The Angular Intensity Correlation Functions $C^{(1)}$ and $C^{(10)}$ for the Scattering of S-Polarized Light from a One-Dimensional Randomly Rough Dielectric Surface

Ingve Simonsen[a,b], Alexei A. Maradudin[b], and Tamara A. Leskova[c]

[a]Department of Physics, Theoretical Physics Group,
The Norwegian University of Science and Technology,
N-7491 Trondheim, Norway

[b]Department of Physics and Astronomy
and Institute for Surface and Interface Science,
University of California, Irvine, CA 92697 USA

[c]Institute of Spectroscopy, Russian Academy of Sciences,
Troitsk 142092 Russia

ABSTRACT

We calculate the short–range contributions $C^{(1)}$ and $C^{(10)}$ to the angular intensity correlation function for the scattering of s-polarized light from a one-dimensional random interface between two dielectric media. The calculations are carried out on the basis of a new approach that separates out explicitly the contributions $C^{(1)}$ and $C^{(10)}$ to the angular intensity correlation function. The contribution $C^{(1)}$ displays peaks associated with the memory effect and the reciprocal memory effect. In the case of a dielectric–dielectric interface, which does not support surface electromagnetic surface waves, these peaks arise from the coherent interference of multiply-scattered lateral waves supported by the interface. The contribution $C^{(10)}$ is a structureless function of its arguments.

Keywords: correlations, memory effect, rough surfaces, scattering

1. Introduction

Recent theoretical studies [1,2] of angular intensity correlation functions of light scattered incoherently from a weakly rough random metal surface have predicted new correlation functions in addition to the short–range ($C^{(1)}$), long–range ($C^{(2)}$), and infinite–range ($C^{(3)}$) correlation functions predicted in earlier studies of angular intensity correlation functions of light scattered from volume disordered media [3], and subsequently observed experimentally [4-6]. The new correlation functions predicted in the context of rough surface scattering have been labeled the $C^{(10)}$ and

Other author information: (Send correspondence to A.A.M)

A.A.M.: Email: aamaradu@uci.edu; Telephone: 949-824-5943; Fax: 949-824-2174

Part of the SPIE Conference on Scattering and Surface Roughness III • Denver, Colorado • July 1999
SPIE Vol. 3784 • 0277-786X/99/$10.00

$C^{(1.5)}$ correlation functions. The former is of the same order of magnitude as the $C^{(1)}$ correlation function, which describes the memory effect and the reciprocal memory effect, so named because of the wave vector constraints that govern their occurrence. The $C^{(10)}$ correlation effect had been overlooked in an earlier study of angular intensity correlation functions in rough surface scattering [7] because it was based on the factorization approximation [8], an approximation that was not used in Ref. [1, 2]. The $C^{(1.5)}$ correlation function, which also cannot be obtained within the factorization approximation, was predicted to display peaks associated with the roughness-induced excitation of the surface plasmon polaritons supported by the metal surface, just as the $C^{(1)}$ correlation function does when the conditions for the occurrence of the memory effect and the reciprocal memory effect are satisfied. However the $C^{(1.5)}$ correlation function is significantly weaker than the $C^{(1)}$ correlation function. In very recent experimental work [9] the envelopes of the $C^{(1)}$ and $C^{(10)}$ correlation functions have been measured experimentally for the scattering of p-polarized light from weakly rough, one-dimensional gold surfaces. The $C^{(1.5)}, C^{(2)}$, and $C^{(3)}$ correlation functions have yet to be observed experimentally.

In contrast with the theoretical and experimental studies of angular intensity correlation functions of light scattered from randomly rough metal surfaces cited above, there have been no theoretical studies to date of these correlation functions for light scattered from dielectric surfaces, and the only experimental studies [10] of these correlation functions, which are limited to $C^{(1)}$, demonstrate the existence of the memory effect and of the reciprocal memory effect, but do not present results for the envelope of $C^{(1)}$.

In this paper we present a theoretical study of the angular intensity correlation functions $C^{(1)}$ and $C^{(10)}$ of s-polarized light scattered from a one-dimensional random interface between two dielectric media. Although a dielectric-dielectric interface does not support surface electromagnetic waves, it does support lateral waves [11], and one of our aims in this work is to show that these waves can give rise to features in $C^{(1)}$, namely peaks when the conditions for the occurrence of the memory and the reciprocal memory effects are satisfied, just as surface plasmon polaritons do in the case of a vacuum-metal interface. A second aim of this work is to present an approach to the calculation of an angular intensity correlation function that explicitly separates out the contributions $C^{(1)}$ and $C^{(10)}$ to it, allowing each to be calculated separately. The remainder of the correlation function, which represents the sum of the contributions labeled $C^{(1.5)}, C^{(2)}$, and $C^{(3)}$, also allows each of these contributions to be treated separately, and will be the subject of a separate paper.

