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# Localization of surface plasmon polaritons on a random surface

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**Abstract**

We study the possibility of the strong localization of surface plasmon polaritons propagating along a metal surface a finite part of which is randomly rough. The surface roughness is such that the roughness-induced conversion of a surface plasmon polariton propagating on it into volume electromagnetic waves in the vacuum above the surface is suppressed. © 2001 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

Since the prediction of localization of electrons in a disordered random system [1], there has been a great interest in localization phenomenon in the physics community. Although this phenomenon was predicted for "quantum" waves, it is not restricted to these kinds of waves, and should in particular also apply to classical waves in random media. For example, the experimental observation of the localization of light was reported recently in a bulk disordered semiconductor [2,3]. In the present work we discuss the Anderson localization of another type of classical wave by disorder of a different nature, namely the localization of surface plasmon polaritons on a randomly rough metal surface in contact with vacuum. This effect has been believed to be difficult to observe due to its being masked by competing effects such as roughness-induced conversion of the surface plasmon polariton into volume waves in the vacuum above the surface (leakage), and ohmic losses due to the non-vanishing imaginary part of the dielectric function of the metal [4–6]. In this work we show how to circumvent this problem by using a specially designed randomly rough surface that suppresses leakage.

The Anderson localization length of a surface plasmon polariton of frequency $\omega$ propagating along a one-dimensional randomly rough surface of a metal in contact with vacuum can be determined by calculating the amplitude $t(\omega, L)$ of the surface plasmon polariton transmitted through a finite length $L$ of the random surface. The surface plasmon polariton transmission coefficient is then given by $T(\omega, L) = |t(\omega, L)|^2$. For large $L$ the average of the self-averaging quantity $\ln T(\omega, L)$ over

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the ensemble of realizations of the random surface, $\langle \ln T(\omega, L) \rangle$, is expected to display a linear dependence on $L$,

$$\langle \ln T(\omega, L) \rangle = \text{const.} - L/\ell_T(\omega), \tag{1.1}$$

where the characteristic length $\ell_T(\omega)$ is called the Lyapunov exponent. It is not the localization length of the surface plasmon polariton $\ell(\omega)$, but can instead be related to the latter by

$$\ell_T^{-1}(\omega) = \ell^{-1}(\omega) + \ell_\varepsilon^{-1}(\omega) + \ell_{\text{rad}}^{-1}(\omega), \tag{1.2}$$

where $\ell_\varepsilon(\omega)$ and $\ell_{\text{rad}}(\omega)$ are, respectively, the characteristic decay lengths associated with the leakage and ohmic losses [7].

In the present paper we analyze analytically and numerically the propagation of surface plasmon polaritons along a metal surface a finite part of which is randomly rough. This random roughness is chosen to constitute a random process that suppresses leakage, i.e. the roughness-induced conversion of a surface plasmon polariton propagating on it into volume electromagnetic waves in the vacuum above the surface. The use of a random surface that suppresses leakage facilitates the investigation of the strong localization of surface plasmon polaritons by random surface roughness by removing the contribution $\ell_{\text{rad}}^{-1}(\omega)$ from the expression for $\ell_T^{-1}(\omega)$ (Eq. (1.2)).

In the approach to the suppression of leakage taken by Sornette and his colleagues [4,5], it was assumed that the random surface was not planar on average, but periodic, so that the dispersion curve of the surface plasmon polaritons supported by the mean surface displays a gap at the boundary of the one-dimensional first Brillouin zone defined by the period of the mean surface. Leakage should then either vanish or decrease significantly for the surface plasmon frequency at the band edge. However, this was not observed in the numerical simulation calculations of leakage carried out in Refs. [6,8].

In this work we first present an approach to designing a one-dimensional random surface that suppresses the leakage of a surface plasmon polariton as it propagates across it that differs from that proposed by Sornette et al. [4,5]. Although the power spectrum of the resulting surface is nonzero in a narrow range of wave numbers, that surface is not periodic on average. However, as with the surface proposed by Sornette et al. our surface is specific to the frequency of the surface plasmon polariton propagating across it: if that frequency is changed, a new surface has to be designed.

For a weakly rough random surface of this nature we analyze the possibility of the localization of surface plasmon polaritons by an analytic approach. In the case of a strongly rough surface we solve the problem of surface polariton propagation numerically.

## 2. The transmitted field

We study the scattering of a p-polarized surface plasmon polariton of frequency $\omega$ propagating in the $x_1$-direction that is incident on a segment of a one-dimensional randomly rough surface defined by the equation $x_3 = \zeta(x_1)$. The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of $x_1$ that is nonzero only in the interval $-L/2 < x_1 < L/2$ (Fig. 1).

The region $x_3 > \zeta(x_1)$ is vacuum and the region $x_3 < \zeta(x_1)$ is a metal characterized by an isotropic, frequency-dependent, complex dielectric function $\varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega)$. We are interested in the frequency range in which $\varepsilon_1(\omega) < -1$, $\varepsilon_2(\omega) > 0$, within which surface plasmon polaritons exist. We write the surface profile function $\zeta(x_1)$ in the form

$$\zeta(x_1) = \Gamma(x_1)s(x_1), \tag{2.1}$$

where $s(x_1)$ is a single-valued function of $x_1$ that is differentiable and constitutes a stationary,

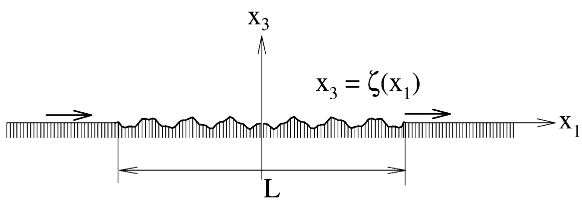


Fig. 1. The scattering system considered in the study of Anderson localization of surface plasmon polaritons on a random surface.

zero-mean, Gaussian random process defined by

$$\langle s(x_1)\rangle = 0, \tag{2.2a}$$

$$\langle s(x_1)s(x'_1)\rangle = \delta^2 W(|x_1 - x'_1|), \tag{2.2b}$$

$$\langle s^2(x_1)\rangle = \delta^2. \tag{2.2c}$$

The angle brackets in Eqs. (2.2) denote an average over the ensemble of realizations of $s(x_1)$, $\delta$ is the rms height of the roughness, and $W(|x_1 - x'_1|)$ is the surface height autocorrelation function. The form of the power spectrum of the surface roughness, which is defined by

$$g(|Q|) = \int_{-\infty}^{\infty} dx_1\, e^{-iQx_1} W(|x_1|), \tag{2.3}$$

appropriate for our purposes will be specified below. The function $\Gamma(x_1)$ serves to restrict the nonzero values of $\zeta(x_1)$ to the interval $-L/2 < x_1 < L/2$. One form $\Gamma(x_1)$ can have is

$$\Gamma(x_1) = \theta\left(\frac{L}{2} + x_1\right)\theta\left(\frac{L}{2} - x_1\right), \tag{2.4a}$$

where $\theta(x_1)$ is the Heaviside unit step function. A smoother, differentiable version of $\Gamma(x_1)$ is provided by

$$\Gamma(x_1) = \frac{1 + \cosh(1/2)\kappa L}{\cosh \kappa x_1 + \cosh(1/2)\kappa L}, \tag{2.4b}$$

where the parameter $\kappa$ controls the range of $x_1$ values over which $\Gamma(x_1)$ decreases from 1 to 0. In view of the factor $\Gamma(x_1)$ in Eq. (2.1), the surface profile function $\zeta(x_1)$ is not a stationary random process even though $s(x_1)$ is.

