Random surfaces that suppress single scattering

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We present a method for numerically generating a one-dimensional random surface, defined by the equation $x_3 = \zeta(x_1)$, that suppresses single-scattering processes in the scattering of light from the surface within a specified range of scattering angles. Rigorous numerical calculations of the scattering of light from surfaces generated by this approach show that the single-scattering contribution to the mean scattered intensity is indeed suppressed within that range of angles. © 1999 Optical Society of America

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In theoretical and experimental studies of multiple-scattering effects in the scattering of light from randomly rough surfaces it is often desirable to be able to suppress the contribution to the mean scattered intensity from single-scattering processes: Effects such as enhanced backscattering or the presence of satellite peaks become more readily observable in the absence of the background provided by single-scattering processes.

In theoretical studies it is possible to separate the contribution of single-scattering processes to the mean intensity of the light that is scattered incoherently from the contribution of multiple-scattering processes.[1,2] However, it is not so easy to achieve experimentally. In the case of scattering of light from two-dimensional random surfaces the in-plane, cross-polarized scattering of p-polarized light suppresses the single-scattering contribution to the mean intensity of the light that is scattered incoherently. In the case of scattering of light that is incident normally upon a weakly rough one-dimensional random metal surface the use of a surface whose roughness is characterized by a power spectrum $g(|k|)$ that vanishes identically for $|k| < k_{\min} \leq \omega/c$ eliminates the contribution of single-scattering processes to the mean intensity of the incoherent component of the scattered light for scattering angles that are smaller in magnitude than $\sin^{-1}(ck_{\min}/k\omega)$.[3] However, such surfaces are difficult to fabricate.

In this Letter we explore a different approach to the design of random surfaces that suppress the single-scattering contribution to the incoherent component of the light that is scattered from them. This approach is not restricted to weakly rough surfaces, and such surfaces appear to be easier to fabricate than surfaces characterized by a West–O'Donnell[3] power spectrum.

To explain the motivation for this approach, let us consider the scattering of an s-polarized plane wave of frequency ω from a one-dimensional, perfectly conducting random surface, where the plane of incidence is perpendicular to the generators of the surface. We recall that, if the inhomogeneous Fredholm equation for the normal derivative of the single nonzero component of the electric field in vacuum (evaluated on the surface) is solved by iteration, the inhomogeneous term yields the Kirchhoff approximation to the mean scattered intensity (a single-scattering approximation), the first iterate yields the pure double-scattering contribution, and so on.[2] Consequently, if a surface can be designed with the property that the Kirchhoff approximation to the mean intensity of the light that is scattered from it vanishes for the scattering angle θ_s in the interval $(-\theta_m,\ \theta_m)$, all the scattered intensity within this range of scattering angles will be the result of multiple-scattering processes. Consequently, our aim is to design a one-dimensional, perfectly conducting random surface for which the Kirchhoff approximation to the mean differential reflection coefficient vanishes identically for θ_s in the interval $(-\theta_m,\ \theta_m)$. We simplify the required analysis significantly by working in the geometrical-optics limit of the Kirchhoff approximation. However, the results still display the behavior that was sought.

Thus, we consider a one-dimensional, randomly rough, perfectly conducting surface defined by $x_3 = \zeta(x_1)$ that is illuminated by an s-polarized plane wave of frequency ω. The surface-profile function $\zeta(x_1)$ is[4]

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$$\zeta(x_1) = \sum_{l=-\infty}^{\infty} c_l s(x_1 - l2b), \tag{1}$$

where $\{c_l\}$ are independent, positive, random deviates, b is a characteristic length, and the function $s(x_1)$ is defined by[4]

$$s(x_1) = \begin{cases} 0 & x_1 < -(m+1)b \\ -(m+1)bh - hx_1 & -(m+1)b < x_1 < -mb \\ -bh & -mb < x_1 < mb \\ -(m+1)bh + hx_1 & mb < x_1 < (m+1)b \\ 0 & (m+1)b < x_1 \end{cases}, \tag{2}$$

where m is a positive integer. Owing to the positivity of the coefficient c_l, its probability-density function (pdf) $f(\gamma) = \langle \delta(\gamma - c_l)\rangle$ is nonzero only for $\gamma > 0$.

It has been shown that for the random surfaces defined by Eqs. (1) and (2) the mean differential reflection coefficient in the geometrical-optics limit of the Kirchhoff approximation is given by[4]

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{2h} \frac{[1 + \cos(\theta_0 + \theta_s)]^2}{\cos\theta_0(\cos\theta_0 + \cos\theta_s)^3} \times \left\{ f\left[\frac{\sin\theta_0 - \sin\theta_s}{h(\cos\theta_0 + \cos\theta_s)}\right] + f\left[\frac{\sin\theta_s - \sin\theta_0}{h(\cos\theta_0 + \cos\theta_s)}\right]\right\}, \tag{3}$$

where θ_0 and θ_s are the angles of incidence and scattering, measured counterclockwise and clockwise from the normal to the mean scattering surface, respectively. Equation (3) is simplified greatly in the case of normal incidence ($\theta_0 = 0°$):