2. The Scattering System

The physical system we consider in this work consists of a dielectric medium, characterized by a real positive dielectric constant ϵ_0, in the region $x_3 > \zeta(x_1)$ and a dielectric medium, characterized by a real positive dielectric constant ϵ,

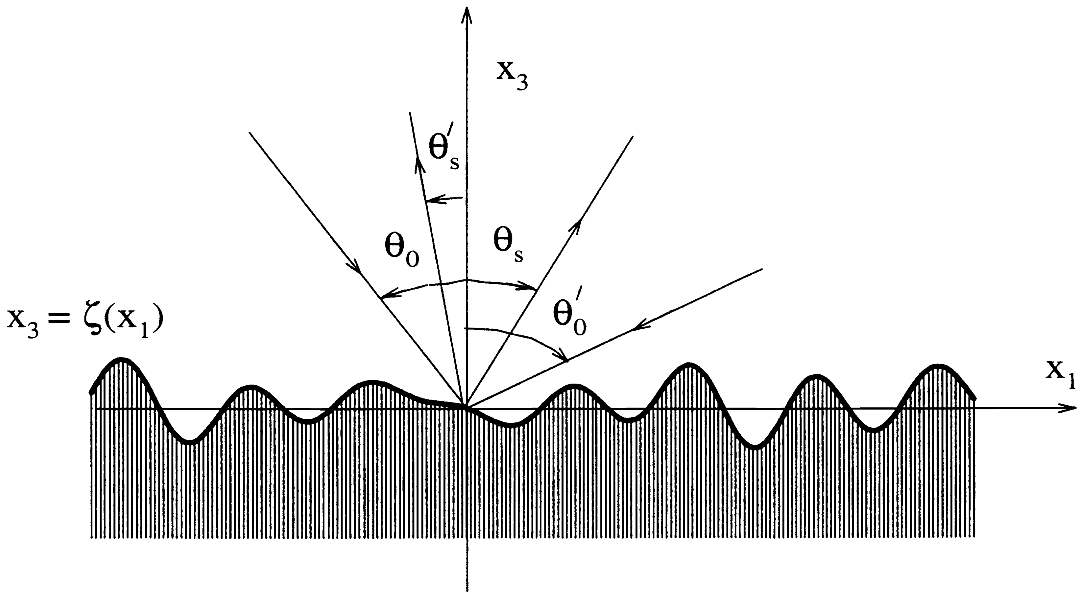


Fig.1 The scattering system studied in this paper.

in the region $x_3 < \zeta(x_1)$ (Fig. 1). The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1 that is differentiable and that constitutes a zero-mean, stationary, Gaussian random process defined by the properties

$$\langle\zeta(x_1)\rangle = 0, \quad \langle\zeta(x_1)\zeta(x_1')\rangle \quad = \quad \delta^2 W(|x_2 - x_1'|). \tag{2.1}$$

In Eqs.(2.1) the angle brackets denote an average over the ensemble of realizations of $\zeta(x_1)$, and $\delta = \langle\zeta^2(x_1)\rangle^{\frac{1}{2}}$ is the rms height of the surface. In numerical calculations we will use the Gaussian form

$$W(|x_1|) = \exp(-x_1^2/a^2) \tag{2.2}$$

for the surface height autocorrelation function. The characteristic length a appearing in the expression is called the transverse correlation length of the surface roughness.

The Fourier coefficient $\hat{\zeta}(k)$ of the surface profile function is also a zero-mean Gaussian random process characterized by the following statistical properties:

$$\langle\hat{\zeta}(k)\rangle = 0, \quad \langle\hat{\zeta}(k)\hat{\zeta}(k')\rangle = 2\pi\delta(k+k')\delta^2 g(|k|), \tag{2.3}$$

where $g(|k|)$, the power spectrum of the surface roughness, is defined by

$$g(|k|) = \int_{-\infty}^{\infty} dx_1 W(|x_1|)e^{-ikx_1}. \tag{2.4}$$

The form of $g(|k|)$ that corresponds to the choice (2.2) for $W(|x_1|)$ is

$$g(|k|) = \sqrt{\pi} a \exp(-a^2/k^2/4). \tag{2.5}$$

3. Scattering Theory

We assume that the surface $x_3 = \zeta(x_1)$ is illuminated from the medium whose dielectric constant is ϵ_0 by an s-polarized electromagnetic wave of frequency ω, whose plane of incidence is the x_1x_3-plane. In the region $x_3 > \zeta(x_1)_{max}$ the single nonzero component of the electric vector of this wave is the sum of an incident wave and a scattered field,

$$E_2^{>}(x_1, x_3|\omega) = \exp[ikx_1 - i\alpha_0(k)x_3] + \int_{-\infty}^{\infty} \frac{dq}{2\pi} R(q|k) \exp[iqx_1 + i\alpha_0(q)x_3], \tag{3.1}$$

