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Inverse statistics in the foreign exchange market

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Abstract

We investigate intra-day foreign exchange (FX) time series using the inverse statistic analysis developed by Simonsen et al. (Eur. Phys. J. 27 (2002) 583) and Jensen et al. (Physica A 324 (2003) 338). Specifically, we study the time-averaged distributions of waiting times needed to obtain a certain increase (decrease) ρ in the price of an investment. The analysis is performed for the Deutsch Mark (DM) against the US\$ for the full year of 1998, but similar results are obtained for the Japanese Yen against the US\$. With high statistical significance, the presence of "resonance peaks" in the waiting time distributions is established. Such peaks are a consequence of the trading habits of the market participants as they are not present in the corresponding tick (business) waiting time distribution in the form of a power law Pdf. This result is achieved by rescaling of the physical waiting time by the corresponding tick time thereby partially removing scale-dependent features of the market activity.

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1. Introduction

Per Bak was a great scientist and a fantastic source of inspiration for many of us over many years. Through numerous lively and exciting discussions with him, one

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always felt that a project or a calculation was brought back on track again by his clever comments and suggestions. He applied his ingenious idea of "self-organized criticality" to many different systems ranging from sand piles, earthquakes to the brain and even finance. As he said: "It's all the same", meaning that in the end the paradigm of the sand pile model would after all describe the behavior of the particular system he considered. The idea of applying inverse statistics to turbulence data was the subject of the discussion between Per Bak and one of us (MHJ) several times. He liked the idea, and as such, we are quite sure that he would have liked our application of inverse statistics to financial data. This in particular applies to the scale invariant power-law scaling that is being observed for the normalized waiting time distribution. It is therefore our pleasure to dedicate this paper to his memory.

With the financial industry becoming fully computerized, the amount of recorded data, from daily close all the way down to tick-to-tick level, has exploded. Nowadays, such tick-to-tick high-frequency data are readily available for practitioners and researchers alike. In general, such high-frequency data are irregularly spaced in (physical) time, since an actual trade is a negotiation between sellers and buyers through a bid and ask process highly influenced by the irregular flow of information reaching the market. Hence, in order to apply the classic return approach to such data, the asset price has to be re-sampled equidistantly in physical time. This has been suggested in the seminal paper on high-frequency foreign exchange (FX) data analysis published by the Olsen & Associates Research Institute [3], but in many ways such a re-sampling violates the true dynamics of the market. Consequently, there has been an increasing interest over the past decade in studying variations in the market over a *variable* time span opposed to that of a *fixed* time span as for the return distribution [1,2,4]. One such approach is to consider drawdowns/ups, where an increasing or decreasing trend is followed to the end [6,7]. Recently, the present authors MHJ, AJ and IS introduced another such time varying approach—the *inverse statistics approach* [1,2]. At the heart of this technique, lies the waiting time needed to cross a pre-described return barrier.¹ The distribution of these waiting times, also termed investment horizons, characterize the inverse statistics [5] and has successfully been applied to daily close stock index data [1,2].

The purpose of the present paper is to follow up on these studies and investigate the corresponding statistical distributions for the FX market using high-frequency data. In particular, this work focuses on the exchange rate for the full year of 1998 between the two major currencies of the world, namely the US\$ and the Deutsch Mark (DM), the latter in 2000 replaced by the Euro.

2. Formalism

Before we present the results of our analysis, we will set the stage by recapitulating a few important definitions and properties of inverse statistics. A more detailed

¹ One may also consider the completion process of a trade as the crossing of the bid and ask random walks.

introduction can be found in Refs. [1,2,5]. Let us assume the value of the asset under study is described by the time varying asset price S(t).² Here, the time variable t can in principle be any time variable and below we will use both physical and tick time. The log-return at time t calculated over a time interval Δt , is defined as

$$r_{\Delta t}(t) = s(t + \Delta t) - s(t) , \qquad (1)$$

where $s(t) = \ln S(t)$. The waiting time for an investment made at time t at log-price s(t), is defined as the time interval $\Delta t = t' - t$, t' > t, where the relation $r_{\Delta t}(t) \ge \rho$ is fulfilled for the *first* time. If physical time is used as the time scale, then the waiting time for return level ρ is denoted by $\tau_{\rho}(t)$. If tick time is used instead, the corresponding (dimensionless) waiting time is denoted by T_{ρ} . The investment horizon, or waiting time distributions are the probability density functions of $\tau_{\rho}(t)$ and T_{ρ} when using physical or tick time, respectively.