We assume that the surface roughness is sufficiently weak that the surface profile function $\zeta(x_1)$ satisfies the conditions for the validity of the Rayleigh hypothesis [9]. In this case the single nonzero component of the magnetic field in the vacuum region $x_3 > \zeta(x_1)_{\max}$ can be written as the sum of the fields of the incident and scattered waves

$$H_2^{>}(x_1, x_3|\omega) = \exp[ikx_1 - \beta_0(k,\omega)x_3] + \int_{-\infty}^{\infty} \frac{dq}{2\pi} R^{>}(q,\omega)\exp[iqx_1 - \beta_0(q,\omega)x_3], \tag{2.5a}$$

while in the region of the metal, $x_3 < \zeta(x_1)_{\min}$,

$$H_2^{<}(x_1, x_3|\omega) = \exp[ikx_1 + \beta(k,\omega)x_3] + \int_{-\infty}^{\infty} \frac{dq}{2\pi} R^{<}(q,\omega)\exp[iqx_1 + \beta(q,\omega)x_3]. \tag{2.5b}$$

In Eqs. (2.5) $k$ is the wave number of the incident surface plasmon polariton,

$$k = \frac{\omega}{c}\left[\frac{\varepsilon(\omega)}{\varepsilon(\omega)+1}\right]^{1/2} = k_1(\omega) + ik_2(\omega), \tag{2.6}$$

while the functions $R^{>}(q,\omega)$ and $R^{<}(q,\omega)$ are the scattering amplitudes of the surface plasmon polariton in the vacuum and in the metal, respectively, and

$$\beta_0(q,\omega) = \left(q^2 - \frac{\omega^2}{c^2}\right)^{1/2}, \quad \mathrm{Re}\,\beta_0(q,\omega) > 0, \quad \mathrm{Im}\,\beta_0(q,\omega) < 0, \tag{2.7a}$$

$$\beta(q,\omega) = \left(q^2 - \varepsilon(\omega)\frac{\omega^2}{c^2}\right)^{1/2} \mathrm{Re}\,\beta(q,\omega) > 0, \quad \mathrm{Im}\,\beta(q,\omega) < 0. \tag{2.7b}$$

The scattering amplitude $R^{>}(q,\omega)$ satisfies the reduced Rayleigh equation [10]

$$R^{>}(p,\omega) = G_0(p)\left\{v(p|k)J(\beta(p,\omega) - \beta_0(k,\omega)|p-k) + \int_{-\infty}^{\infty}\frac{dq}{2\pi} v(p|q)J(\beta(p,\omega) - \beta_0(q,\omega)|p-q)R^{>}(q,\omega)\right\}, \tag{2.8}$$

where

$$J(\gamma|Q) = \int_{-\infty}^{\infty} dx_1\, e^{-iQx_1}\frac{e^{\gamma\zeta(x_1)} - 1}{\gamma} \tag{2.9}$$

and

$$v(p|q) = \frac{1-\varepsilon(\omega)}{\varepsilon(\omega)}[pq - \beta(p,\omega)\beta_0(q,\omega)], \tag{2.10}$$

while

$$G_0(p) = \frac{\varepsilon(\omega)}{\varepsilon(\omega)\beta_0(p,\omega) + \beta(p,\omega)} \tag{2.11}$$

is the Green's function of surface plasmon polaritons associated with the planar surface. We note that $G_0(p)$ has simple poles at $p = \pm k$.

If we introduce the transition matrix $T(p|k)$ by the relation

$$R^{>}(p,\omega) = G_0(p)T(p|k), \tag{2.12}$$

then Eq. (2.8) takes the form

$$T(p|k) = V(p|k) + \int_{-\infty}^{\infty} \frac{dq}{2\pi} V(p|q)G_0(q)T(q|k), \tag{2.13}$$

where the scattering potential $V(p|q)$ is given by

$$V(p|q) = v(p|q)J(\beta(p,\omega) - \beta_0(q,\omega)|p-q). \tag{2.14}$$

From Eqs. (2.5a) and (2.12) we see that the scattered field in the vacuum region can be written in the form

$$H_2^{>}(x_1, x_3|\omega)_{sc} = \int_{-\infty}^{\infty} \frac{dq}{2\pi} G_0(q)T(q|k)e^{iqx_1 - \beta_0(q,\omega)x_3}. \tag{2.15}$$

The field of the scattered surface plasmon polariton in the region $x_1 > L/2$ is given by the contribution from the pole in the integrand of the integral in Eq. (2.15), and has the form

$$H_2^{>}(x_1 > L/2, x_3|\omega)_{sc} = iCT(k|k)e^{ikx_1}e^{-\beta_0(k,\omega)x_3}, \tag{2.16}$$

where

$$C = \frac{\sqrt{-\varepsilon^3(\omega)}}{\varepsilon^2(\omega) - 1} \tag{2.17}$$

is the residue of the Green's function $G_0(q)$ at $q = \pm k$. The amplitude of the transmitted surface polariton is therefore

$$t(\omega, L) = 1 + iCT(k|k). \tag{2.18}$$

The surface plasmon polariton transmission coefficient $T(\omega, L)$ is then defined by

$$T(\omega, L) = \frac{P_{tr}(L/2)}{P_{inc}(-L/2)} = |t(\omega,L)|^2 \exp\left(-\frac{L}{\ell_\varepsilon(\omega)}\right), \tag{2.19}$$

where

$$\ell_\varepsilon(\omega) = \frac{1}{2k_2(\omega)} \tag{2.20}$$

is the propagation length of the surface plasmon polaritons due to the ohmic losses in the metal, and gives the fraction of the flux entering the random segment of the metal surface at $x_1 = -L/2$, $P_{inc}(-L/2)$, that leaves it at $x_1 = L/2$, $P_{tr}(L/2)$.

