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Volatility of power markets

Ingve Simonsen*

Department of Physics, NTNU, NO-7491 Trondheim, Norway

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Abstract

Volatility features of the Nordic day ahead power spot market for a 12-year period up till May 2004 are studied. The daily logarithmic volatility was measured for this period to be about 16%. This level is well above what is observed for most other well-studied financial markets. Volatility clustering, log-normal distribution, and long-range correlations are found to be striking features of the volatility of power markets. In addition, a cyclic behavior of the time-dependent volatility can be observed for the Nordic power market. Furthermore, the volatility shows a dependence on the price level, and this is pronounced mostly when the spot price is low. The correlation in volatility is consistent with an inverse power-law decay, $\tau^{-\nu}$, superposed on an oscillating term. The numerical value of the exponent ν is similar to what has been reported previously for stock markets.

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Keywords: Volatility; Electricity markets; Deregulation; Econophysics; Interdisciplinary physics

1. Introduction

Electricity is one of the most important commodities on which contemporary societies depend. It powers many of the modern tools that we encounter in our daily life. Over the last two decades, the power sector has gradually been transformed

*Fax: +33 1 4834 7416.
E-mail address: Ingve.Simonsen@phys.ntnu.no.

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from an exclusive monopolistic state to a competitive market place. This has exposed market players to more uncertainty, and as a consequence, separate option and futures markets have emerged. Like in more traditional financial markets, there is always the question of how to price such derivatives. At the very heart of any such pricing scheme lies a series of assumptions regarding the stochasticity of the underlying variable. This is, not surprisingly, also the case for power markets, where the underlying usually is the spot electricity price. It is therefore of importance to have good knowledge of the stochastic properties of the spot price process. This, and related questions, have recently prompted research efforts into uncovering the structures and features of power markets and how to analyze them [1–8].

A number of empirical studies, spanning different market types, have established certain *stylized facts* of financial time series [8,10–16]. These features are to some extent universal, i.e., they are independent of the market type (stocks, currencies, interest rates, different commodities, etc.). For a collection of characteristic features of deregulated power markets, the reader is referred to Ref. [8].

One well-known stylized fact of financial data is the so-called *volatility clustering* phenomenon [10–12,16–18]. It expresses itself in the price process as localized outbursts of volatility (standard deviation of returns), much like the intermittent nature of the velocity fluctuations found in turbulence [11,19]. In other words, this clustering is observed as periods of high volatility followed by extended periods of low volatility.

The purpose of the present work is to study volatility features of the deregulated Nordic power market. In particular, we point out similarities and differences in the properties of the volatility of this market as compared to similar findings for more traditional financial markets. Such a detailed volatility study for the power sector has, to the best of our knowledge, not been reported previously.

2. The data set

The time series that will be studied in this paper is depicted in Fig. 1. It shows the Nord Pool day ahead system spot price for the period from the beginning of May 1992 up to the same month of the year 2004 [20]. The inset to Fig. 1 presents the price variations over an arbitrarily chosen week from which the daily and weekly structure of the spot price can readily be observed. Because of the way the market is organized, the system price data are sampled regularly at an hourly interval. The analyzed data set consists in total of $N = 105,216$ samples. It was a similar data set from the same market, but for a somewhat different time period, that recently was used to study the scaling properties of spot price by wavelets [5,21].

3. The analysis

Let $S(t)$ denote the spot price process at time t. The logarithmic price (again at time t) is then $s(t) = \ln S(t)$. Logarithmic *returns* over a time horizon Δt are