where $\alpha_0(q) = \sqrt{\epsilon_0(\omega^2/c^2) - q^2}$, $Re\alpha_0(q) > 0$, $Im\alpha_0(q) > 0$. A time dependence of the form of $\exp(-i\omega t)$ has been assumed in Eq. (3.1) but explicit reference to it has been suppressed. The scattering amplitude $R(q|k)$ satisfies the reduced Rayleigh equation

$$\int_{-\infty}^{\infty} \frac{dq}{2\pi} M(p|q) R(q|k) = N(p|k), \tag{3.2}$$

where

$$M(p|q) = \frac{I(\alpha(p) - \alpha_0(q)|p - q)}{\alpha(p) - \alpha_0(q)}, \quad N(p|k) = -\frac{I(\alpha(p) + \alpha_0(k)|p - k)}{\alpha(p) + \alpha_0(k)}, \tag{3.3}$$

with $\alpha(q) = \sqrt{\epsilon(\omega^2/c^2) - q^2}$, $Re\alpha(q) > 0$, $Im\alpha(q) > 0$, and

$$I(\gamma|Q) = \int_{-\infty}^{\infty} dx_1 \, e^{-iQx_1} e^{-i\gamma\zeta(x_1)}. \tag{3.4}$$

The solution of Eq. (3.2) can be written in the form [12]

$$R(q|k) = -2\pi\delta(q - k) - 2iG(q|k)\alpha_0(k), \tag{3.5}$$

where $G(q|k)$ is the Green's function associated with the random rough interface. It is defined as the solution of the equation

$$G(q|k) \quad = \quad 2\pi\delta(q - k)G_0(k) + \int_{-\infty}^{\infty} \frac{dp}{2\pi} G_0(q) V(q|p) G(p|k), \tag{3.6}$$

where

$$G_0(k) = \frac{i}{\alpha_0(k) + \alpha(k)} \tag{3.7}$$

is the Green's function for a planar dielectric-dielectric interface. The scattering potential $V(q|k)$ is defined by Eqs. (3.2) and (3.6), and is found to satisfy the integral equation

$$\int_{-\infty}^{\infty} \frac{dp}{2\pi} [M(q|p) + N(q|p)] \frac{V(p|k)}{2i\alpha_0(p)} = \{M(q|k)[\alpha_0(k) - \alpha(k)] - N(q|k)[\alpha_0(k) + \alpha(k)]\} \frac{1}{2\alpha_0(k)}. \tag{3.8}$$

In what follows we will need the averaged Green's function $\langle G(q|k)\rangle$. Due to the stationarity of the surface profile function $\zeta(x_1)$, $\langle G(q|k)\rangle$ must be diagonal in q and k, $\langle G(q|k)\rangle = 2\pi\delta(q-k)G(k)$. An application of the smoothing method [12] to Eq. (3.6) yields the result that $G(k)$ is given by

$$G(k) = \frac{1}{G_0^{-1}(k) - M(k)} = \frac{i}{\alpha_0(k) + \alpha(k) - iM(k)}. \tag{3.9}$$

The proper self-energy $M(k)$ appearing in these expressions is defined by the relation $\langle M(q|k)\rangle = 2\pi\delta(q-k)M(k)$, where the unaveraged self-energy $M(q|k)$ is the solution of

$$M(q|k) = V(q|k) + \int_{-\infty}^{\infty} \frac{dp}{2\pi} M(q|p)G_0(p)[V(p|k) - \langle V(p|k)\rangle]. \tag{3.10}$$

In order to calculate $M(k)$ and the Green's function $G(q|k)$ we need the scattering potential $V(q|k)$. We can solve the equation it satisfies, Eq. (3.8), as an expansion in powers of the surface profile function $\zeta(x_1)$. In what follows we will make the small roughness approximation [13]. This consists of approximating $V(q|k)$ by the term of first order in the surface profile function $V^{(1)}(q|k)$:

$$V^{(1)}(q|k) = (\epsilon - \epsilon_0)\frac{\omega^2}{c^2}\hat{\zeta}(q-k). \tag{3.11}$$

The results we obtain are therefore limited to weakly rough surfaces. In the small roughness approximation the self-energy $M(k)$ obtained by Eq. (3.10) is given to lowest nonzero order in $\zeta(x_1)$ by

$$M(k) = W^2 \int_{-\infty}^{\infty} \frac{dp}{2\pi} g(|k-p|)G_0(p), \tag{3.12}$$

where

$$W = \delta(\epsilon - \epsilon_0)\frac{\omega^2}{c^2}. \tag{3.13}$$

4. The Correlation Function $C(q,k|q',k)$

The angular intensity correlation function of interest to us is defined by

$$C(q,k|q',k') = \langle I(q|k)I(q'|k')\rangle - \langle I(q|k)\rangle\langle I(q'|k')\rangle. \tag{4.1}$$