From Eq. (2.19) we find that

$$\langle \ln T(\omega, L)\rangle = \langle \ln |t(\omega, L)|^2\rangle - \frac{L}{\ell_\varepsilon(\omega)}. \tag{2.21}$$

We can rewrite Eq. (2.21) in the form

$$\langle \ln T(\omega, L)\rangle = 2\mathrm{Re}\langle \ln[t(\omega, L)]\rangle - \frac{L}{\ell_\varepsilon(\omega)}. \tag{2.22}$$

Therefore, in view of Eqs. (1.1) and (1.2), and in the absence of leakage, we expect that

$$2\mathrm{Re}\langle \ln[t(\omega, L)]\rangle = \mathrm{const.} - \frac{L}{\ell(\omega)}. \tag{2.23}$$

## 3. The random surface

Before proceeding, several remarks on the properties of the rough surface have to be made. From Eq. (2.5a) it follows that the total power scattered into the vacuum above the surface is

$$P_{sc} = L_2 \frac{\omega}{16\pi^2} \int_{-\pi/2}^{\pi/2} d\theta_s \cos^2\theta_s \left| R^{>}\left(\frac{\omega}{c}\sin\theta_s, \omega\right)\right|^2, \tag{3.1}$$

where $L_2$ the length of the surface along the $x_2$-axis. The scattering angle $\theta_s$, measured clockwise from the $x_3$-axis, is related to the wavenumber $q$ by $q = (\omega/c)\sin\theta_s$. Since the integrand in Eq. (3.1) is non-negative, we see that the only way in which leakage can be suppressed, i.e. the only way in which $P_{sc}$ can be made to vanish, is to design a one-dimensional random surface for which the amplitude $R^{>}(q,\omega)$ is identically zero for $-(\omega/c) < q < (\omega/c)$. Several ways to design surfaces that give rise to specified angular distributions of the scattered intensity have been proposed recently [11,12]. To suppress the leakage we will use a surface characterized by the power spectrum [13]

$$g(|Q|) = \frac{\pi}{2\Delta k}[\theta(Q - k_{\min})\theta(k_{\max} - Q) + \theta(-Q - k_{\min})\theta(k_{\max} + Q)], \tag{3.2}$$

where

$$k_{\min} = 2k_1(\omega) - \Delta k, \tag{3.3a}$$

$$k_{\max} = 2k_1(\omega) + \Delta k, \tag{3.3b}$$

and $\Delta k$ must satisfy the inequality

$$\Delta k \ll k_1(\omega) - (\omega/c). \tag{3.4b}$$

That a surface characterized by the power spectrum (3.2) suppresses leakage can be seen from the following argument: The incident surface plasmon polariton has a wave number whose real part is $k_1(\omega)$. After its first interaction with the surface roughness it will be scattered into waves the real parts of whose wave numbers lie in the two intervals $(3k_1(\omega) - \Delta k, 3k_1(\omega) + \Delta k)$ and $(-k_1(\omega) - \Delta k, -k_1(\omega) + \Delta k)$. This is because the wave numbers in the spectrum of the surface roughness with which $k_1(\omega)$ can combine lie in the intervals $(2k_1(\omega) - \Delta k, 2k_1(\omega) + \Delta k)$ and $(-2k_1(\omega) - \Delta k, -2k_1(\omega) + \Delta k)$. For the same reason, after the scattered waves interact again with the surface roughness the real parts of the wave numbers of the doubly scattered waves will lie in the three intervals $(5k_1(\omega) - 2\Delta k, 5k_1(\omega) + 2\Delta k)$, $(k_1(\omega) - 2\Delta k, k_1(\omega) + 2\Delta k)$, and $(-3k_1(\omega) - 2\Delta k, -3k_1(\omega) + 2\Delta k)$. After the third interaction with the surface roughness the real parts of the wave number of the scattered waves will lie in the four intervals $(7k_1(\omega) - 3\Delta k, 7k_1(\omega) + 3\Delta k)$, $(3k_1(\omega) - 3\Delta k, 3k_1(\omega) + 3\Delta k)$, $(-k_1(\omega) - 3\Delta k, -k_1(\omega) + 3\Delta k)$, and $(-5k_1(\omega) - 3\Delta k, -5k_1(\omega) + 3\Delta k)$, and so on. Thus, for example, if $-k_1(\omega) + 3\Delta k < -(\omega/c)$, so that $\Delta k < \frac{1}{3}(k_1(\omega) - (\omega/c))$, after triple scattering the surface plasmon polaritons will not be converted into volume electromagnetic waves. In general, if we wish the surface plasmon polariton to scatter $n$ times from the surface roughness without being converted into volume electromagnetic waves, we must require that $\Delta k < (1/n)(k_1(\omega) - (\omega/c))$. It is clear that the strongest scattering processes are those whose final states are backward or forward propagating surface plasmon polaritons, since they result in propagating excitations while all others final states are strongly decaying electromagnetic waves.

A randomly rough surface with a power spectrum of the form of two Gaussian peaks centered at $q = \pm 2k_1(\omega)$ has been assumed in Ref. [14] in a search for the localization-induced enhancement of the surface plasmon polariton field. However, such a power spectrum is nonzero in the range $|q| < (\omega/c)$ and, therefore, such a surface does not suppress leakage.

## 4. Analytical arguments

Let us consider Eq. (2.13) for the transition matrix $T(q|k)$. In deriving this equation we have assumed that the conditions for the validity of Rayleigh hypothesis are satisfied. The scattering potential $V(q|p)$, given by Eq. (2.14), does not have any poles in the complex plane of the variables $q$ and $p$, and the inequality $(\mathrm{e}^{(\beta(k,\omega) - \beta_0(k,\omega))\zeta(x_1)} - 1) \ll 1$ is satisfied. In view of the power spectrum of the surface roughness assumed, the main contribution to the integral term in Eq. (2.13) comes from the poles of the Green's function $G_0(p)$. In the pole approximation for the Green's function [15] we can rewrite Eq. (2.13) as

$$T(q|k) = V(q|k) + \mathrm{i}CV(q|k)T(k|k) + \mathrm{i}CV(q|-k)T(-k|k), \tag{4.1a}$$

$$T(k|k) = \mathrm{i}CV(k|-k)T(-k|k), \tag{4.1b}$$

$$T(-k|k) = V(-k|k) + \mathrm{i}CV(-k|k)T(k|k). \tag{4.1c}$$

From Eqs. (4.1) we obtain

$$T(q|k) = \frac{V(q|k) + \mathrm{i}CV(q|-k)V(-k|k)}{1 + C^2V(k|-k)V(-k|k)} \tag{4.2a}$$

and

$$T(k|k) = \frac{\mathrm{i}CV(k|-k)V(-k|k)}{1 + C^2V(k|-k)V(-k|k)}. \tag{4.2b}$$

Therefore, from Eq. (2.18) we find that the amplitude of the transmitted field is given by

$$t(\omega, L) = \frac{1}{1 + C^2 V(k|-k)V(-k|k)}. \tag{4.3}$$

Using Eq. (4.3) we can calculate the desired quantity $\langle \ln T(\omega, L)\rangle$,

$$\begin{aligned}\langle \ln T(\omega, L)\rangle &= 2\mathrm{Re}\langle \ln t(\omega, L)\rangle \\ &= -2\mathrm{Re}\langle \ln[1 + C^2 V(k|-k)V(-k|k)]\rangle. \end{aligned}\tag{4.4}$$