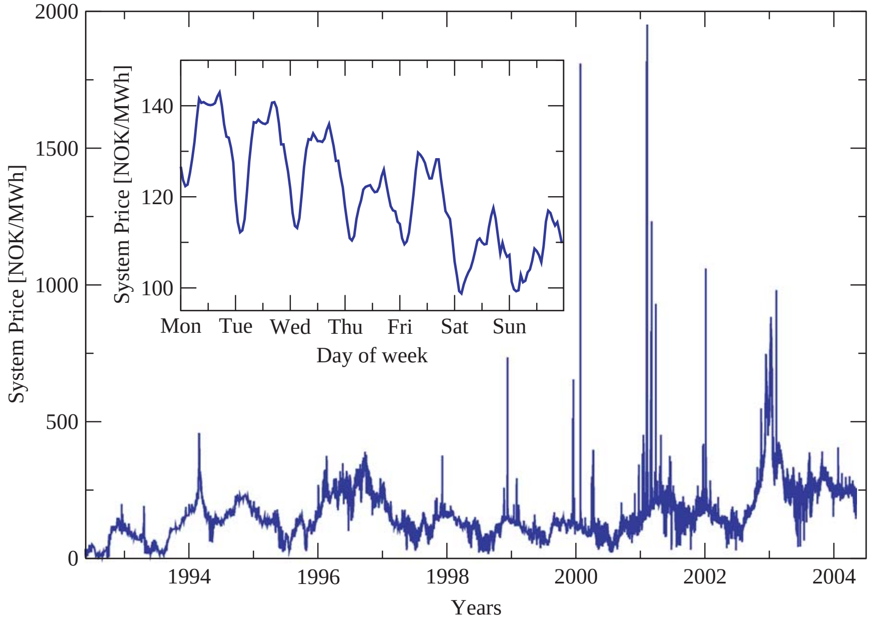

Fig. 1. Hourly system prices for the spot market (Elspot) at the Nordic power exchange (Nord Pool) from May 1992 up to May 2004 (12 full years of data). In total, the data set consists of 105, 216 data points. The inset depicts the system price for a typical week chosen arbitrarily (1st week of the year 2000).

defined as [10–12]

$$r_{\Delta t}(t) = s(t+\Delta t) - s(t) = \ln\left(\frac{S(t+\Delta t)}{S(t)}\right) . \tag{1}$$

It is an empirical fact of financial time series that returns are uncorrelated after a relatively short period of time [10–12]. The explicit time dependence of this quantity is therefore often suppressed. The daily ($\Delta t = 24\,\mathrm{h}$) logarithmic returns of the Nordic power spot market data (Fig. 1) are presented in the inset to Fig. 2. Due to the daily price cycles, it should be noted that it is beneficial to use values of Δt that correspond to full days.

Returns can alternatively be defined as the relative price change (simple returns), $R_{\Delta t}(t) = \Delta S(t)/S(t)$ with $\Delta S(t) = S(t+\Delta t) + S(t)$. As long as $\Delta S(t) \ll S(t)$, it follows readily that $R_{\Delta t}(t) \approx r_{\Delta t}(t)$ [11]. For the financial markets most frequently considered in empirical studies, the assumption $\Delta S(t) \ll S(t)$ is normally well satisfied, and it is thus *not* crucial to enforce a strict distinction between these two quantities. For power markets, on the other hand, the spiky nature of the price process (cf. Fig. 1) requires a distinction to be made between $R_{\Delta t}(t)$ and $r_{\Delta t}(t)$. In the present study, we will exclusively focus on $r_{\Delta t}(t)$. The benefit of considering logarithmic returns is that they are additive stochastic variables. Simple returns, on the other hand, do not share this property. Interested readers are urged to consult the recent book by