The intensity $I(q|k)$ entering this expression is defined in terms of the scattering matrix $S(q|k)$ for the scattering of light of frequency ω from a one-dimensional random surface by

$$I(q|k) = \frac{\sqrt{\epsilon_0}}{L_1}\left(\frac{\omega}{c}\right)|S(q|k)|^2, \tag{4.2a}$$

where $S(q|k)$ is given in terms of the scattering amplitude $R(q|k)$ by

$$S(q|k) = \frac{\alpha_0^{\frac{1}{2}}(q)}{\alpha_0^{\frac{1}{2}}(k)} R(q|k). \tag{4.2b}$$

The wavenumbers k and q in Eqs. (4.2a) and (4.2b) are related to the angles of incidence and scattering, θ_0 and θ_s, respectively, by (Fig. 1)

$$k = \sqrt{\epsilon_0}(\omega/c)\sin\theta_0, \quad q = \sqrt{\epsilon_0}(\omega/c)\sin\theta_s; \tag{4.3}$$

L_1 is the length of the x_1-axis covered by the random surface; and the angle brackets in Eq. (4.1) denote an average over the ensemble of realizations of the surface profile function $\zeta(x_1)$.

We note that the definition of the intensity used by West and O'Donnell in their study of angular intensity correlation functions in the scattering of light from a random vacuum-metal interface [9] differs by a factor from that given by Eq. (4.2a), namely

$$I(q|k)_{\text{wo}} = \frac{\cos\theta_s}{2\pi\sqrt{\epsilon_0}} I(q|k). \tag{4.4}$$

With the definition of $I(q|k)$ given by Eq. (4.2a) we write the correlation function $C(q,k|q',k')$ in the form

$$C(q,k|q',k') = \epsilon_0 \frac{1}{L_1^2}\left(\frac{\omega}{c}\right)^2 \frac{\alpha_0(q)\alpha_0(q')}{\alpha_0(k)\alpha_0(k')}[\langle|R(q|k)|^2|R(q'|k')|^2\rangle - \langle|R(q|k)|^2\rangle\langle|R(q'|k')|^2\rangle]. \tag{4.5}$$

If we substitute into this expression the result for $R(q|k)$ given by Eq. (3.6), and omit all terms proportional to $\delta(q-k)$ and/or $\delta(q'-k')$, as uninteresting specular contributions, we obtain

$$C(q,k|q',k') = \epsilon_0 \frac{16\epsilon_0}{L_1^2}\left(\frac{\omega}{c}\right)^2 \alpha_0(q)\alpha_0(k)\alpha_0(q')\alpha_0(k')[\langle|G(q|k)|^2|G(q'|k')|^2\rangle - \langle|G(q|k)|^2\rangle\langle|G(q'|k')|^2\rangle]. \tag{4.6}$$

Now $G(q|k)$ satisfies [14]

$$G(q|k) = 2\pi\delta(q-k)G(k) + G(q)t(q|k)G(k), \tag{4.7}$$

where the function $t(q|k)$ is the solution of [14]

$$t(q|k) = w(q|k) + \int_{-\infty}^{\infty}\frac{dp}{2\pi} w(q|p)G(p)t(p|k), \tag{4.8}$$

with

$$w(q|k) = V(q|k) - \langle M(q|k)\rangle. \tag{4.9}$$

Then, again omitting all terms proportional to $\delta(q-k)$ and/or $\delta(q'-k')$, we obtain from Eq. (4.6)

$$C(q,k|q',k') = \epsilon_0 \frac{16\epsilon_0}{L_1^2}\left(\frac{\omega}{c}\right)^2 \alpha_0(q)|G(q)|^2\alpha_0(k)|G(k)|^2 D(q,k|q',k')\alpha_0(q')|G(q')|^2\alpha_0(k')|G(k')|^2, \tag{4.10}$$

where

$$D(q,k|q',k') = \langle t(q|k)t^*(q|k)t(q'|k')t^*(q'|k')\rangle - \langle t(q|k)t^*(q|k)\rangle\langle t(q'|k')t^*(q'|k')\rangle. \tag{4.11}$$

Before proceeding we also introduce the normalized angular intensity correlation function defined by

$$\Xi(q,k|q',k') = \frac{\langle I(q|k)I(q'|k')\rangle - \langle I(q|k)\rangle\langle I(q'|k')\rangle}{\langle I(q|k)\rangle\langle I(q'|k')\rangle}, \tag{4.12}$$

which in terms of the function $t(q|k)$ takes the form

$$\Xi(q,k|q',k') = \frac{D(q,k|q',k')}{\langle|t(q|k)|^2\rangle\langle|t(q'|k')|^2\rangle}. \tag{4.13}$$

We note that this definition of the normalized angular intensity correlation function differs from that introduced by West and O'Donnell [9]. If we note that $\langle t(q|k)\rangle = 0$ (see Eq. (4.7)), we can rewrite $D(q,k|q',k')$ equivalently as

$$D(q,k|q',k') = |\langle t(q|k)t^*(q'|k')\rangle|^2 + |\langle t(q|k)t(q'|k')\rangle|^2 + \{t(q|k)t^*(q|k)t(q'|k')t^*(q'|k')\}, \tag{4.14}$$

where $\{\cdots\}$ denotes the cumulant average [15].