In order to calculate the average in the second line of Eq. (4.4) we shall use the Taylor expansion of the logarithm

$$\begin{aligned}&\langle \ln[1 + C^2 V(k|-k)V(-k|k)]\rangle \\ &= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} C^{2n}\langle [V(k|-k)V(-k|k)]^n\rangle,\end{aligned}\tag{4.5}$$

and calculate the moments $\langle [V(k|-k)V(-k|k)]^n\rangle$. From Eq. (2.14) it follows that

$$\begin{aligned}&\langle [V(k|-k)V(-k|k)]^n\rangle \equiv [v(k|-k)v(-k|k)]^n \\ &\times \langle J^n(\beta(k,\omega) - \beta_0(k,\omega)|2k)J^n(\beta(k,\omega) \\ &- \beta_0(k,\omega)|-2k)\rangle.\end{aligned}\tag{4.6}$$

To calculate these moments we use their representation in terms of cumulant averages [16,17]. In doing so we will use the fact that due to the non-stationarity of the surface the cumulant average of $J(\gamma|Q)$ is given by

$$\langle J(\gamma|Q)\rangle_c = L\,\mathrm{sinc}\left[Q\frac{L}{2}\right]\frac{1}{\gamma}(\mathrm{e}^{\gamma^2\delta^2/2} - 1), \tag{4.7}$$

the cumulant average of the product $J(\gamma|Q_1)J(\gamma|Q_2)$ is given by

$$\begin{aligned}\langle J(\gamma|Q_1)J(\gamma|Q_2)\rangle_c &= j(Q_1, Q_2) \\ &\equiv L^2\int_{-\infty}^{\infty}\frac{\mathrm{d}r}{2\pi}\mathrm{sinc}\left[\frac{(Q_1 - r)L}{2}\right] \\ &\times \mathrm{sinc}\left[\frac{(Q_2 + r)L}{2}\right]\hat{g}(r),\end{aligned}\tag{4.8}$$

where

$$\hat{g}(r) = \frac{1}{\gamma^2}\mathrm{e}^{\gamma^2\delta^2}\int_{-\infty}^{\infty}\mathrm{d}u\,\mathrm{e}^{-iru}[\mathrm{e}^{\delta^2\gamma^2 W(|u|)} - 1] \tag{4.9}$$

and so on. As a result, the cumulants of any odd numbers of $J$ functions from Eq. (4.6) are found to be proportional to at least one factor $\mathrm{sinc}(k_1 L)$, which is small when $k_1 L \gg 1$. Only the cumulant averages of products of even numbers of $J$ functions, which contain equal numbers of $J(\gamma|2k)$ and $J(\gamma|-2k)$, do not contain this small factor.

When the length of the rough part of the surface is not very large, so that $L\delta^2 \ll |\varepsilon(\omega)|\lambda^3$, where $\lambda = (2\pi c/\omega)$, the main contribution to the average of the product of $J$ functions comes from the products of pair cumulant averages, so that the moments are found to be given by

$$\begin{aligned}&\langle J^n(\beta(k,\omega) - \beta_0(k,\omega)|2k)J^n(\beta(k,\omega) - \beta_0(k,\omega)|-2k)\rangle \\ &\approx n![\langle J(\beta(k,\omega) - \beta_0(k,\omega)|2k) \\ &\times J(\beta(k,\omega) - \beta_0(k,\omega)|-2k)\rangle_c]^n \\ &= n!j^n(2k, -2k).\end{aligned}\tag{4.10}$$

With the use of Eq. (4.10) we can rewrite the infinite series in Eq. (4.5) as

$$\begin{aligned}&\langle \ln[1 + C^2 V(k|-k)V(-k|k)]\rangle \\ &= C^2 v(k|-k)v(-k|k)\sum_{n=0}^{\infty}(-1)^n n! C^{2n} \\ &\times [v(k|-k)v(-k|k)j(2k, -2k)]^n \\ &= C^2 v(k|-k)v(-k|k)j(2k, -2k) \\ &\times \int_0^{\infty}\mathrm{d}u\frac{\mathrm{e}^{-u}}{1 + C^2 v(k|-k)v(-k|k)j(2k, -2k)u}.\end{aligned}\tag{4.11}$$

In the limit of a weakly rough surface, $\delta \ll \lambda\sqrt{|\varepsilon(\omega)|}$, the function $\hat{g}(r)$ can be approximated by $\hat{g}(2k) = \delta^2 g(|r|)$, so that $j(2k, -2k)$ takes the form

$$\begin{aligned}j(2k, -2k) = L^2\delta^2\int_{-\infty}^{\infty}\frac{\mathrm{d}r}{2\pi}g(|r|) \\ \times \mathrm{sinc}^2\left[(2k - r)\frac{L}{2}\right].\end{aligned}\tag{4.12}$$

When the length of the rough part of the surface is small, so that the conditions $L\Delta k \ll 1$ and $k_2 L \ll 1$ are satisfied, the function $j(2k, -2k)$ becomes $j(2k, -2k) = L^2\delta^2/2$. Since in this case

$$C^2 v(k|-k)v(-k|k)j(2k, -2k) \ll 1, \tag{4.13}$$

the integral in Eq. (4.11) can be replaced by unity, and therefore, we obtain

$$\langle \ln T(\omega, L)\rangle \approx -\mathrm{Re}[C^2L^2\delta^2 v(k|-k)v(-k|k)]. \tag{4.14}$$