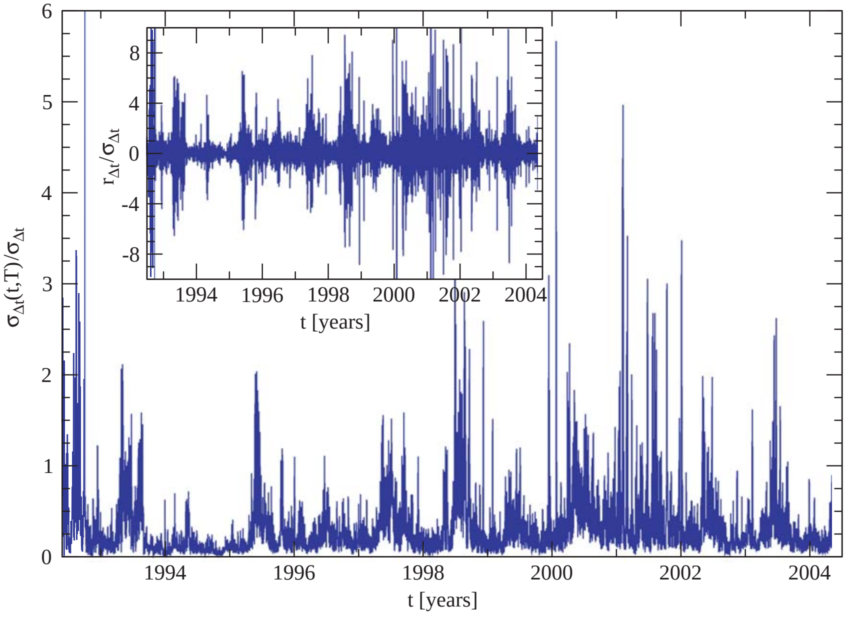

Fig. 2. The time-dependent daily volatility (main figure), $\sigma_{\Delta t}(t,T)$, and the daily returns (inset), $r_{\Delta t}(t)$, for $\Delta t = T = 24\,\mathrm{h}$. The underlying data set used to produce the results of this figure is that of Fig. 1. The volatility was calculated according to Eq. (2). The (sample) averaged daily volatility used in the normalizations of both returns and volatility was $\sigma_{\Delta t} = 16\%$.

McCauley [9] for further information regarding this issue. We have also undertaken a parallel analysis in terms of $R_{\Delta t}(t)$. Therefrom, it was found that doing so would *not* alter the main conclusions that could be drawn from the present study.

In finance and economics, (historical) *volatility* is used to measure the fluctuations, or risk, that can be associated with holding an asset [10–12]. Formally, it is defined as the standard deviation of returns[1] [10–12]:

$$\sigma_{\Delta t}(t,T) = \langle [r_{\Delta t}(t) - \langle r_{\Delta t}(t)\rangle_T]^2 \rangle_T^{1/2} \; . \tag{2}$$

Here, $\langle\cdot\rangle_T$ denotes the (sample) average taken over a time window of size T (about time t). If T (or t) is not explicitly given, the averages in Eq. (2) are assumed to be taken over all available data (sample averages). Empirically, one frequently finds that the average return is rather small[2] so that $\sigma_{\Delta t}(t,T) \approx \langle r^2_{\Delta t}(t)\rangle_T^{1/2}$. For the Nord Pool data set studied in this paper, the (sample) average of $r_{\Delta t}(t)$, for $\Delta t = 24\,\mathrm{h}$, is about $1/500$ of its standard deviation (that later will be denoted by $\sigma_{\Delta t}$).

With Eq. (2), one can readily calculate the time-dependent volatility $\sigma_{\Delta t}(t,T)$. This quantity for the Nordic power market is depicted in Fig. 2 for $\Delta t = T = 24\,\mathrm{h}$. Here, one has chosen to normalize the results by the (sample) averaged daily volatility $\sigma_{\Delta t}$,

[1] A similar definition of volatility can be associated with returns as defined by $R_{\Delta t}(t)$.

[2] This is a result of the return distribution being close to symmetric.

found to be about 16% when using logarithmic returns. If simple returns ($R_{\Delta t}(t)$) had been used instead, the corresponding volatility would have been 28%. Strictly speaking, some hours of the day are more volatile than others. For instance, if the averages in Eq. (2) are limited to a *given* hour h of the day, 24 volatility levels $\sigma_{\Delta t;h}$ will result (Fig. 3). A local volatility peak is observed in the morning and a less pronounced one in the afternoon, while the lowest level of $\sigma_{\Delta t;h}$ occurs during midnight. Note that the volatility profile seems partly to follow the daily price (and consumption) profiles [8].

In Fig. 4, the empirical probability distribution function of the time-dependent volatility is presented (solid bars). The continuous solid line in Fig. 4 represents a maximum likelihood fit of the log-normal probability distribution function to the empirical histogram. The resulting parameters are found in the figure caption. At the 5% significance level, the volatility of the Nord Pool spot price for $\Delta t = T = 24$ h is consistent with the hypothesis of a log-normal volatility distribution. A similar conclusion was recently reached for the distribution of volatility in the Italian MIB30 stock index [16].