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \left(1 + \tan^2 \frac{\theta_2}{2}\right) \times \frac{f\left(-\frac{1}{h}\tan\frac{\theta_s}{2}\right) + f\left(\frac{1}{h}\tan\frac{\theta_s}{2}\right)}{4h}, \tag{4}$$

and we restrict ourselves to this case in what follows. From Eq. (4) we find that if we wish $\langle \partial R/\partial\theta_s\rangle$ to have, say, the form

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \begin{cases} 0 & 0 < |\theta_s| < \theta_m \\ \dfrac{\cos\theta_s}{2(1 - \sin\theta_m)} & \theta_m < |\theta_s| < \pi/2 \end{cases}, \tag{5}$$

we must choose, for $f(\gamma)$,

$$f(\gamma) = \begin{cases} 0 & 0 < \gamma < \gamma_m \\ 2h \dfrac{1 + h^2{\gamma_m}^2}{(1 - h\gamma_m)^2} \dfrac{1 - h^2\gamma^2}{(1 + h^2\gamma^2)^2} & \gamma_m < \gamma < \dfrac{1}{h} \end{cases}, \tag{6}$$

where $\gamma_m = [\tan(\theta_m/2)]/h$. From this form for $f(\gamma)$ a long sequence of $\{c_l\}$ can be generated, e.g., by the rejection method,[5] and the surface-profile function generated by use of Eqs. (1) and (2).

The surface-profile functions $\zeta(x_1)$ generated in this way are not zero-mean Gaussian random processes and are not stationary. Indeed, the mean-square height of the surface, $\delta^2 = \langle\zeta^2(x_1)\rangle - \langle\zeta(x_1)\rangle^2$, is a periodic function of x_1 with a period $2b$ and for $m = 1$ is given by $\delta^2 = [\langle c^2\rangle - \langle c\rangle^2]h^2b^2[1 + (x_1/b)^2]$ for $-b \le x_1 \le b$. The average of this function over a period, ${\delta_{\mathrm{av}}}^2 = [\langle c^2\rangle - \langle c\rangle^2]4h^2b^2/3$, can be used to estimate the rms height of the surface. Similarly, the mean-square slope of the surface is given by $s^2 = \langle[\zeta'(x_1)]^2\rangle - \langle\zeta'(x_1)\rangle^2 = [\langle c^2\rangle - \langle c\rangle^2]h^2$, from which the rms slope can be determined. The averages $\langle c\rangle$ and $\langle c^2\rangle$ that appear in these expressions, the first two moments of $f(\gamma)$, are given by

$$\langle c\rangle = \frac{1}{h}\frac{1 + h^2{\gamma_m}^2}{(1 - h\gamma_m)^2}\left[\cos\theta_m + 2\ln\frac{\cos(\pi/4)}{\cos(\theta_m/2)}\right], \tag{7a}$$

$$\langle c^2\rangle = \frac{2}{h^2}\frac{1 + h^2{\gamma_m}^2}{(1 - h\gamma_m)^2}\left\{\frac{\pi}{2} - \theta_m + \tan\left(\frac{\theta_m}{2}\right) \times \left[1 + \cos^2\left(\frac{\theta_m}{2}\right)\right] - \frac{3}{2}\right\}. \tag{7b}$$

An example of a surface generated in this way is presented in Fig. 1. The pdf $f(\gamma)$ used in the generation of this surface is defined by Eq. (6), with $\theta_m = 40.1°$. The parameters entering the definition of the function $s(x_1)$ are $b = 3\lambda$, $m = 3$, and $h = 0.2$. For these values of the parameters we find that $\delta_{\mathrm{av}} = 0.9\lambda$ and $s = 0.57$, so the surface is moderately rough. The surface was sampled at points $x_p = [(p + 1/2)b]/N$, where $p = 0, \pm1, \pm2, \ldots$ and $N = 100$ and can be seen to consist of a succession of triangular peaks and valleys.

To show that this random surface suppresses single-scattering processes for $|\theta_s| < 40.1°$, in Fig. 2 we plot the contribution to the mean differential reflection coefficient of the incoherent component of the scattered light, $\langle\partial R/\partial\theta_s\rangle_{\mathrm{incoh}}$, for scattering from this surface, calculated by a computer-simulation approach in the Kirchhoff approximation,[2] with and without invoking the geometrical-optics limit of the latter. We obtained these numerical results by averaging over 2000 surface realizations. It can be seen that in the

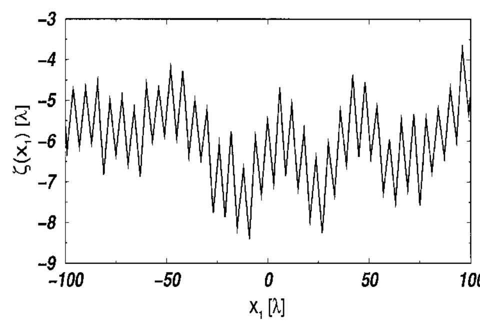

Fig. 1. One-dimensional random surface-profile function $\zeta(x_1)$ obtained from Eq. (1) by the use of the pdf given by Eq. (6), together with a function $s(x_1)$ defined by Eq. (2), with $b = 3\lambda$, $m = 3$, $h = 0.2$, and $\theta_m = 40.1°$.

