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Frustration driven stock market dynamics: Leverage effect and asymmetry

Peter Toke Heden Ahlgren[a,*], Mogens H. Jensen[a], Ingve Simonsen[b], Raul Donangelo[c], Kim Sneppen[a]

[a]The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
[b]Department of Physics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway
[c]Instituto de Fisica da UFRJ, Caixa Postal 68528, 21941-972 Rio de Janeiro, Brazil

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Abstract

By applying inverse statistics to financial data it has recently been found from empirical studies that indices exhibit a pronounced *gain-loss asymmetry* [M.H. Jensen, Phys. Rev. Lett. 83 (1999) 76; I. Simonsen, M.H. Jensen, A. Johansen, Eur. Phys. J. B 27 (2002) 583; M.H. Jensen, A. Johansen, I. Simonsen, Physica A 324 (2003) 338]. This gain-loss asymmetry appears to have some similarities with the stylized fact *leverage effect* and we investigate if they could be of same origin. For this purpose we introduce the *Frustration Governed Market* model which includes correlations in time between a model index and its individual stocks. It is shown that the model reproduces very well the empirical findings with respect to gain-loss asymmetry and leverage. In special cases, however, the model may produce leverage without a pronounced gain-loss asymmetry.

Keywords: Stochastic processes; Inverse statistics; Gain-loss asymmetry; Leverage

1. Introduction

All investors share one major concern: optimization of profit. Though simplistic, this viewpoint has subtle details to it due to the different approaches that can be applied to reach this goal. For instance, one could risk all for what one believes to be the most profitable investments or alternatively pay attention to minimization of portfolio risk. The first strategy may be a fruitful, but also very hazardous one. As known, investments with high profit potential also inherit high risk. The alternative amounts to calculate realistic measures of risk and to find other assets that may cover up possible losses created by the initial investment—a very celebrated method known as hedging [1].

Recently the concept of fear as a driving mechanism for the stock market was introduced and investigated in Ref. [2]. The empirical work leading to the so-called *Fear Factor* model showed the existence of an inherent *gain-loss asymmetry* in stock indices, while the constituting stocks show little, or no, sign of a similar effect [3].

*Corresponding author.
E-mail address: ahlgren@nbi.dk (P.T.H. Ahlgren).

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The Fear Factor model [2] has brought knowledge about the importance of collective behavior in the stock market and its consequences concerning the asymmetry.

The gain-loss asymmetry share common features with another stylized fact—the *leverage effect* [4,5]—which is also most pronounced for indices. Still it is an open question if the asymmetry and leverage effect are measures of the same property: The Fear Factor model has shown that leverage is not necessary to produce the observed gain-loss asymmetry. To shed light on this discussion, we shall in the following introduce a model constructed to incorporate the leverage effect; this model we term the *Frustration Governed Market* model.

2. Inverse statistics and leverage

Inverse statistics, as recently introduced in econophysics [3,6,7], determines the distribution of waiting times for a given, asset specific, return level. Fig. 1(a) depicts these *investment horizon distributions* for the Dow Jones Industrial Average (DJIA). The positions of the *optimal investment horizons* (the maximum of each distribution) show that one most likely will be able to realize a loss faster than a corresponding gain. This is the inherent gain-loss asymmetry also found in indices like NASDAQ and S&P500 [8]. The constituting stocks themselves show *no* asymmetry which can be seen from the insert in Fig. 1(a). It seems reasonable that the phenomenon is a result of collective behavior among investors, the basic mechanism of the Fear Factor model [2].

Recently the leverage effect has attracted some attention in the literature. According to Bouchaud et al. [4], leverage is defined as the correlation, with time lag τ, between future volatility and past return, $r_{\Delta t}(t)$, of a studied asset

$$\mathscr{L}_{\Delta t}(\tau) = \frac{\langle r_{\Delta t}(t+\tau)^2 r_{\Delta t}(t)\rangle}{\langle r_{\Delta t}(t)^2\rangle^2}, \tag{1}$$

with $r_{\Delta t}(t)^2$ as a proxy for the square of the local volatility σ^2. Notice that with this particular choice of normalization, $\mathscr{L}_{\Delta t}(\tau)$ has dimension σ^{-1} making leverage comparison of heterovolatile assets non-trivial. For consistency, we will adopt the definition Eq. (1).