The result given by Eq. (4.14) is very convenient. Due to the stationarity of the surface profile function $\zeta(x_1)$, $\langle t(q|k)t^*(q'|k')\rangle$ is proportional to $2\pi\delta(q-k-q'+k')$. It therefore gives rise to the contribution to $C(q,k|q',k')$ called $C^{(1)}(q,k|q',k')$, and describes the memory effect and the reciprocal memory effect. Similarly, $\langle t(q|k)t(q'|k')\rangle$ is proportional to $2\pi\delta(q-k+q'-k')$, and therefore contributes the correlation function $C^{(10)}(q,k|q',k')$ to $C(q,k|q',k')$. The third term on the right hand side of Eq. (4.14) $\{t(q|k)t^*(q|k)t(q'|k')t^*(q'|k')\}$ due to the stationarity of the surface profile function $\zeta(x_1)$ is proportional to $2\pi\delta(0) = L_1$, and gives rise to the long- and infinite-range contributions to $C(q,k|q',k')$ given by the sum $C^{(1.5)}(q,k|q',k') + C^{(2)}(q,k|q',k') + C^{(3)}(q,k|q'|k')$. Thus, the approach presented here separates explicitly the contributions to $C(q,k|q',k')$ that have been named $C^{(1)}(q,k|q',k')$ and $C^{(10)}(q,k|q',k')$. What is more, this approach explicitly shows the relative magnitudes of the different contributions to the correlation function. Indeed, since $2\pi\delta(0) = L_1$, when the arguments of the delta-functions vanish the $C^{(1)}(q,k|q',k')$ and $C^{(10)}(q,k|q',k')$ correlation functions are independent of the length of the surface L_1, because they contain $[2\pi\delta(0)]^2$. At the same time the remaining term in Eq. (4.14), that yields the sum $C^{(1.5)}(q,k|q',k') + C^{(2)}(q,k|q',k') + C^{(3)}(q,k|q'|k')$, is inversely proportional to the surface length, due to the lack of the second delta function. Therefore, in the limit of a long surface or a large illumination area the long-range and infinite-range correlations vanish.

The explicit expressions for the short–range contributions to the angular intensity correlation functions are

$$C^{(1)}(q,k|q',k') = \epsilon_0\frac{16\epsilon_0}{L_1^2}\left(\frac{\omega}{c}\right)^2\alpha_0(q)|G(q)|^2\alpha_0(k)|G(k)|^2|\langle t(q|k)t^*(q'|k')\rangle|^2\alpha_0(q')|G(q')|^2\alpha_0(k')|G(k')|^2 \tag{4.15}$$

$$C^{(10)}(q,k|q',k') = \epsilon_0\frac{16\epsilon_0}{L_1^2}\left(\frac{\omega}{c}\right)^2\alpha_0(q)|G(q)|^2\alpha_0(k)|G(k)|^2|\langle t(q|k)t(q'|k')\rangle|^2\alpha_0(q')|G(q')|^2\alpha_0(k')|G(k')|^2, \tag{4.16}$$

while the normalized angular intensity correlation functions are given by

$$\Xi^{(1)}(q,k|q',k') = \frac{\langle|\langle t(q|k)t^*(q'|k')\rangle|^2}{\langle|t(q|k)|^2\rangle\langle|t(q'|k')|^2\rangle}, \tag{4.17}$$

$$\Xi^{(10)}(q,k|q',k') = \frac{|\langle t(q|k)t(q'|k')\rangle|^2}{\langle|t(q|k)|^2\rangle\langle|t(q'|k')|^2\rangle}. \tag{4.18}$$

The property of a speckle pattern that is characterized by the presence of the factor $2\pi\delta(q-k-q'+k')$ in $C^{(1)}(q,k|q',k')$ is that if we change the angle of incidence in such a way that k goes into $k' = k+\Delta k$, the entire speckle pattern shifts in such a way that any feature initially at q moves to $q' = q+\Delta k$. This is the reason why the

$C^{(1)}$ correlation function was originally named the memory effect. In terms of the angles of incidence and scattering, we have that if θ_0 is changed to $\theta_0' = \theta_0 + \Delta\theta_0$, any feature in the speckle pattern originally at θ_s is shifted to $\theta_s' = \theta_s + \Delta\theta_s$, where $\Delta\theta_s = \Delta\theta_0(\cos\theta_0/\cos\theta_s)$ to first order in $\Delta\theta_0$.

The property of a speckle pattern that is characterized by the presence of the factor $2\pi\delta(q-k+q'-k')$ in $C^{(10)}(q,k|q',k')$ is that if we change the angle of incidence in such a way that k goes into $k' = k + \Delta k$, a feature at $q = k - \Delta q$ will be shifted to $q' = k' + \Delta q$, i.e. to a point as much to one side of the new specular direction as the original point was on the other side of the original specular direction. For one and the same incident beam the $C^{(10)}$ correlation function therefore reflects the "symmetry" of the speckle pattern with respect to the specular direction.

