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Inverse statistics in economics: the gain–loss asymmetry

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Abstract

Inverse statistics in economics is considered. We argue that the natural candidate for such statistics is the investment horizons distribution. This distribution of waiting times needed to achieve a predefined level of return is obtained from (often detrended) historic asset prices. Such a distribution typically goes through a maximum at a time called the *optimal investment horizon*, τ_ρ^*, since this defines the most likely waiting time for obtaining a given return ρ. By considering equal positive and negative levels of return, we report on a quantitative gain–loss asymmetry most pronounced for short horizons. It is argued that this asymmetry reflects the market dynamics and we speculate over the origin of this asymmetry.

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Financial time series have been recorded and studied for many decades. With the appearance of the computer, this development has accelerated, and today large amounts of financial data are recorded daily. These data are used in the financial industry for statistical studies and for benchmarking. In particular, they can be used to measure the performance of a financial instrument. Traditionally this has been done by studying the distribution of returns [1–3] calculated over a *fixed* time period Δt. Such distributions measure how much an initial investment, made at time t, has gained or lost by the

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time $t + \Delta t$. Numerous empirical studies have demonstrated that for not too large Δt's, say from a few seconds to weeks, the corresponding (return) distributions are characterized by so-called fat tails [1–4]. This is to say that the probability for large price changes are much larger than what is to be expected from Gaussian statistics, an assumption typically made in theoretical and mathematical finance [1–3]. However, as Δt is increased even further, the distribution of returns gradually converge to the Gaussian distribution.

In the context of economics, it was recently suggested [5], partly inspired by earlier work in turbulence [6], to alternatively study the distribution of waiting times needed to reach a *fixed* level of return. These waiting times, for reasons to be clarified in the discussion below, were termed *investment horizons*, and the corresponding distributions the *investment horizon distributions*. Furthermore, it was shown for positive levels of return, that the distributions of investment horizons had a well-defined maximum followed by a power-law tail scaling like [2] $p(t) \sim t^{-3/2}$. The maximum of this distribution signifies the *optimal investment horizon* for an investor aiming for a given return.

In order to present the method, let us start by letting $S(t)$ denote the asset price. Then the logarithmic return at time t, calculated over a time interval Δt, is defined as [1–3]

$$r_{\Delta t}(t) = s(t + \Delta t) - s(t) , \tag{1}$$

where $s(t) = \ln S(t)$. Hence the log-return is nothing but the log-price change of the asset. We consider a situation where an investor is aiming for a given return level denoted ρ, which may be both positive (being "long" on the market) or negative (being "short" on the market). If the investment is made at time t, then the investment horizon is defined as the time $\tau_\rho(t) = \Delta t$ so that the inequality $r_{\Delta t}(t) \geqslant \rho$ when $\rho \geqslant 0$, or $r_{\Delta t}(t) \leqslant \rho$ when $\rho < 0$, is satisfied for the *first* time. The investment horizon distribution, $p(\tau_\rho)$, is then the distribution of investment horizons τ_ρ (see Fig. 1) averaged over the data.

A classic assumption made in theoretical finance is that the asset prices follow a geometrical Brownian motion, i.e., $s(t) = \ln S(t)$ is just a Brownian motion. For a Brownian motion, the solutions to the investment horizon (first passage time) problem is known analytically [7,8]. It can be shown that the investment horizon distribution is given by the Gamma-distribution: $p(t) = |a| \exp(-a^2/t)/(\sqrt{\pi} t^{3/2})$, where $a \propto \rho$. Note, that in the limit of large (waiting) times, one recovers the well-known first return probability $p(t) \sim t^{-3/2}$. As the empirical logarithmic stock price process is known not to be Brownian [1–4], we instead suggest to use a generalized (shifted) Gamma distribution of the form:

$$p(t) = \frac{\nu}{\Gamma(\alpha/\nu)} \frac{|\beta|^{2\alpha}}{(t + t_0)^{\alpha+1}} \exp\left\{ -\left(\frac{\beta^2}{t + t_0} \right)^{\nu} \right\} \tag{2}$$

[2] Notice that this scaling behavior implies that the first (average investment horizon), and higher, moments of this distribution do not exist.