The level of (sample averaged) daily volatility of the Nordic power market, $\sigma_{\Delta t} \approx 16\%$, is rather high compared to many other financial markets. Such a level is indeed a characteristic feature of deregulated power markets [8]. Moreover, it ought to be mentioned that the spot market at Nord Pool actually is known for its “low” volatility. Other liberalized power markets may be considerably more volatile than

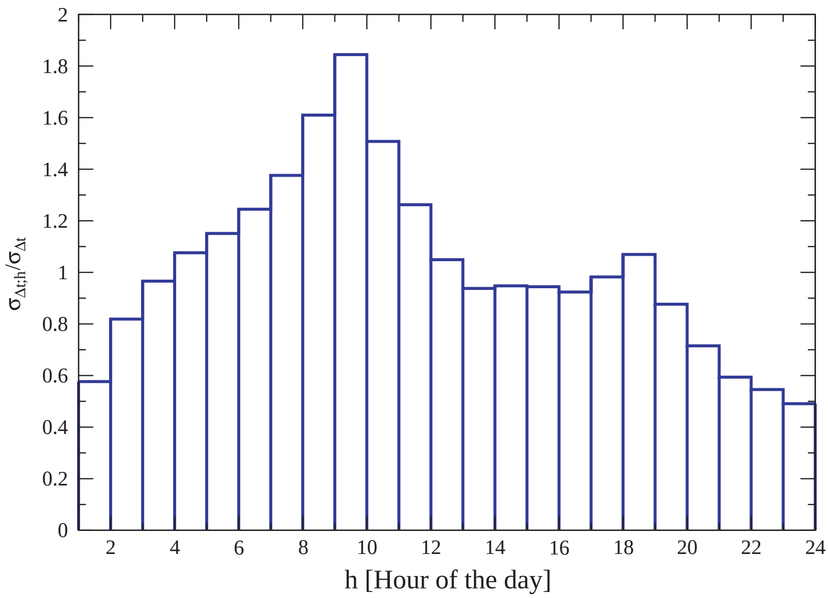

Fig. 3. The sample-averaged hourly volatility $\sigma_{\Delta t;h}$ for the hth hour of the day for the Nord Pool data set of Fig. 1. This quantity is calculated by limiting the averages to data points belonging to a given hour h of the day. This is in contrast to the way $\sigma_{\Delta t}$ was calculated for which all hours of the day were included into the average.

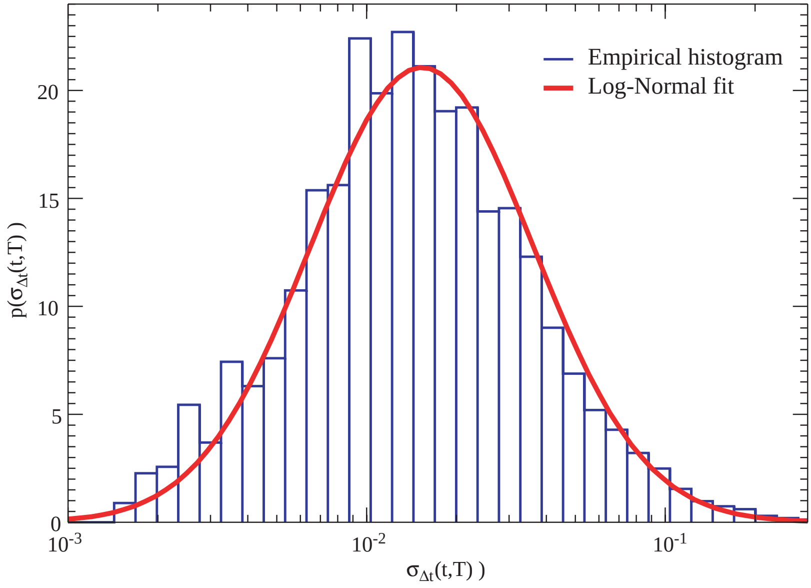

Fig. 4. The empirical histogram, with 50 bins, for the daily, time-dependent volatility, $\sigma_{\Delta t}(t,T)$, for $\Delta t = T = 24\,\mathrm{h}$. The thick continuous solid line represents a log-normal fit to the empirical distribution. The estimated parameters were 0.032 (mean) and 0.856 (standard deviation). The corresponding χ^2 goodness-of-fit parameter was found to be a little less than 58, and the number of degrees of freedom 47. Hence, at the 5% significance level ($\chi^2_{0.95,45} = 61.7$), the hypothesis that the Nord Pool volatility is log-normal distributed for the values of Δt and T considered can be accepted.