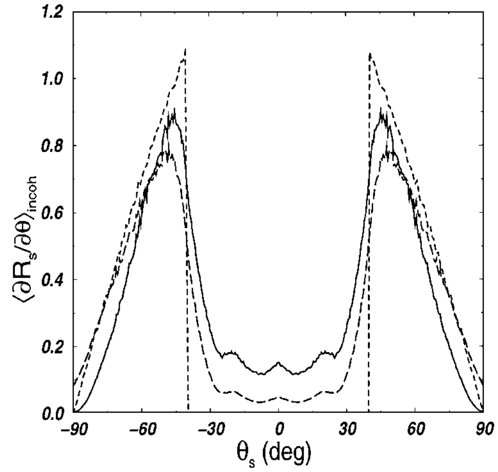

Fig. 2. $\langle \partial R/\partial\theta_s\rangle_{\text{incoh}}$ calculated by a computer-simulation approach for the random surface displayed in Fig. 1, when s-polarized light of wavelength λ is incident normally upon it. Short-dashed curve, geometrical-optics limit of the Kirchhoff approximation; long-dashed curve, Kirchhoff approximation; solid curve, result with all multiple-scattering contributions included.

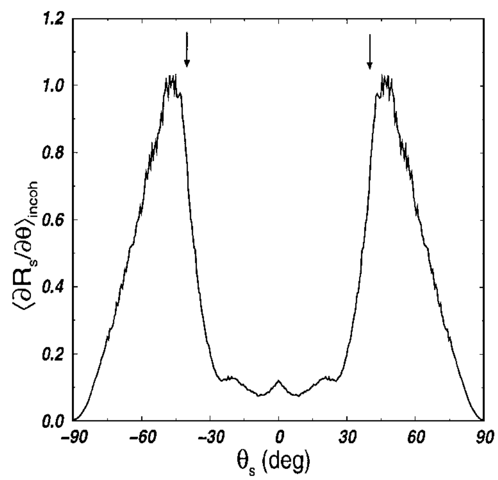

Fig. 3. $\langle \partial R/\partial\theta_s\rangle_{\text{incoh}}$ calculated by a computer-simulation approach for the case in which s-polarized light of wavelength $\lambda = 612.7$ nm is incident normally upon a one-dimensional random silver surface. The arrows indicate the positions of the angles $\pm\theta_m$.

geometrical-optics limit of the Kirchhoff approximation $\langle \partial R/\partial\theta_s\rangle_{\text{incoh}}$ vanishes for $|\theta_s| < 40.1°$. In the Kirchhoff approximation $\langle \partial R/\partial\theta_s\rangle_{\text{incoh}}$ is not identically zero in this region of scattering angles but is quite small. The difference between these two results shows how well the geometrical-optics limit of the Kirchhoff approximation reproduces the result of the Kirchhoff approximation itself. In Fig. 2 we also plot the total contribution to the mean differential reflection coefficient from the incoherent component of the scattered light, including all multiple-scattering contributions. This result for $\langle \partial R/\partial\theta_s\rangle_{\text{incoh}}$ was calculated exactly by a computer-simulation approach.[2] One can see from Fig. 2 that there is now a low background for $|\theta_s| < 40.1°$, owing to multiple scattering, upon which is superimposed an enhanced backscattering peak in the retroreflection direction ($\theta_s = 0°$), whose height is nearly twice that of the background at its position. The latter result is expected when the contribution from single-scattering processes has been subtracted.[6]

Although the theory underlying the approach to generating random surfaces that suppress single scattering that has been presented here was based on the assumption that the scattering surface is perfectly conducting, the resulting approach also works well for finitely conducting surfaces. In Fig. 3 we plot a rigorous computer-simulation result for $\langle \partial R/\partial\theta_s\rangle_{\text{incoh}}$ in the case in which s-polarized light of wavelength $\lambda = 612.7$ nm is incident normally upon a one-dimensional random silver surface $[\epsilon(\omega) = -17.2 + i0.498]$ defined by the parameters described above. The strong suppression of $\langle \partial R/\partial\theta_s\rangle_{\text{incoh}}$ in the interval $|\theta_s| < 40.1°$ can be clearly seen, and an enhanced backscattering peak at $\theta_s = 0°$ rises to approximately twice the height of the background at its position.

In this Letter we have presented a method for generating numerically a one-dimensional random surface-profile function $\zeta(x_1)$ that suppresses single scattering of s-polarized light from it, a property that in the case of a perfectly conducting surface is independent of the wavelength of the incident light. The extension of the present approach to the generation of one-dimensional random surfaces that suppress the single scattering of p-polarized light is straightforward. This method is not restricted to generation of weakly rough surfaces. Surfaces defined by Eqs. (1) and (2), with a different form of the pdf $f(\gamma)$, have been fabricated successfully in the laboratory,[4] and their fabrication appears to be simpler than that of surfaces characterized by a West–O'Donnell[3] power spectrum.

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