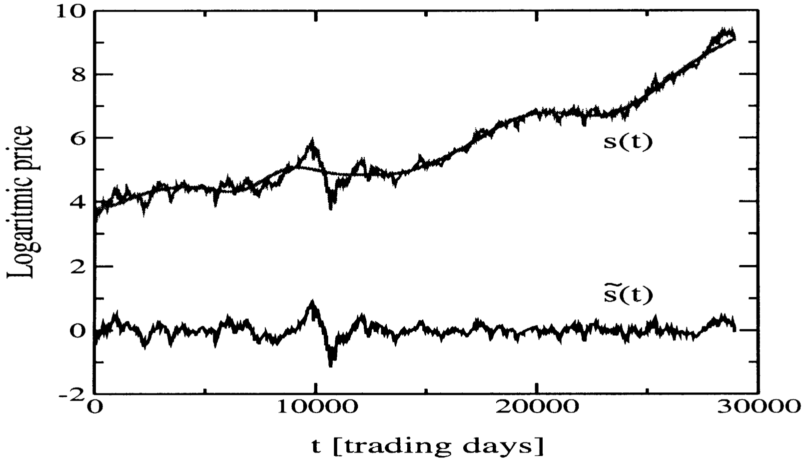

Fig. 1. The historic daily closure prices, $S(t)$, of the Dow Jones industrial average (DJIA) over the period from May 26, 1896 to June 5, 2001. The upper curly curve is the raw logarithmic DJIA price $s(t) = \ln S(t)$, while the smooth curve represents the drift on a scale larger then 1000 trading days. The lower curly curve represents the wavelet filtered logarithmic DJIA data, $\tilde{s}(t)$, defining the fluctuations of $s(t)$ around the drift.

as a basis for fitting the empirical investment horizon distributions. It will be seen below, that this form parametrize the empirical data excellently. Note, that the distribution, Eq. (2), reduces to the Gamma-distribution (given above) in the limit of $\alpha = 1/2, \beta = a, \nu = 1$, and $t_0 = 0$. Furthermore, the maximum of this distribution, i.e., the *optimal investment horizon*, is located at $\tau_\rho^* = \beta^2(\nu/(\alpha+1))^{1/\nu} - t_0$ for a given level of return ρ. If the underlying asset price process is geometric Brownian, then one would have $\tau_\rho^* \sim \rho^2$ for *all* values of ρ. We will later see that this is far from what is observed empirically.

It is well-known that many historic financial time series posses an (often close to exponential) positive drift over long time scales. If such a drift is present in the analyzed time series, one can obviously not compare directly the histograms for positive and negative levels of return. Since we in this paper mainly will be interested in making such a comparison, one has to be able to reduce the effect of the drift significantly. One possibility for detrending the data is to use deflated asset prices. However, in the present study we have chosen an alternative strategy for drift removal based on the use of wavelets [9], which has the advantages of being non-parametric and does not rest on any economic theory whatsoever. This technique has been described in detail elsewhere [5], and will therefore not be repeated here. It suffices to say that this wavelet technique enables a separation of the original time series into a short scale (detrended) time series $\tilde{s}(t)$ and a drift term $d(t)$ so that $s(t) = \tilde{s}(t) + d(t)$. In Fig. 1, we see the effect of this procedure on the whole history of one of the major US economical indicators, namely the Dow Jones industrial average (DJIA). In this particular example, which is the one used in the analysis, the separation is set to 1000 trading days, corresponding to roughly 4 calendar years.

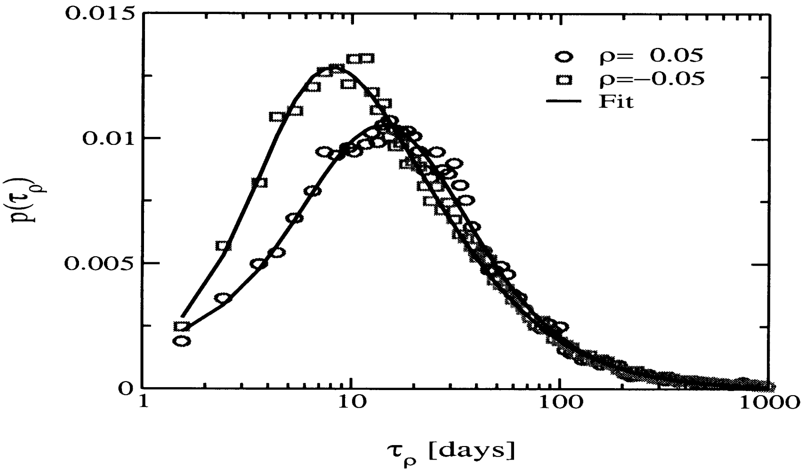

Fig. 2. The investment horizon distributions for the DJIA logarithmic closing prices at a return level $|\rho|$=0.05. The open symbols correspond to the empirical distributions, while the solid lines represents the maximum likelihood fit of these distributions to the functional form given by Eq. (2). The fitting parameters used to obtain these fits are for $\rho = 0.05$: $\alpha = 0.50$, $\beta = 4.5$ days$^{1/2}$, $\nu = 2.4$, and $t_0 = 11.2$ days; and for $\rho = -0.05$: $\alpha = 0.50, \beta = 5.0$ days$^{1/2}$, $\nu = 0.7$, and $t_0 = 0.6$ days.

Based on $\tilde{s}(t)$ for the DJIA, the empirical investment horizon distributions, $p(\tau_\rho)$, can easily be calculated for various levels of return ρ. In Fig. 2 these empirical distributions for $\rho = 0.05$ (open circles) and $\rho = -0.05$ (open squares) are presented. The solid lines in this figure are the maximum likelihood fits of the empirical data to the functional form (2). It is indeed observed that the generalized Gamma distribution, Eq. (2), fits the empirical data well for both positive and negative levels of return. It has been checked separately that the quality of the fits are of comparable quality for other values of ρ. However, as $|\rho|$ becomes large, the empirical distributions are hampered by low statistics that makes the fitting procedure more difficult.

