJIA, and subsequently averaging over the stocks. Notice that the asymmetry is absent for individual stocks.

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randomly:

$$s_i(t+1) = s_i(t) + \varepsilon_i(t)\delta. \tag{1}$$

Notice that this update rule implies that the logarithmic return is

$$r_i(t) = \ln(S_i(t+1)/S_i(t)) = \varepsilon_i(t)\delta. \tag{2}$$

With such a price process applying to all the constituting stocks, the value of the (price-weighted) stock index (like the DJIA) is calculated according to

$$I(t) = \frac{1}{d(t)} \sum_{i=1}^{N} S_i(t) = \frac{1}{d(t)} \sum_{i=1}^{N} \exp s_i(t), \tag{3}$$

where $d(t)$ denotes the divisor of the index (at time t). This quantity is adjusted over time to take into account structural changes on the index, for instance stock splits and mergers. However, in this work, for simplicity, we have not considered such possibilities and instead have fixed its value to $d(t) = N$ (the initial value originally used by the DJIA). This price-weighted way of calculating the index, as already mentioned, is consistent with the DJIA, but is, however, more the exception than the rule. A more common scenario is construction of the index from the sum of the capitalizations of the constituent companies (a market capitalization-weighted index). This is obtained by summing, for each company, the product of the number of shares and the share price. This is the way that e.g. the NASDAQ and the SP500 indices are calculated. Notice, however, that the gain–loss asymmetry does not depend on the way that the index is calculated, since the same type of behaviour is found in both cases.

With these definitions in place, the increments of the index itself between two consecutive days can be written as

$$\Delta I(t) = I(t+1) - I(t) = \frac{1}{d(t)} \sum_{i=1}^{N} S_i(t) \left[\mathrm{e}^{\varepsilon_i(t)\delta} - 1 \right], \tag{4}$$

where we have simply substituted the expression (1) into the definition (3). Notice that this expression contains a price dependent weight factor that comes about due to the geometric Brownian motion assumption for the stock prices. In the limit of small δ, the expression for the increments of the index may be well approximated by its first-order expansion, so that

$$\Delta I(t) \simeq \frac{\delta}{d(t)} \sum_{i=1}^{N} \varepsilon_i(t)\, S_i(t), \qquad \delta \ll 1. \tag{5}$$

Synchronization is introduced into the model via simultaneous down movements of all stocks at some time steps. The frequency of such events is given by a 'fear factor' parameter p. Therefore, at each time step, with probability p *all* stocks move down synchronously, i.e. for that time step $\varepsilon_i = -1$ for all i, and with probability $1-p$ each stock makes an independent and random adjustment to its logarithmic stock price. The process is illustrated in figure 2(a). To guarantee that the logarithmic prices of the individual stocks behave like standard random walks without any drift, the forced down movements are compensated with a slight tendency of up movements in the calm periods between