In this paper we will be concerned only with the short-range correlation functions. The extraction of $C^{(1.5)}, C^{(2)}$, and $C^{(3)}$ from the third term on the right hand side of Eq. (4.14) will be described in a separate paper.

5. The Correlation functions $C^{(1)}$ and $\Xi^{(1)}$.

It has been shown in Ref. [14] that if we rewrite the product $t(q|k)t^*(q'|k')$ in a direct product notation as $\tau^{(1)}(q,q'|k,k')$ the ensemble average of $\tau^{(1)}(q,q'|k,k')$ satisfies the equation

$$\langle\tau^{(1)}(q,q'|k,k')\rangle = \langle\Gamma^{(1)}(q,q'|k,k')\rangle + \int_{-\infty}^{\infty}\frac{dp}{2\pi}\int_{-\infty}^{\infty}\frac{dp'}{2\pi}\langle\Gamma^{(1)}(q,q'|p,p')\rangle G(p)G^*(p')\langle\tau^{(1)}(p,p'|k,k')\rangle. \tag{5.1}$$

In this equation $\langle\Gamma^{(1)}(q,q'k,k')\rangle$ is an irreducible vertex function. In the present work we will approximate it by the sum of all maximally-crossed diagrams in the small roughness approximation. It is the contributions associated with these diagrams that describe the phase-coherent multiple-scattering processes that give rise to the effects we seek. Due to the stationarity of the surface profile function $\zeta(x_1)$ each term in the expansion of $\langle\Gamma^{(1)}(q,q'|k,k')\rangle$ is proportional to $2\pi\delta(q-k-q'+k')$, so that $\langle\Gamma^{(1)}(q,q'|k,k')\rangle$ is given by

$$\begin{aligned}\langle\Gamma^{(1)}(q,q'|k,k')\rangle &= 2\pi\delta(q-k-q'+k')\{W^2g(|k-q|)\\ &\quad+\int_{-\infty}^{\infty}\frac{dp_1}{2\pi}W^2g(|q-p_1|)G(p_1)G^*(q'+k-p_1)W^2g(|p_1-k|)\\ &\quad+\int_{-\infty}^{\infty}\frac{dp_1}{2\pi}\int_{-\infty}^{\infty}\frac{dp_2}{2\pi}W^2g(|q-p_2|)G(p_2)G^*(q'+k-p_2)W^2g(|p_2-p_1|)G(p_1)\\ &\qquad\times G^*(q'+k-p_1)W^2g(|p_1-k|)+\cdots\}.\end{aligned} \tag{5.2}$$

To evaluate this sum we proceed as follows. We write the power spectrum of the surface roughness $g(|q-k|)$ in the separable form

$$g(|q-k|) = \sqrt{\pi}a\exp\left[-\frac{a^2(q-k)^2}{4}\right] = \sum_{\ell=0}^{\infty}\phi_\ell(q)\phi_\ell(k), \tag{5.3a}$$

where

$$\phi_\ell(q) = \left(\frac{\sqrt{\pi}a^{2\ell+1}}{2^\ell\ell!}\right)^{\frac{1}{2}}q^\ell e^{-\frac{a^2}{4}q^2}. \tag{5.3b}$$

In numerical calculations we will replace the upper limit on the sum in Eq. (5.3a) by an integer N, which is increased until a convergent result for $\langle\Gamma^{(1)}(q,q'|k,k')\rangle$ is obtained. With the use of this representation we find that

$$\begin{aligned}\langle\Gamma^{(1)}(q,q'|k,k')\rangle &= 2\pi\delta(q-k-q'+k') \\ &\times\left\{W^2 g(|q-k|)+W^2\sum_{\ell=0}^{N}\sum_{\ell'=0}^{N}\phi_\ell(q)\{[\mathbf{I}-\mathbf{K}(q'+k)]^{-1}\mathbf{K}(q'+k)]\}_{\ell\ell'}\phi_{\ell'}(k)\right\}, \quad (5.4)\end{aligned}$$

where the elements of the $(N+1)\times(N+1)$ matrix $\mathbf{K}(Q)$ are given by

$$K_{\ell\ell'}(Q)=W^2\int_{-\infty}^{\infty}\frac{dp}{2\pi}\phi_\ell(p)G(p)G^*(Q-p)\phi_{\ell'}(p). \quad (5.5)$$