For a geometrical Brownian motion this distribution, known as the *first passage* distribution, is known analytically [9–11] to be $p(t) = a \exp(-a^2/t)/\sqrt{\pi}t^{3/2}$, where a depends on the return barrier ρ . In Refs. [1,2] it was shown that this distribution is too "primitive" to fit the waiting time distributions for the three major US stock market indexes (DJIA, SP500 and NASDAQ) and instead a type of generalized Gamma distribution was found to give an excellent parameterization of the data, see Refs. [1,2] for details. As the daily close of the stock indexes was analyzed, which by definition are regularly sampled with the exception of weekends and public holidays, the distinction between physical time and "tick-time" was not made.

3. Analysis of the FX market

We have been able to obtain FX data (cf. Ref. [3] for "stylized facts" of FX-markets) for the DM against the US\$ for the full year of 1998. The data set consists of 1,620,944 ticks irregularly distributed in physical time. This corresponds to an average time between ticks of roughly 20 s. However, as we will see, there are hours during the day where the trading activity is much higher than during the remaining of the 24 h day. This will play an important role for our results.

For high-frequency data, the results depend highly on how "time" is defined [3,8]. Two obvious choices for a time scale are *physical time* (or "wall time" displayed on the trading floor) and *tick time* (also referred to as "business time" by some authors) as mentioned previously. As we see in Fig. 2 the average physical time interval between ticks will decrease during active market periods and on the other hand increase when the market comes less active.

In Figs. 1a and b, the *physical waiting time* distribution $p(\tau_{\rho})$ and the *tick waiting time* distribution $p(T_{\rho})$ for the DM against the US\$ are shown for the year 1998. The return level used to obtain these results was $\rho = 0.005$. We did also check that our

² As the true trading price is not publicly disclosed, we have chosen to calculate the price as $S(t) = (S_{bid}(t) + S_{ask}(t))/2$. Other options are to use $s(t) = (\log S_{bid}(t) + \log S_{ask}(t))/2$ suggested in Ref. [3] or the algorithm proposed in Ref. [8].



Fig. 1. Inverse statistics $(p(\tau_{\rho}) \text{ or } p(T_{\rho}))$ vs. waiting time using *physical waiting time* τ_{ρ} (a) or *tick time* T_{ρ} (b) for the 1998 DM/US\$ data. The return level used to obtain these results was $\rho = 0.005$. The vertical dashed lines in Fig. 1a indicate physical waiting times of 1–3 days (from left-to-right). Notice the apparent resonances seem to coincide with the daily structure, while such resonances are not present in the corresponding tick time distribution.

findings were not affected in any significant way by instead considering a return level of $\rho = -0.005$. This indicates that drift is not an important component to the analyzed data set, as opposed to the daily data analyzed in Refs. [1,2], and hence no need for detrending is present. The two graphs of Fig. 1 both go through a single global maximum and for waiting times smaller then these maxima the two distributions are similar. However, for longer waiting times, there are some notable differences between the two distributions. First, the tick time distribution, $p(T_{\rho})$, falls off faster, actually as 1/T, than the corresponding distribution using physical time. Secondly, and more important, the "resonance peak" structure present in $p(\tau_{\rho})$ has vanished in $p(T_{\rho})$. The first of these peaks is located roughly at the daily scale (indicated by the left vertical dashed line in Fig. 1a) and we clearly see the second and third "harmonics".



Fig. 2. The tick frequency probability distribution, p(h), vs. UTC-hour, h, at which the tick took place for the DM/US\$ of the year 1998.

The origin of these "resonances" is to be found in the varying activity of the market. The main difference between the two ways of quantifying time is that tick time is equidistant, whereas physical time between ticks is not. Mathematically, one may say that the data using physical time is the "convolution" of the data using tick-time with the distribution of ticks as a function of physical time. Hence, a change in market activity will alter the inter-relation between these two time scales. In order to study the daily peak structure of Fig. 1a in more detail, we in Fig. 2 show the Pdf of ticks as a function of the universal time coordinate, (UTC, former GMT) hour of that tick. One observes that this distribution is far from being flat. Thus there exists indeed time periods where the FX-market is semi-closed. In particular, almost 80% of the ticks correspond to a UTC-hour of 6-16 with a local maximum located around UTC-hour 8 and another one at 13 or 14. The active periods defined by UTC-hours from 6 to 16, correspond to working hours in London and the east coast and the mid-west of the US.