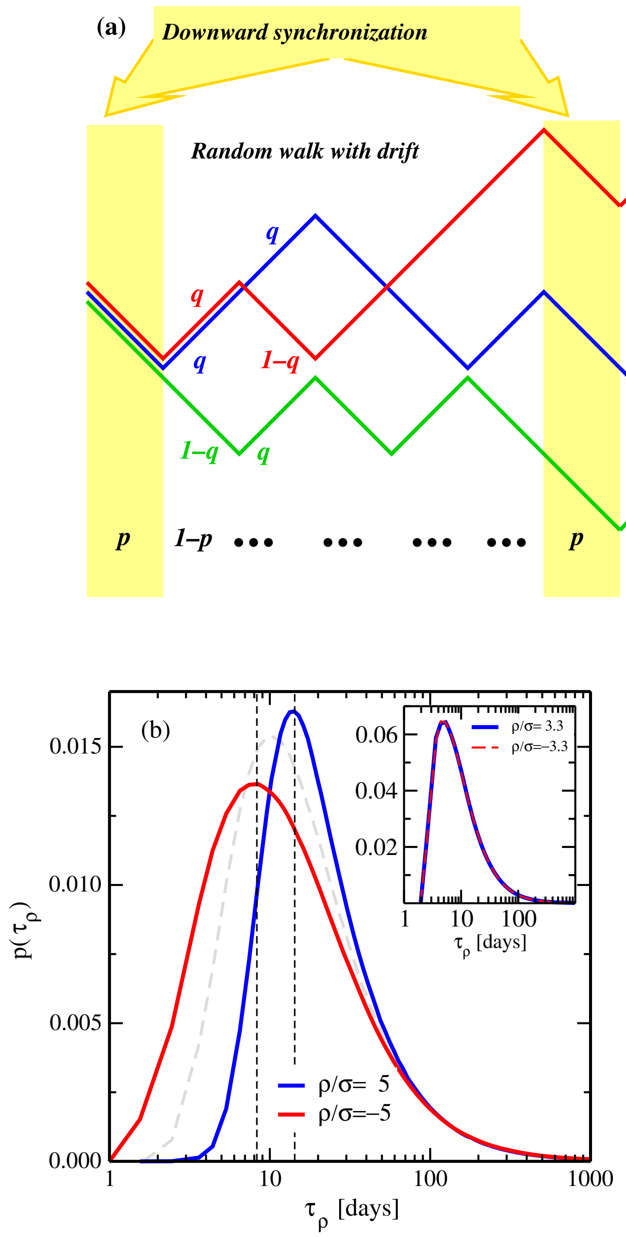

Figure 2. The asymmetric synchronous model. (a) The panel illustrates the time evolution of three stocks, which fall simultaneously with probability p, or move as (biased) random walkers with probability $1-p$ (see the text for additional details). (b) The inverse statistics obtained within this model, for an index consisting of $N = 30$ stocks and a ratio $\rho/\sigma = 5$, where ρ is the return level and σ denotes the daily volatility of the index. In the model the index volatility, σ, should reflect the observed 1% daily volatility of the DJIA, and the $\rho/\sigma = 5$ therefore corresponds to $\rho = 5\%$ in figure 1. A fear factor of $p = 0.05$ was chosen to reproduce the positions of the two asymmetric maxima appearing in figure 1(b) and indicated by dashed vertical lines. The dashed thick line is the result for a fear factor parameter $p = 0$, in which case the asymmetry vanishes. As in figure 1(b), the inset shows the loss and gain distributions for the individual stocks in the model. Notice that here the asymmetry is also absent.

synchronized downwards events. That is, on a day without synchronized movements, the chance for a stock of moving up q is slightly bigger than its chance of moving down, $1-q$. These latter situations correspond to $\varepsilon_i = 1$ and $\varepsilon_i = -1$, respectively. Notice that the precise value of q depends on the fear factor p, and is determined by equating the probabilities of up and down movements. The probability of moving up, $(1-p) \cdot q$, must therefore equal the probability of moving down, $p + (1-p) \cdot (1-q)$, implying that

$$q = \frac{1}{2(1-p)}. \tag{6}$$

We stress that a single stock generated in this way, by construction, will show no gain–loss asymmetry. This is exemplified by e.g. the inset to figure 2(b).

The fear factor parameter p reflects a collective anxiety state of the investors, probably triggered by unexpected events. But how often do such events occur? We have found that a value $p = 0.05$, that corresponds to one collective event every month or so, reproduces the empirical asymmetry. Figure 2(b) depicts the inverse statistics of the model shown in figure 2(a). In obtaining these results we used a fear factor of $p = 0.05$, $N = 30$ stocks, and the return level was set to $\rho/\sigma = \pm 5$, where σ denotes the daily volatility of the index[5]. For the DJIA, the daily volatility is about 1%, and hence the model value $\rho/\sigma = 5$ should be comparable to the $\rho = 5\%$ for the DJIA index used in obtaining the empirical results of figure 1(b). Moreover, it should be noted that when $p = 0$ no asymmetry is predicted by the model (dashed line in figure 2(b)).