Nord Pool. Still, however, the volatility of power markets may be (at least) an order of magnitude higher than what is typically found for more traditional and well-studied financial and commodity markets. For instance, for the daily volatility one finds: 1–1.5% for stock indices, less than 4% for individual stocks, for bonds less than 0.5%, 2–3% for crude oil and about 3–5% for natural gas, and as low as 0.03% for short-term interest rates [15].

From Fig. 2, one readily observes the so-called volatility clustering effect, i.e., time periods where the volatility is consistently higher than in other time periods. Volatility clustering is one of several well-studied stylized facts of, say, stock markets [10–12,17,18]. Fig. 2 explicitly demonstrates that this is indeed also a striking feature of deregulated power markets. Volatility clustering is reminiscent of similar intermittent patterns found in turbulence [11,19]. Historically, this prompted efforts to compare turbulence to finance (see e.g. Ref. [11] and references therein).

To quantify in more detail the volatility clustering observed in Fig. 2, one may study the temporal volatility–volatility (auto) correlation function $C_{\sigma\sigma}(\tau)$. It is defined as [10,11]

$$C_{\sigma\sigma}(\tau) = \frac{\langle \xi(t+\tau)\xi(t)\rangle - \langle \xi \rangle^2}{\sigma_\xi^2}, \quad \xi(t) = \sigma_{\Delta t}(t,T)\,, \tag{3}$$

where we, in order not to clutter the notation, have introduced the short hand $\xi(t) = \sigma_{\Delta t}(t, T)$. Moreover, σ_{ξ} represents the standard deviation of $\xi(t)$, and $\langle \cdot \rangle$ denotes, as above, the sample average. Results for the correlation function, $C_{\sigma\sigma}(\tau)$, are put forward in Fig. 5. Therefrom, significant temporal correlations can be observed up to a time lag of approximately 100 days. Above this scale, only correlations that can be attributed to the weekly cycle of the system price are observable. The overall decay of $C_{\sigma\sigma}(\tau)$ with lag τ is consistent with an inverse power law

$$C_{\sigma\sigma}(\tau) \sim \tau^{-\nu} \tag{4}$$

of a rather small exponent ν. The numerical value of this exponent was found to be approximately $\nu \approx 0.07$ (thick dashed line in Fig. 5). For stock markets, this exponent has previously been reported to lie in the range of 0.1–0.3 [18,22].

From Fig. 5, it should be apparent that $C_{\sigma\sigma}(\tau)$ does not decay as a pure power law. In addition, it has superimposed an oscillating term. The inset to Fig. 5 gives the estimated power spectral density of the volatility. It shows explicitly strong seasonality in the volatility at 7/3, 7/2 and 7 days as well as a well-pronounced