The most interesting feature that can be observed from Fig. 2, is the apparent asymmetry between the empirical investment horizon distributions for $\rho = \pm 0.05$. In particular, for $\rho = -0.05$ there is a higher probability, as compared to what is observed for $\rho = 0.05$, to find short investment horizons, or in other words, *draw-downs* are faster than *draw-ups*. Consequently, one might say that there exists a gain–loss asymmetry! This result is in agreement with the draw-down/draw-up analysis presented in Ref. [10]. Similar results to those presented here have also been obtained for SP500 and NASDAQ.

Fig. 3 depicts the optimal investment horizon vs. level of return. From this figure it is observed that the asymmetry feature found for a return level of 5% is not unique. For the smallest levels considered, $|\rho| \sim 10^{-3}$, no asymmetry can be detected. However, as $|\rho|$ is gradually increased, the asymmetry starts to emerge at $|\rho| \sim 10^{-2}$. By further increasing the level of return, a state of saturation for the asymmetry appears to be reached. In this state the asymmetry in the optimal investment horizon for the DJIA is almost 200 trading days.

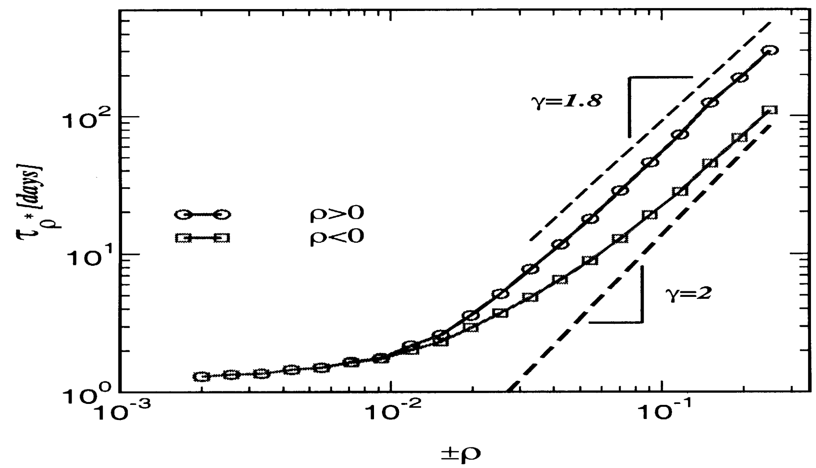

Fig. 3. The optimal investment horizon τ_ρ^* for positive (open circles) and negative (open squares) levels of return $\pm\rho$. In the case where $\rho < 0$ one has used $-\rho$ on the abscissa for reasons of comparison. If a geometrical Brownian price process is assumed, one will have $\tau_\rho^* \sim \rho^\gamma$ with $\gamma = 2$ for all values of ρ. Such a scaling behaviour is indicated by the lower dashed line in the graph. Empirically one finds $\gamma \simeq 1.8$ (upper dashed line), only for large values of the return.

These findings in fact confirms the saying in the financial industry that *it takes time to drive up prices*. From this analysis, one may add *compared to driving them down*, a result that coincides with the common belief that the market reacts more violently to negative information than to positive. To our knowledge, this is the first time that such statements have been founded in a quantitative analysis. The investment horizon distributions are, in fact, ideal tools for addressing such questions quantitatively. Before arriving at the conclusion of this paper, we will take the opportunity to speculate about the reason for this asymmetry. It cannot be due to any residues of the drift left by the wavelet analysis. The economic expansion in the 20th century (corresponding to the time period of the data) has in general been positive. Hence, if this was an effect of the drift, it should be expected that the waiting times for $\rho > 0$ should be the shortest. However, from the data we find it to be the other way around. Hence, we are led to conclude that this asymmetry is reflecting the market dynamics. Both from the point of view of risk management and of market psychology, it makes sense that market participants reacts faster to “bad” news than “good” news.

In conclusion, we have considered inverse statistics in economics. It is argued that the natural candidate for such statistics is what we call the investment horizon distribution. Such a distribution, obtained from the historic data of a given market, indicates the time span an investor historically has to wait in order to obtain a predefined level of return. The distributions are parametrized excellently by shifted generalized Gamma distributions for which the first moment does not exist. The typical waiting time, for a given level of return ρ, can therefore be characterized by, e.g., the time position of the maximum of the distribution, i.e., by the *optimal* investment horizon. By studying the behaviour of this quantity for positive (gain) and negative (loss) levels of return, a very interesting and pronounced gain–loss asymmetry emerges. It is concluded that this asymmetry is part of the market dynamics in contradiction with the symmetric assumption of the efficient market hypothesis.

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