Figure 2(b) shows that the model results in a clear gain–loss asymmetry that is qualitatively very similar to what is found empirically for the DJIA (cf figure 1(b)). In particular, the empirical peak positions are determined rather accurately by the model, as indicated by the vertical dashed lines in figure 2(b). A detailed comparison of the shapes of the empirical and the modelled inverse statistics curves reveals some minor differences, especially for short waiting times. One could find simple explanations for these differences, such as the fact that the model does not consider a realistic jump size distribution, or even that it does not include an 'optimism factor' synchronizing draw-ups. This would result in a wider $\rho > 0$ distribution for short waiting times, and additionally would lower the value of the maximum. However, we have chosen not to include any of these additional issues in the phenomenological model, in order to keep it as simple and transparent as possible, and since it serves well our main aim, which is to address the origin of the asymmetry.

In principle it would be possible to estimate the value of the fear factor p (if it is time independent) from time series of the individual stocks constituting the DJIA. One would have to identify the down movements of each individual stock, estimate the (zero-time-lag) correlations between those events in the different stocks and compare them to the corresponding up trends. This would require long time series of prices for each stock with corresponding time stamps. However, such a procedure is in practice daunting due to the changing composition of the DJIA and the splits and mergers of its stocks as reflected in the time dependent divisor factor used to calculate the value of the index. Therefore, we do not present data for such correlation estimates.

[5] The input parameter of the model was in practice the single-stock volatility (and not that of the index). The volatility of the statistically identical single stocks was adjusted, given the other parameters of the model, to produce the desired volatility of the index σ.

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Our model opens for additional investigations into the effects of the many small synchronous events in the market ('mini-crashes'). In particular, we have studied the probability that the DJIA index goes down or up over a day and have found that there is a 9% larger probability of going down than of going up [11].

This is in perfect agreement with the model where the index has a larger probability of going down because of the synchronizing draw-down events, as quantified by the fear factor p. Moreover, we have found overall quantitative agreement between the empirical DJIA data and the model (with the parameters given above) for the probability of moving up/down for M consecutive days [11]. The peak positions in figures 1(b) and 2(b) are obviously related to the value chosen for ρ. As ρ increases, the peaks move to longer times [1], and their amplitudes decrease.

One might speculate on whether the observed asymmetry could be used to generate profit. It cannot (we believe)! A call (put) option contract gives the holder the right to buy (sell) and obliges the writer to sell (buy) a specified number of shares at a specified strike price, any time before its expiry date. If we implemented a strategy based on a put option at the current price eight days from now (corresponding to the maximum loss curve), and a call option at the current price 15 days from now (corresponding to the maximum probability of gain curve), one can demonstrate that the expected long term gain is mathematically identical to a straightforward hold position. Obviously, the cost of buying the options and any additional transaction costs would render the use of our observed asymmetry unprofitable.

The asymmetry of markets reflects an inherent difference between the value of money and the value of stocks, where crashes reflect the tendency of people to believe in money, rather than stocks, during crises. On this perspective it is interesting to notice that it is the index, i.e. the value of all stocks, that is systematically vulnerable relative to the more fluid money. The buying power of money is complementary to the value of stocks [12], and thus exhibits a mirrored asymmetry with a tendency of an increased buying power for money relative to index at short times. In periods of fear, people prefer money as the more certain asset, while calm periods are characterized by random reshuffling of agent's stock assets with a tendency to push stock values upwards.

We conclude that the asymmetric synchronous market model captures basic characteristic properties of the day-to-day variations in stock markets. The agreement between the empirically observed data, here exemplified by the DJIA index, and the parallel results obtained for the model gives credibility to the point that the presence of a 'fear factor' is a fundamental social ingredient in the dynamics of the overall market.

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