The result given by Eq. (5.4) now has to be substituted into Eq. (5.1), which is then solved by iteration. However, in each of the integral terms in the iterative solution we keep only the contribution associated with the first term on the right hand side of Eq. (5.4), and omit all contributions that contain the second term. The sum of the resulting integral terms is given by

$$\begin{aligned}2\pi\delta(q-k-q'+k')&\left\{\int_{-\infty}^{\infty}\frac{dp_1}{2\pi}W^2 g(|q-p_1|)G(p_1)G^*(q-q'-p_1)W^2 g(|p_1-k|)\right.\\ &+\int_{-\infty}^{\infty}\frac{dp_1}{2\pi}\int_{-\infty}^{\infty}\frac{dp_2}{2\pi}\,W^2 g(|q-p_2|)G(p_2)G^*(q-q'-p_2)W^2 g(|p_2-p_1|)G(p_1)\\ &\left.\times G^*(q-q'-p_1)W^2 g(|p_1-k|)+\dots\right\}. \quad (5.6)\end{aligned}$$

This is nothing more than the sum of the contributions associated with all the ladder diagrams, starting with the two-rung ladder diagram. We sum this infinite series with the use of the representation (5.3a), and obtain

$$2\pi\delta(q-k-q'+k')\;W^2\sum_{\ell=0}^{N}\sum_{\ell'=0}^{N}\phi_\ell(q)\{[\mathbf{I}-\mathbf{K}(q-q')]^{-1}\mathbf{K}(q-q')\}_{\ell\ell'}\phi_{\ell'}(k). \quad (5.7)$$

This contribution equals that of the second term on the right hand side of Eq. (5.4) when $q'=-k'$. Therefore we cannot neglect it in comparison with the latter contribution. On combining this result with the one given by Eq. (5.4) we obtain finally our approximation to $\langle\tau^{(1)}(q,q'|k,k')\rangle$:

$$\begin{aligned}\langle\tau^{(1)}(q,q'|k,k')\rangle &\equiv \langle t(q|k)t^*(q'|k')\rangle \\ &= 2\pi\delta(q-k-q'+k')\{W^2 g(|q-k|)+W^2\sum_{\ell=0}^{N}\sum_{\ell'=0}^{N}\phi_\ell(q)\{\mathbf{I}-\mathbf{K}(q'+k)]^{-1}\mathbf{K}(q'+k)\}_{\ell\ell'}\phi_{\ell'}(k) \\ &\quad +W^2\sum_{\ell=0}^{N}\sum_{\ell'=0}^{N}\phi_\ell(q)\{\mathbf{I}-\mathbf{K}(q-q')]^{-1}\mathbf{K}(q-q')\}_{\ell\ell'}\phi_{\ell'}(k)\}. \quad (5.8)\end{aligned}$$

The substitution of this result into Eq. (4.15) yields our result for $C^{(1)}(q,k|q',k')$. To normalize the correlation function we use the fact that $\langle t(q|k)t^*(q|k)\rangle=\tau^{(1)}(q,q|k,k)$.

6. The Correlation Functions $C^{(10)}$ and $\Xi^{(10)}$

In exactly the same way as Eq. (5.1) was derived in Ref. 14 the correlation function $\langle\tau^{(10)}(q,q'|k,k')\rangle = \langle\tau(q|k)t(q'|k')\rangle$ can be shown to be the solution of the equation

$$\langle\tau^{(10)}(q,q'|k,k')\rangle = \langle\Gamma^{(10)}(q,q'|k,k')\rangle + \int_{-\infty}^{\infty}\frac{dp}{2\pi}\int_{-\infty}^{\infty}\frac{dp'}{2\pi}\Gamma^{(10)}(p,p'|p,p')\rangle G(p)G(p')\langle\tau^{(10)}(q,q'|k,k')\rangle. \tag{6.1}$$

In the present case the averaged irreducible vertex function $\langle\Gamma^{(10)}(q,q'|k,k')\rangle$ will be approximated in the small roughness limit by the sum of the maximally crossed diagrams. Evaluating the contribution associated with the maximally crossed diagrams in a standard manner, we obtain for $\langle\Gamma^{(10)}(q,q'|k,k')\rangle$ the result

$$\begin{aligned}\langle\Gamma^{(10)}(q,q'|k,k')\rangle &= 2\pi\delta(q-k+q'-k')\{W^2g(|q-k|) \\ &+\int_{-\infty}^{\infty}\frac{dp_1}{2\pi}W^2g(|q-p_1|)G(p_1)G(k-q'-p_1)W^2(g(|p_1-k|) \\ &+\int_{-\infty}^{\infty}\frac{dp_1}{2\pi}\int_{-\infty}^{\infty}\frac{dp_2}{2\pi}\,W^2g(|q-p_2|)G(k-q'-p_2)W^2g(|p_2-p_1|) \\ &\qquad\times G(p_1)G(k-q'-p_1)W^2g(|p_1-k|)+\cdots\}\end{aligned} \tag{6.2}$$