Fig. 1(b) depicts the leverage correlation for the DJIA. Strong correlations between negative returns and fluctuating prices can be observed for $\tau = 0$, with decay towards zero as τ is increased. That $\mathscr{L}_{\Delta t}(\tau) < 0$ for positive values of τ implies that a negative return at time t will increase volatility at $t + \tau$. Though statistics is too poor to conclude on any fit to the DJIA alone the study of several indices and stocks in Ref. [4] concludes that exponential fits to the correlations for $\tau > 0$ are preferable. It is found that the leverage effect is much more pronounced for indices than single stocks. Again we observe possible evidence of a stylized fact originating from collective behavior of investors.

3. The Frustration Governed Market model

The previously introduced Fear Factor model [2] utilizes synchronized stock idiosyncrasies to explain the appearance of the empirically observed gain-loss asymmetry. It is also upon the notion of collectivity that the Frustration Governed Market model is to be rationalized. In contrast to the Fear Factor model, the stock dynamics will be controlled via a time-dependent stock volatility, $\sigma(t)$, assumed equivalent for all stocks. A combination of both update rules could also be examined, however, this is not to be considered herein.

To outline the model, consider a stock index $I(t)$ and its N constituting stocks of value $S_i(t)$ at time t. All stocks are assumed to perform geometrical Brownian motions with common volatility $\sigma(t)$ and uncorrelated Gaussian distributed idiosyncrasies $\varepsilon_i(t)$. From the sum of stock prices the corresponding index value $I(t)$ may be constructed according to (with $d(t)$ denoting the divisor of the index—here $d(t) = N$)

$$I(t) = \frac{1}{d(t)} \sum_{i=1}^{N} S_i(t), \quad S_i(t) = \exp[s_i(t)] = \exp[s_i(t-1) + \varepsilon_i(t)\sigma(t)]. \tag{2}$$

In a period of downward trend in $I(t)$ investors may find it necessary to replace some investments trying to adapt to the new unfavorable situation, depending on $|r_{\Delta t}(t)|$, their stop-loss strategy, etc. One may say, that