To illustrate these results we assume that surface polaritons of frequency $\omega$ corresponding to a wavelength $\lambda = (2\pi c/\omega) = 457.9$ nm propagate along the silver surface, whose dielectric function at this wavelength is $\varepsilon(\omega) = -7.5 + \mathrm{i}0.24$. Let the surface roughness be characterized by the parameters $\Delta k = 0.3(k_1(\omega) - (\omega/c)) \propto 0.15\omega/(c|\varepsilon(\omega)|)$ and the rms height $\delta = 3$ nm. For these parameters the two main characteristic lengths of the problem are $\ell_{\mathrm{corr}} \equiv 1/(\Delta k) = 4\lambda$ and $\ell_\varepsilon \equiv 1/2k_2(\omega) = 30.1\lambda$. Therefore, to satisfy the condition $L\Delta k \ll 1$ the length of the surface should be of the order of a few wavelengths. Then, since $L < \ell_{\mathrm{eff}} = 5.3\lambda$, where $\ell_{\mathrm{eff}}(\omega) = \{\mathrm{Re}[C^2 v(k|-k)v(-k|k)]\}^{-1/2}$, the condition at which Eq. (4.14) have been obtained is satisfied, and the average logarithm of the transmission coefficient of surface plasmon polaritons has the form $\langle \ln T(L)\rangle = \mathrm{const.} - (L/\ell_{\mathrm{eff}}(\omega))^2$.

However, when studying the localization of classical waves we are interested in the limit $L \to \infty$. Therefore, in this limit $L\Delta k \gg 1$, and the function $j(2k|-2k)$ has the form

$$j(2k|-2k) = L\frac{2\pi}{\Delta k}\delta^2. \tag{4.15}$$

If, in addition, the condition (4.13) is satisfied, we obtain the expression for the averaged logarithm of the transmission coefficient in the form

$$\langle \ln T(\omega, L)\rangle \approx -\mathrm{Re}\left[C^2L\frac{2\pi}{\Delta k}\delta^2 v(k|-k)v(-k|k)\right], \tag{4.16}$$

i.e.

$$\langle \ln T(\omega, L)\rangle \approx -\frac{L}{\ell(\omega)}, \tag{4.17}$$

where

$$\ell^{-1}(\omega) = \mathrm{Re}\left[C^2\frac{2\pi}{\Delta k}\delta^2 v(k|-k)v(-k|k)\right] \tag{4.18}$$

and coincides with the scattering length

$$\ell_{\mathrm{sc}}^{-1}(\omega) = 2\Delta_{\mathrm{sc}}(\omega), \tag{4.19}$$

where $\Delta_{\mathrm{sc}}(\omega)$ is the roughness-induced decay rate of surface plasmon polaritons. Indeed, let us introduce the exact Green's function $G(q|p)$ in accordance with Ref. [18] by the relation

$$G_0(p)T(p|k) = \int_{-\infty}^{\infty}\frac{\mathrm{d}q}{2\pi}G(p|q)V(q|k). \tag{4.20}$$

In the limit $L\Delta k \gg 1$ the stationarity of the surface is almost restored and the average Green's function $G(q)$, which is then defined by $\langle G(q|p)\rangle = G(q)2\pi\delta(q-p)$, has the form

$$G(q) = \frac{1}{G_0^{-1}(q) - M(q)}, \tag{4.21}$$

where $M(q)$ is the averaged self-energy defined by $\langle M(q|p)\rangle = M(q)2\pi\delta(q-p)$. The self-energy $M(q|p)$ satisfies the equation [18]

$$M(q|k) = V(q|k) + \int_{-\infty}^{\infty}\frac{\mathrm{d}p}{2\pi}V(q|p)G_0(p)[M(p|k) - \langle M(p|k)\rangle]. \tag{4.22}$$

In the pole approximation the averaged self-energy can be obtained in the same manner in which we calculated the transition matrix $T(q|k)$. The result is

$$\langle M(k|k)\rangle = \frac{\langle \mathrm{i}CV(k|-k)V(-k|k)/[1 + C^2V(k|-k)V(-k|k)]\rangle}{\langle [1 + \mathrm{i}CV(k|-k)]/[1 + C^2V(k|-k)V(-k|k)]\rangle}. \tag{4.23}$$

In the limit in which Eq. (4.16) was obtained the averaged self-energy is given by

$$\langle M(k|k)\rangle \approx \mathrm{i}Cv(k|-k)v(-k|k)j(2k|-2k). \tag{4.24}$$

Therefore, $\Delta_{\mathrm{sc}}(\omega) = \mathrm{Im}\,CM(k)$, and is indeed the roughness–induced decay rate of surface plasmon polaritons.

For example, for the case where the rms height of the surface roughness is $\delta = 3$ nm while the length of the rough part of the surface is small $L < 20\lambda$, the length $\ell_{\mathrm{sc}}(\omega)$ turns out to be $\ell \approx 8\lambda$, and is smaller than the propagation length of surface plasmon polaritons associated with the ohmic losses. However, the exponential decay of the transmission coefficient of surface plasmon polaritons does not necessarily imply that they are localized. For such a weakly rough surface, although the scattering by surface roughness leads to a strong damping of the surface waves, nevertheless they remain propagating electromagnetic waves, since $k_2(\omega) + \Delta_{sp}(\omega) \ll k_1(\omega)$. And if the rough surface is not very long, the surface polaritons can escape the rough part of the surface.

The situation is different when the surface is moderately rough, or the length of the rough part of the surface increases, so that $\delta^2 L \gg |\varepsilon(\omega)|\lambda^3$. In this limit the main contribution to the average of the product of $nJ$ functions comes not from the products of pair cumulants but from the cumulant average of largest order that is

$$\langle J^n(\beta(k,\omega) - \beta_0(k,\omega)|2k)J^n(\beta(k,\omega) - \beta_0(k,\omega)| - 2k)\rangle \approx \langle J^n(\beta(k,\omega) - \beta_0(k,\omega)|2k)J^n(\beta(k,\omega) - \beta_0(k,\omega)| - 2k)\rangle_c = Ln\hat{g}^{2n-1}(2k), \quad (4.25)$$

where $\hat{g}(q)$ is given by Eq. (4.9). In this case we can sum the infinite series in Eq. (4.5) with the result

$$\langle \ln[1 + C^2 V(k| -k)V( -k|k)]\rangle = L\sum_{n=0}^{\infty} ( -1)^n C^{2n+2} \times L[v(k| - k)v( - k|k)]^{n+1}\hat{g}^{2n+1}(2k) = 2L\frac{C^2 v(k| - k)v( - k|k)\hat{g}(2k)}{1 + C^2 v(k| - k)v( - k|k)\hat{g}^2(2k)}. \quad (4.26)$$