In view of the results of Figs. 1 and 2, one might suspect that the daily peak structure observed in $p(\tau_{\rho})$ is a result of this uneven trading activity during the day of the global FX-market. If it is (partly) true that (tick time) returns calculated from two consecutive ticks are only weakly correlated, one would naively expect that the volatility of a given *physical time interval* is larger in a high market activity period than in a low one. Under this assumption, the tick time distribution $p(T_{\rho})$ should not be sensitive to whether or not one is in a high or low activity region, since tick time by construction is equidistant. On the other hand, for the physical waiting time distribution, $p(\tau_{\rho})$, the market activity does indeed matter. Here the pre-described return level will more likely be reached during the highly active periods. If the return level is not reached within one and the same period of high activity, there is higher chance that it will do so in the next one than in the intermediate low activity period, simply because



Fig. 3. The probability distribution of *normalized* waiting time τ_{ρ}/T_{ρ} needed to reach a return level $\rho = 0.005$ for the 1998 DM against the US\$ exchange rate data (open circles). The power-law dependence $p(\tau_{\rho}/T_{\rho}) \sim (\tau_{\rho}/T_{\rho})^{-\gamma}$ with $\gamma \simeq 2.4$ is indicated by the solid line.

there are fewer ticks during this low activity periods. Such a behavior will therefore result in an enhanced physical waiting time probability corresponding to an integer numbers of days, just like we see in Fig. 1a.

To investigate this further, we introduce a new type of time scale, specifically a normalized waiting time that aims at partly suppressing the effect of varying market activity. This time scale is defined as τ_{ρ}/T_{ρ} , or in words, as the average *physical* waiting time per tick needed to break through the return level ρ . As we will see, normalizing the physical waiting time with the (corresponding) number of ticks needed to cross the ρ return barrier, reduces the effect of varying market activity. It should also be noted that one naively would expect the inverse statistics, as characterized by the normalized waiting time distribution, to be less sensitive to the level of return ρ then their unnormalized partners. This is so since an increase in $|\rho|$ will increase the overall waiting time measured both in physical or tick time units.

In Fig. 3, the probability distribution function for the normalized waiting time, τ_{ρ}/T_{ρ} , is presented. As suspected, there seems to be little, or no, effect of the change in market activity throughout the day. For instance, the daily peaks that are so marked features of $p(\tau_{\rho})$ are not observable in Fig. 3. However, more surprisingly, the behavior of $p(\tau_{\rho}/T_{\rho})$ for not too low normalized waiting times, seems to be well fitted by a single power law. In particular one finds

$$p\left(\frac{\tau_{\rho}}{T_{\rho}}\right) \sim \left(\frac{\tau_{\rho}}{T_{\rho}}\right)^{-\gamma}$$
, (2)

with $\gamma \simeq 2.4$ when $\rho = 0.005$, spanning nearly three orders of magnitude in normalized time τ_{ρ}/T_{ρ} . (The question of how sensitive γ is to the return level ρ will be addressed in

a separate forthcoming publication.) The conclusion is that the proposed rescaled time is the most natural one to use when analyzing high-frequency data in terms of inverse statistics and makes the inverse statistics approach well suited for high-frequency data.

4. Conclusions

In conclusion, we have studies high-frequency FX data for the DM against the \$US from an inverse statistics point of view. It is found that the change in market activity makes it more challenging to define an appropriate and unique time scale, since the change in activity level of the market causes certain resonances to emerge in some quantities. In particular, when physical time is used as the time scale it is demonstrated that daily peaks emerge in the inverse statistics as quantified by the physical waiting time distribution function. Such peaks are, however, not fond to be present in the corresponding inverse statistics for tick time. The trading activity effect is partly removed from the inverse statistics by studying the new time scale defined as the average physical waiting time per tick, τ_{ρ}/T_{ρ} needed to reach a given level of return ρ . In terms of this normalized time variable a new type of power law is observed for the inverse statistics. Over a nearly three orders of magnitude in normalized waiting times, excluding the smallest ones, the waiting time distribution for $\rho = 0.005$ was found to be well characterized by a single power law of exponent $\gamma \simeq -2.4$. This scaling law represents a new type of *stylized fact* for the FX-market which, to the best of our knowledge, has not been reported before.

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