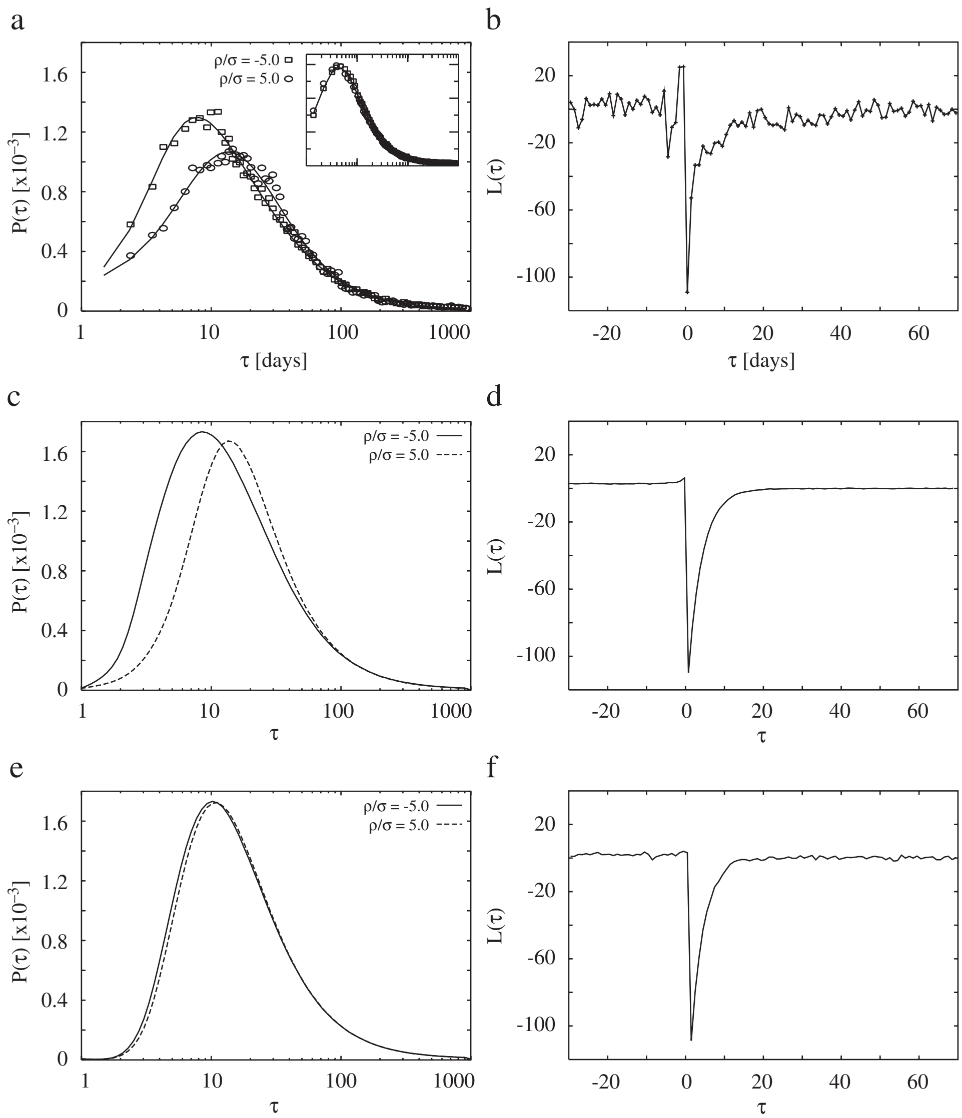

Fig. 1. The investment horizon distributions (first column) and the corresponding leverage correlations (second column) for empirical and model generated index data. In (a) and (b), the statistics obtained for the wavelet detrended daily close of the DJIA are depicted. (c) and (d), show results obtained with the frustration governed market model with $A = 1.8$, $\sigma_0 = 0.0375$, $\Delta t = 1$ and $\kappa = 3.1$. These results seem to show good consistency with the empirical findings of the DJIA. Finally (e) and (f) show that within the present model, leverage does not necessarily imply a pronounced gain-loss asymmetry. Parameter values are $A = 0.2$, $\sigma_0 = 0.0049$, $\Delta t = 1$ and $\kappa = 3.7$.

during periods of dropping prices, investors get frustrated and as a consequence the volatility of single stocks, $\sigma(t)$, increases [5]. On the other hand, when the trend is upward, it is not a bad idea to keep a certain position; why make a change when investments are heading in a profitable direction? Hence, "excited frustrated states" are gradually relaxed by lowering $\sigma(t)$ towards a more fundamental long-term level, σ_0. With this *ansatz*, the equation describing the dynamics of the volatility $\sigma(t)$ becomes

$$\frac{\partial\sigma(t)}{\partial t} = -\frac{(\sigma(t) - \sigma_0)}{\kappa} - A\Theta[-r_{\Delta t}(t)]r_{\Delta t}(t), \tag{3}$$

where Θ is the Heaviside step-function, $r_{\Delta t}(t)$ the logarithmic return of $I(t)$, κ the characteristic volatility decay time and A a positive amplitude. Notice that Eq. (3) partly resembles a mean-reverting Ornstein–Uhlenbeck

(OU) process [9,10]. However, the nature of the last term of Eq. (3) makes it different from a classic OU-process.

4. Results and discussion

Even though the model defined by Eqs. (2) and (3) is rather simple, space limitations refrain us from a systematic parameter study. Instead we will only present some highlights.

Fig. 1(c) depicts the investment horizons obtained from the average of an ensemble of indices generated by the model described above (parameter values are given in the figure caption). The solid line in Fig. 1(b) shows the investment horizon distribution corresponding to negative return barriers while the dashed line represents the corresponding positive values. From the position of the optimal investment horizons, τ^*_ρ, it is clear that the model shows a gain-loss asymmetry resembling the one found for the DJIA.

Fig. 1(d) presents the corresponding leverage correlations showing very good agreement with the empirical findings of Fig. 1(b). The correlations show that negative returns cause volatility to increase, not very surprisingly though, since it is the basis of the very construction of the model. Positive correlations for $\tau < 0$ can also be observed in the figure, consistent with empirical leverage correlations described in Ref. [4] and Fig. 1(b). While positive index increments contribute to the positive correlations for negative τ-values they also lowers the volatility. As an effect only short ranged correlations appear. It is well documented that days of negative return are followed by so-called positive rebound days capable of causing the positive peak [1].

Figs. 1(e) and (f) show the inverse statistics and leverage correlations, respectively, for another set of parameter values. It is observed that the leverage correlations are comparable to those found above, however, the gain-loss asymmetry has almost vanished. One could be tempted to conclude that the two phenomena do not have the same origin, which may be erroneous due to the normalization in Eq. (1). Further work is needed to clarify this interesting issue.

5. Conclusion and outlook

We have briefly reviewed the stylized facts *gain-loss asymmetry* and *leverage* that both seem to result from collective behavior in stock market dynamics. We have presented the new Frustration Governed Market model that conceptually ascribe these stylized facts to frustrated investors. It is shown that the model is capable of reproducing the empirical findings, but also that leverage within this framework does not necessarily imply the existence of well-pronounced gain-loss asymmetry. Future studies are called upon to resolve this question.

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