The average of the logarithm of the transmission coefficient in this case takes the form

$$\langle \ln T(\omega, L)\rangle = -2L\,\mathrm{Re} \times \left\{\frac{C^2 v(k| - k)v( - k|k)\hat{g}(2k)}{1 + C^2 v(k| - k)v( - k|k)\hat{g}^2(2k)}\right\}. \quad (4.27)$$

Therefore, we obtained the linear dependence

$$\langle \ln T(\omega, L)\rangle \approx -\frac{L}{\ell(\omega)}, \quad (4.28)$$

where

$$\ell^{-1}(\omega) = 2\mathrm{Re}\left\{\frac{C^2 v(k| - k)v( - k|k)\hat{g}(2k)}{1 + C^2 v(k| - k)v( - k|k)\hat{g}^2(2k)}\right\}. \quad (4.29)$$

For a rough surface with an rms height $\delta = 10$ nm, we find that $C^2 v(k| - k)v( - k|k)\hat{g}^2(2k) \gg 1$. Therefore, instead of Eq. (4.27) we can write

$$\langle \ln T(\omega, L)\rangle \approx -2L\,\mathrm{Re}\left[\frac{1}{\hat{g}(2k)}\right], \quad (4.30)$$

so that

$$\ell^{-1}(\omega) = 2\mathrm{Re}\left[\frac{1}{\hat{g}(2k)}\right]. \quad (4.31)$$

Thus, in this case the localization length is $\ell(\omega) \approx 0.1\lambda$. In this case, the surface polariton field is overdamped and the waves cease to propagate. As in the case of a weakly rough surface the length $\ell(\omega)$ coincides with the roughness induced decay length of surface plasmon polaritons, $\ell(\omega) = 1/(2\Delta_{sc}(\omega))$, where $\Delta_{sc}(\omega) = \mathrm{Im}\, CM(k)$, since in this limit the average self energy, Eq. (4.23) is

$$M(k) = -\frac{\mathrm{i}}{C\hat{g}(2k)}. \quad (4.32)$$

Several remarks have to be made concerning the pole approximation we have used here. When using the pole approximation and, thus, reducing the integral equations (2.13) to a system of algebraic equation, we lose the contributions from the nonsingular part of the integrand. This might be significant if the transition matrix $T(q|k)$ has strong peaks. However, the heights of the peaks can be expected to be of the order of $1/\Delta_{sc}(\omega)$ and $1/\Delta k$. Since $\Delta_{sc}(\omega) \gg k_2(\omega)$ and $\Delta k \gg k_2(\omega)$, the contributions of these possible peaks to the integral part of Eq. (2.13) are much smaller than the contribution from the poles of the Green's function $G_0(q)$ and can be neglected.

In the analytical approach described above we assumed that the surface is weakly rough, so that $\delta\sqrt{\varepsilon(\omega)} \ll \lambda$. In the case when the surface roughness is quite strong, so that $\delta\sqrt{\varepsilon(\omega)} \geqslant \lambda$, but the conditions for the validity of the Rayleigh hypothesis are, nevertheless, satisfied, to study the localization of surface plasmon polaritons we solved Eq. (2.13) numerically.

## 5. Numerical solution

In order to solve the equation for the transition matrix care has to be taken. We recall that our ultimate goal is calculate the transmission amplitude $t(\omega, L)$ defined in Eq. (2.18), i.e. essentially to obtain $T(\omega, L)$ from a numerical solution of the inhomogeneous Fredholm integral equation of the second kind satisfied by the transition matrix. In doing so one is facing at least two major challenges: (i) How to calculate the transition matrix $T(p|k)$ at the wave number of the surface plasmon polariton $p = k$, which is complex due to the non-vanishing imaginary part of the dielectric function of the metal, and (ii) how to handle the poles of $G_0(q)$ at $q = \pm k$. The numerical technique used to calculate $T(k|k)$ numerically is a two step process. It is started by observing that for a real (absorbing) metal there will be no poles located directly on the real axis. Therefore, at least in principle, one can calculate $T(p|k)$ for all real arguments $p$. Since the kernel is well defined for all real $q$'s, this is done by converting the integral equation into a set of linear equation that can be solved by standard techniques [19]. For step 2 we notice that the integral equation (2.13) is valid for all momenta $p$ and $q$, both real and complex. Thus, one can calculate the desired transition matrix at $p = k$ by integrating along the real $q$-axis because here $T(q|k)$ is already known from the preceding step, i.e. one calculates

$$T(k|k) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} V(k|q) G_0(q) T(q|k). \tag{5.1}$$

Note that the scattering potential vanishes at $q = k$, $V(q|k) = 0$, since $\beta(k,\omega)\beta_0(k,\omega) \equiv k^2$.

Although the poles are not located on the real axis, they are still rather close to it. Therefore, in the vicinity of $q = \pm k_1$, where the poles are closest to the real axis, the integrand in Eq. (5.1) changes rapidly. Furthermore, $T(q|k)$ might have weaker peaks, due to multiply scattered surface plasmon polaritons, located at $q = \pm(2n+1)k_1(\omega)$ $n = 1, 2, 3 \ldots$. It is therefore beneficial not to use a uniform discretization grid, so that a higher density of points can be used around these wave numbers. This was done by first replacing the upper and lower limits in the integral in Eq. (5.1) by finite values and then subdividing this resulting region of integration into subintervals where different densities of points were allowed for. The integration range was divided into a total of 27 subintervals and, in particular, small intervals with high densities of points were chosen around $\pm k_1(\omega)$, $\pm 3k_1(\omega)$, and $\pm 5k_1(\omega)$. Within each subinterval the grid points corresponding to different densities were obtained by the classic Gauss–Legendre method [19]. The total number of points used in the integration was $N = 1850$.

In this way we solved the integral equation (5.1) satisfied by the transmission matrix $T(k|k)$ needed to calculate e.g. the transmission coefficient of surface plasmon polaritons as a function of the length $L$ of the rough part of the surface, for each particular realization of the surface. In numerical calculations the function $\Gamma(x_1)$ which serves to restrict the nonzero values of $s(x_1)$ to the interval $-L/2 < x_1 < L/2$, was taken in the form given by Eq. (2.4b) with $\kappa = 100L$ so that $s(x_1)$ was cut off smoothly.