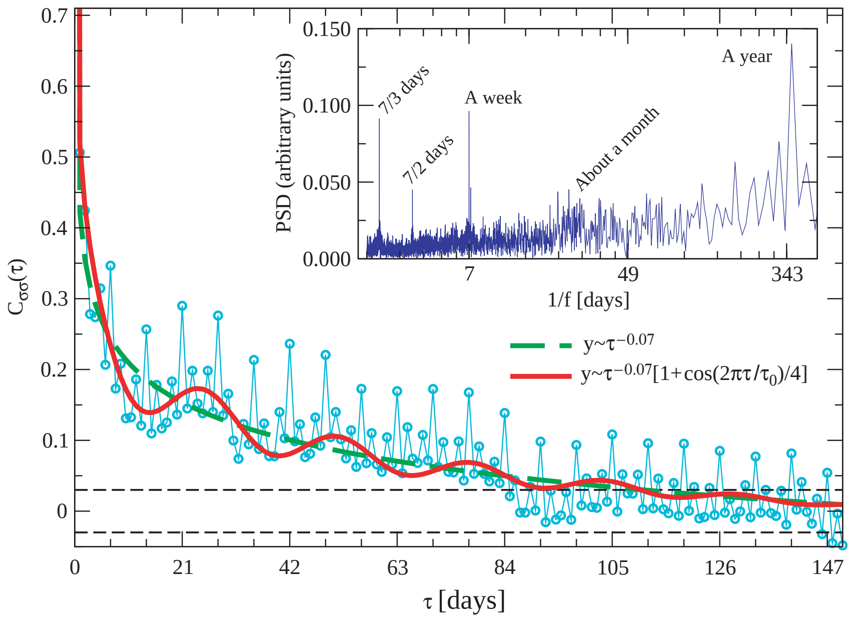

Fig. 5. The volatility–volatility correlation function, $C_{\sigma\sigma}(\tau)$, *vs.* lag τ for the Nord Pool spot price volatility. It is observed that after about 100 days, no significant correlation is present in the volatility except for what can be attributed to the strong weekly cycle. The thick, dashed line represents an inverse power-law fit to the data: $y(\tau) \sim \tau^{-\nu}$ with $\nu = 0.07$. In the solid line, an oscillating term is in addition included into the fit: $y(\tau) \sim \tau^{-\nu}[1 + \cos(2\pi\tau/\tau_0)/4]$, where the time constant was $\tau_0 \approx 26$ days (and ν as given above). The thin, horizontal dashed lines represent the 95% confidence intervals. The inset depicts an estimate of the power spectrum density (PSD) *vs.* period defined as one over frequency (f).

yearly cycle. Moreover, from the volatility auto-correlation, a longer-than-a-week period component seems to be present. The period of these oscillations is at a close-to-monthly scale. If one inspects $C_{\sigma\sigma}(\tau)$ closely, it is realized that this "period" is in fact not absolutely constant, but instead depends weakly on the lag τ. This view is also supported by the spectral analysis (inset to Fig. 5), from which no well-pronounced cycle is apparent around the monthly scale. Instead, a local enhancement of the spectral components corresponding to periods between 3 and 4 weeks can be observed. We speculate that this enhancement, "smeared out" over a range of periods, may be related to (social) trading patterns (e.g. clearing at the end of the month). However, further work is needed to settle this issue. If a single oscillating term is included into the fitting function, one obtains

$$C_{\sigma\sigma}(\tau) \sim \tau^{-\nu}\left[1 + \frac{1}{4}\cos\left(2\pi\frac{\tau}{\tau_0}\right)\right], \tag{5}$$

with $\nu \approx 0.07$, as before, and a time constant $\tau_0 \approx 26$ days (solid line in Fig. 5).

A detailed inspection of Fig. 2 reveals that rather consistently the volatility is at a local high during the summer months, i.e., the volatility has an annual cycle (cf. the inset to Fig. 5). Recall that during the summer season, the spot prices in the Nordic market are generally low due to the lower consumption (cf. Fig. 1). Hence, consumption constraints can be ruled out as causing this phenomenon since such constraints are most frequent during the winter period [8]. However, we speculate that the reason instead may be found in the structure of the Nordic power market itself. Large fractions of the power generation in the Nordic region come from hydro-power. During summer and autumn, the filling fraction of the water reservoirs feeding the hydro-power plants is normally at its maximum. If, over time, the inflow of water to the reservoirs is larger than the outflow needed to generate the power to satisfy demand, one may end up in a situation of the so-called *forced production*. Under such circumstances, the power generators will produce electricity whatever the price might be, just to prevent the reservoirs from flooding. The dam owners are normally supposed to regulate the water flow, and they will be liable for potential damage caused by flooding. When many power generators go into a state of forced production, the system price may drop to a very low level, making such periods rather volatile.

To enable a more detailed investigation into the dependence of (time-dependent) volatility, $\sigma_{\Delta t}(t, T)$, on spot price, $S(t)$, we present a scatter plot (crosses) of these two quantities in Fig. 6. The open circles of Fig. 6 result from applying a median filter of length $N = 25$ to a version of the data that has been sorted according to increasing spot price. Fig. 6 explicitly shows that there is a price level dependence on the volatility for the lowest spot prices. For higher prices, however, the volatility shows little or no dependence on the spot price. In particular, there is no drop in volatility for the highest price levels (the spikes). A reasonable approximation of the volatility–price dependence is represented by a stretched exponential plus a constant (solid line in Fig. 6). Spot price variations within a typical range amount to about an order of magnitude variation in the level of volatility (cf. Fig. 6).