The traditional way of generating randomly rough surfaces with a well-defined power spectrum and Gaussian height distribution is to use the so-called Fourier filtering method [20, Appendix A]. This method consists of generating Gaussian uncorrelated random variables that are filtered with the desired (decaying) power spectrum. By Fourier transforming this filtered sequence back into real space one obtains a randomly rough surface with the desired statistical properties. In most implementations of this algorithm, it is beneficial to take advantage of the fast Fourier transform for performing the inverse transform needed. However, to generate numerically surfaces that suppress leakage as defined in the preceding sections, the use of the fast Fourier transform (FFT) is not necessarily the best option. The reason for this is that the power spectrum, according to its definition, is nonvanishing only in a very narrow interval of width $2\Delta k$ about $\pm 2k_1(\omega)$, where $k_1(\omega)$ is the real part of the wave number of the surface plasmon polaritons supported by the planar surface. This has the consequence that the number of points needed in order to resolve the nonvanishing part of the power spectrum in a satisfactory matter is very large. Since it is the widths of the rectangles contained in the power spectrum that makes the surface randomly rough, we want a rather good resolution here. For example for the numerical results for silver to be shown later, $\Delta k \sim 10^{-2}\omega/c$ and the number of points needed in order to generate surfaces in a satisfactory manner by using the FFT was $N \geqslant 10^4$. The FFT for this number of points is a computationally costly algorithm, and we therefore calculated the Fourier transform by straightforward numerical integration for which a high-density discretization in momentum space is possible at lower computational costs. Another advantage of this numerical integration approach is, as we will see below, that the surface now may be generated directly on a non-uniform grid without any need for any interpolation. In Fig. 2 an example of a surface realization generated by the method just described is presented.

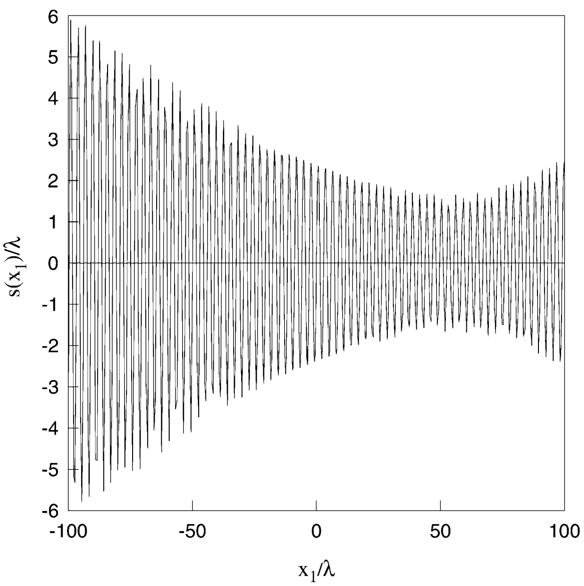


Fig. 2. Example of a random surface.

## 6. Numerical results

We start our discussion of the numerical results by presenting a result that explicitly shows that the surfaces generated in the way described above suppress leakage, i.e. that the scattering amplitude vanishes in the radiative region $|q| < \omega/c$. In Fig. 3 we present a plot of $(\omega/c)^2\langle|R^>(q,\omega)|^2\rangle$ as a function of $cq/\omega$ for a silver surface where the rough portion had a length $L = 20\lambda$. The result plotted in Fig. 3 was calculated from the analytical expression for the transition matrix $T(q|k)$, Eq. (4.2a), obtained in the pole approximation for the Green's function. No expansion in powers of the surface profile function $s(x_1)$ was used when averaging $|R^>(q,\omega)|^2$. The vacuum wavelength of the surface plasmon

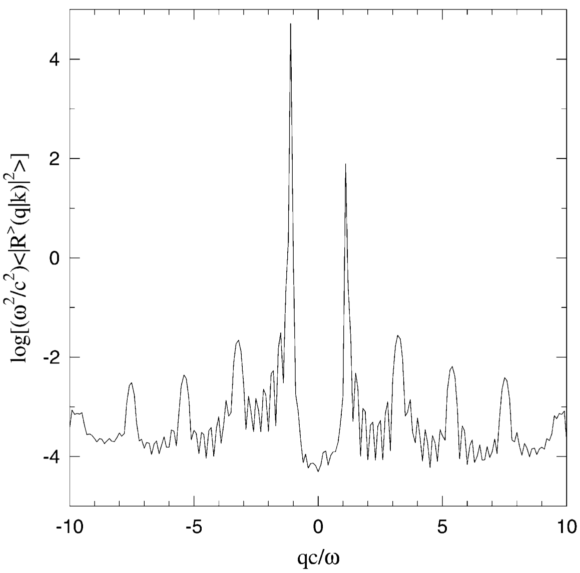


Fig. 3 . Plot of $(\omega/c)^2\langle|R^>(q,\omega)|^2\rangle$ as a function of $cq/\omega$ calculated by averaging analytically the expression (5.1) for the transition matrix $T(q|k)$ for a silver surface characterized by the parameters $\Delta k = 0.3(k_1(\omega) - (\omega/c))$ and $\delta = 3$ nm. The rough portion of the surface has length $L = 20\lambda$. The wave number of the surface plasmon polariton, $k(\omega) = k_1(\omega) + ik_2(\omega) = (1.0741 + i0.0026)\omega/c$, corresponds to a vacuum wavelength of $\lambda = 457.9$ nm, and the dielectric function of silver at this frequency is $\varepsilon(\omega) = -7.5 + i0.24$.

polaritons was taken to be $\lambda = (2\pi c/\omega) = 457.9$ nm, so that the dielectric function of silver at this frequency is $\varepsilon(\omega) = -7.5 + i0.24$. The corresponding wave number of the surface plasmon polariton is $k(\omega) = k_1(\omega) + ik_2(\omega) = (1.0741 + i0.0026)\omega/c$. The surface roughness was characterized by the parameters $\Delta k = 0.3(k_1(\omega) - (\omega/c))$ and $\delta = 3$ nm. With this value of $\Delta k$ the surface should suppress leakage due to scattering processes of up to, and including, third order. We observe from Fig. 3 that $\langle |R^>(q,\omega)|^2 \rangle$ is indeed suppressed in the radiative region. The analogous results obtained by means of a numerical solution of Eq. (2.13) is presented in Fig. 4 for the case where the rms height of the surface roughness was taken to be $\delta = 30$ nm. From this figure we see that although $\langle |R^>(q,\omega)|^2 \rangle$ is heavily suppressed in the region of small values of $q \ll (\omega/c)$, it is far from zero for almost grazing directions of radiation $q \leqslant (\omega/c)$. This is due to the strong higher-order scattering processes which are possible for such a strongly rough surface. Only six peaks corresponding to $\pm k_1(\omega)$, $\pm 3k_1(\omega)$, and $\pm 5k_1(\omega)$ are easily seen in this figure. It should also be noted that when $|q| \geqslant 7\omega/c$, $(\omega/c)^2 \langle |R^>(q|k)|^2 \rangle$ becomes flatter. This flatness is due to leakage setting in for such scattering wave numbers, and they are no longer restricted to well-defined values as is the case for $|q| \leqslant 7\omega/c$. The reason for the rapid dip at $q \approx k_1(\omega)$ is caused by the vanishing of the scattering potential $V(k|q)$ at $q = k$.

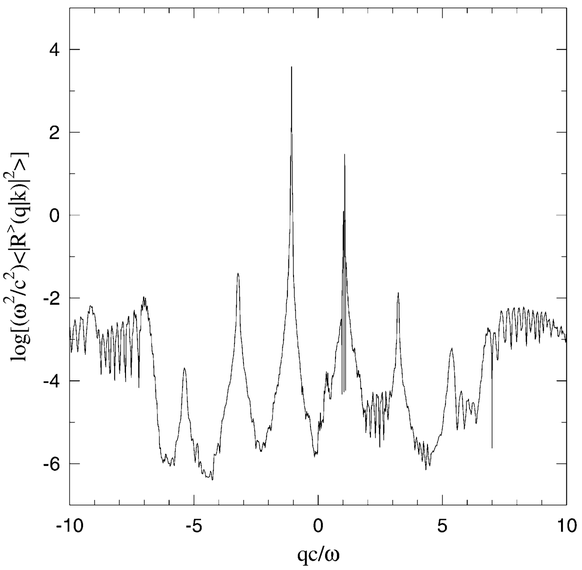


Fig. 4. The same as in Fig. 3 but calculated by means of a numerical solution of Eq. (2.13). The rms height of the surface roughness is $\delta = 30$ nm. The results for 50 realizations of the surface profile function were averaged numerically to obtain the results plotted in this figure.

By numerically solving Eq. (2.13), and calculating $T(k|k)$ by numerical integration in Eq. (5.1), the transmission amplitude $t(\omega, L)$ defined in Eq. (2.18), and the transmission coefficient $T(\omega, L)$ defined by Eq. (2.19) could be calculated for different values of the length $L$ of the rough portion of the surface. From this equation we also recall that the decay due to ohmic losses in the metal could be factored out leaving only possible leakage or Anderson localization in the quantity $|t(\omega, L)|^2$. In Fig. 5 we show numerical simulation results (filled circles) for $\langle \ln T(\omega, L) \rangle$ as a function of the length of the rough portion of the surface. The remaining parameters are those used to obtain the results of Fig. 4. The error bars indicate errors due to the use of a finite number of samples. These errors tend to increase with increasing system size $L$ because of numerical difficulties related to the peaks that can be seen in Fig. 4 becoming narrower and higher.

We observe from Fig. 5 that the behavior of $\langle \ln T(\omega, L) \rangle$ within the error bars is consistent with the behavior predicted in Eq. (2.23), i.e. with an exponential decay of the surface plasmon polariton

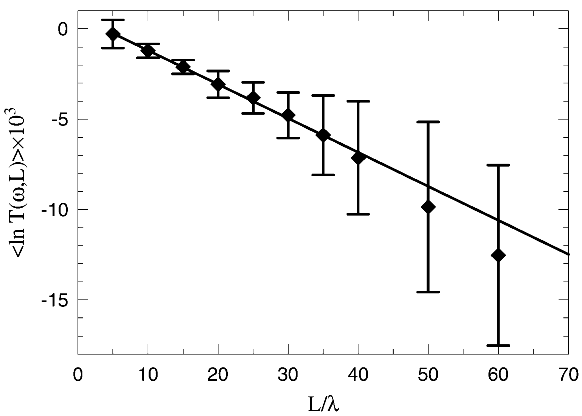


Fig. 5. Numerical simulation results for $\langle \ln T(\omega, L) \rangle$ versus the length of the rough portion of the surface $L$.

transmission coefficient. The solid line in Fig. 5 represents a $\chi^2$-fit [19] to the simulation data. We recall that in the absence of leakage the slope of this straight line gives according to Eq. (2.23) the inverse of the Anderson localization length, $\ell(\omega)$. The numerical value that we obtain in this way is

$$\ell(\omega) = (5319 \pm 905)\lambda. \tag{6.2}$$

Thus, by large-scale numerical simulations we have shown that for such a strongly rough random surface the average logarithm of the transmission coefficient is a linear function of the length of the rough part of the surface. However, in this case the characteristic length of the decay of the transmission coefficient $\ell(\omega)$ is of the order of many thousands of wavelengths. As we have seen in Fig. 4 in this case the bulk electromagnetic waves are quite efficiently radiated into the vacuum in the directions almost parallel to the surface. These scattered bulk waves are, in their turn, scattered by the surface roughness and excite surface plasmon polaritons. Just these processes of reexcitation of surface plasmon polaritons lead to such a long decay length $\ell(\omega)$. Thus, in this case, although the scattering length $\ell_{sc}(\omega)$ might be quite small, any possibility of localization is destroyed by the strong reexcitation of surface plasmon polaritons.

## 7. Conclusions

In this paper we have presented an approach to generating a one-dimensional random surface that suppresses leakage. The suppression of leakage is essential for being able to observe the localization of surface plasmon polaritons on a randomly rough surface. We have shown that in the case of a weakly and moderately rough surface the transmission coefficient $T(\omega, L)$ of surface plasmon polaritons decays exponentially with the length $L$ of the rough part of the surface. The inverse of the characteristic length of the decay $\ell_{sc}^{-1}(\omega)$ is determined by the roughness-induced decay rate of the surface plasmon polaritons $\ell_{sc}^{-1}(\omega) = 2\Delta_{sc}(\omega) = 2\,\mathrm{Im}\,CM(k)$. In the case of a weakly rough surface and when the length $L$ of the rough part of the surface is small, although the localization length is smaller than the propagation length $\ell_{\varepsilon}(\omega)$, which is due to the ohmic losses in the metal, it is large enough to ensure the propagative nature of the surface plasmon polariton field. With the increase of the strength of the surface roughness, the localization length becomes considerably smaller than the vacuum wavelength of the surface plasmon polariton. In this case the surface plasmon polaritons lose their wave-like nature and their field is localized. With a further increase of the rms height, the scattering processes of higher order (higher than the third) become efficient. These processes lead to the appearance of leakage and, more important, to the processes of reexcitation of surface plasmon polaritons. By large-scale numerical simulations we showed that the localization length in this case is of the order of many thousands of wavelengths. This is because in this case the surface plasmon polaritons which propagate along the planar surface away from the rough part of the surface ($x_1 > L/2$) are actually not the transmitted surface plasmon polaritons but can be regarded as the surface plasmon polaritons excited by the effective modes of the rough surface: coupled multiply scattered surface plasmon polaritons and bulk electromagnetic waves.

## Acknowledgements

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