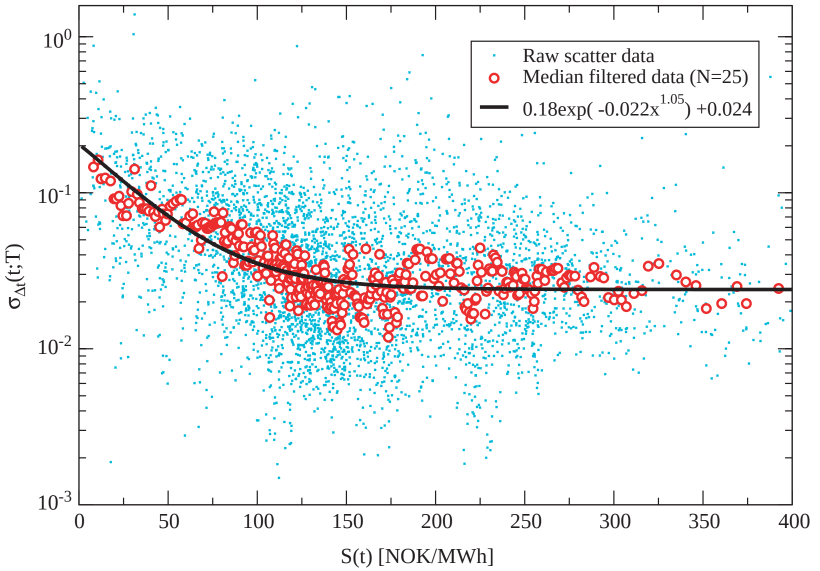

Fig. 6. A scatter plot (crosses) of daily volatility, $\sigma_{\Delta t}(t, T)$ *vs* spot price $S(t)$. The volatility was calculated from the logarithmic returns via Eq. (2) with $\Delta t = T = 24\,\text{h}$. The open circles correspond to smoothed data obtained after applying a median filter of length $N = 25$ to a spot price sorted version of the original data set. As a guide to the eye, the solid line represents a fit to the median filtered data. The fitting function used, and given in the graph, was a stretched exponential plus a constant.

Logarithmic returns, $r_{\Delta t}(t) = \ln(R_{\Delta t}(t) + 1)$, in contrast to simple returns, $R_{\Delta t}(t) = \Delta S(t)/S(t)$, are not symmetric with respect to a price increase and a drop of the same magnitude if $\Delta S(t)$ is comparable to $S(t)$. We would like to stress that the price dependence observed for the volatility in Fig. 6 is *not* simply an artifact of the use of $r_{\Delta t}(t)$. A similar dependence is also found when $R_{\Delta t}(t)$ is used to define returns.

Before closing this paper, we would like to mention that we have tried to empirically identify the so-called leverage effect for the Nord Pool data [6,15,23–25]. This effect was first observed by F. Black in the mid-1970s. He observed that volatility of stocks tends to increase when the price drops [23]. Nowadays, the leverage effect is usually stated as the existence of a negative correlation between past returns and future volatility, but *not* the other way around. From the empirical (Nord Pool) volatility-return correlation function it has proven difficult to be conclusive on this issue due to the seasonality of the spot price data. Further work is needed to clarify this issue.

4. Conclusions

We have investigated the properties of the volatility function of the deregulated Nordic power market over a 12-year period. It has been demonstrated that the volatility of power markets shares many of the features of more well-studied financial and commodity markets. Examples are volatility clustering, log-normal distribution, and long-range volatility–volatility correlations. Not so commonly observed features can also be found: an overall high level of (sample) volatility, oscillating volatility–volatility correlations, daily volatility profiles, multi-seasonality, and price level-dependent volatility. Moreover, the study is inconclusive when it comes to determining if a leverage effect is present for the Nordic power market. The paramount volatility features of the Nordic power market that differ the most from, say, a stock market, are the consistently high level of volatility and that the volatility may depend on the price level itself.

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