In writing this expansion we have used the fact that $G(k)$ is an even function of k. We sum this series with the aid of the decomposition (5.3a) and obtain

$$\begin{aligned}\langle\Gamma^{(10)}(q,q'|k,k')\rangle &= 2\pi\delta(q-k+q'-k') \\ &\times\left\{W^2g(|q-k|)+W^2\sum_{\ell=0}^{N}\sum_{\ell'=0}^{N}\phi_\ell(q)\left\{[\mathbf{I}-\mathbf{L}(k-q')]^{-1}\mathbf{L}(k-q')\right\}_{\ell\ell'}\phi_{\ell'}(k)\right\},\end{aligned} \tag{6.3}$$

where the elements of the $(N+1)\times(N+1)$ matrix $\mathbf{L}(Q)$ are given by

$$L_{\ell\ell'}(Q) = W^2\int_{-\infty}^{\infty}\frac{dp}{2\pi}\phi_\ell(p)G(p)G(Q-p)\phi_{\ell'}(p). \tag{6.4}$$

As in the calculation of $\langle\tau^{(1)}(q,q'|k,k')\rangle$, when the result given by Eq. (6.3) is substituted into Eq. (6.1), and the resulting integral equation is solved by iteration, in each integral term in the resulting expansion only the contribution associated with the first term on the right hand side of Eq. (6.3) is kept, and all contributions that contain the second are omitted. The sum of the resulting integral terms is given by

$$\begin{aligned}2\pi\delta(q-k+q'-k')&\left\{\int_{-\infty}^{\infty}\frac{dp_1}{2\pi}W^2g(|q-p_1|)G(p_1)G(q+q'-p_1)W^2g(|p_1-k|)\right. \\ &+\int_{-\infty}^{\infty}\frac{dp_1}{2\pi}\int_{-\infty}^{\infty}\frac{dp_2}{2\pi}\,W^2g(|q-p_2|)G(p_2)G(q+q'-p_2)W^2g(|p_2-p_1|) \\ &\left.\times\, G(p_1)G(q+q'-p_1)W^2g(|p_1-k|)+\cdots\right\}.\end{aligned} \tag{6.5}$$

This is just the sum of the contributions associated with all the ladder diagrams starting with the two-rung ladder diagram. We sum the infinite series (6.5) with the aid of the representation Eq. (5.3a) to obtain the result that it is given by

$$2\pi\delta(q-k+q'-k')\,W^2\sum_{\ell=0}^{N}\sum_{\ell'=0}^{N}\phi_\ell(q)\{[\mathbf{I}-\mathbf{L}(q+q')]^{-1}\mathbf{L}(q+q')\}_{\ell\ell'}\phi_{\ell'}(k). \tag{6.6}$$

Thus, our approximation to $\langle \tau^{10}(q,q'|k,k')\rangle$ is

$$\begin{aligned}\langle \tau^{(10)}(q,q'|k,k')\rangle &\equiv \langle t(q|k)t(q'|k')\rangle \\ &= 2\pi\delta(q-k+q'-k')\Bigg\{W^2 g(|q-k|) + W^2\sum_{\ell=0}^{N}\sum_{\ell'=0}^{N}\phi_\ell(q)\{[\mathbf{I}-\mathbf{L}(k-q')]^{-1}\mathbf{L}(k-q')\}_{\ell\ell'}\phi_{\ell'}(k) \\ &\qquad + W^2\sum_{\ell=0}^{N}\sum_{\ell'=0}^{N}\phi_\ell(q)\{[\mathbf{I}-\mathbf{L}(q+q')]^{-1}\mathbf{L}(q+q')\}_{\ell\ell'}\phi_{\ell'}(k)\Bigg\}. \end{aligned} \quad (6.7)$$

The second and third terms on the right hand side of this equation are equal when $k'=-k, q'=-q$, and $q=-k$. The substitution of this expression for $\langle\tau^{(10)}(q,q'|k,k')\rangle$ into Eq. (4.16) yields our approximation to $C^{(10)}(q,k|q',k')$. The correlation function $\Xi^{(10)}(q,k|q',k')$ is then readily obtained.

7. Results

To illustrate the results presented in the preceding Sections it is convenient to represent the correlation functions in the form

$$C^{(1)}(q,k|q',k') = 2\pi\delta(q-k-q'+k')\frac{1}{L_1}C_0^{(1)}(q,k|q',q'-q+k), \quad (7.1a)$$

$$\Xi^{(1)}(q,k|q',k') = 2\pi\delta(q-k-q'+k')\frac{1}{L_1}\Xi_0^{(1)}(q,k|q',q'-q+k), \quad (7.1b)$$

and

$$C^{(10)}(q,k|q',k') = 2\pi\delta(q-k+q'-k')\frac{1}{L_1}C_0^{(10)}(q,k|q',q'+q-k), \quad (7.2a)$$

$$\Xi^{(10)}(q,k|q',k') = 2\pi\delta(q-k+q'-k')\frac{1}{L_1}\Xi_0^{(10)}(q,k|q',q'+q-k), \quad (7.2b)$$

where $C_0^{(1)}(q,k|q',q'-q+k)$, $\Xi_0^{(1)}(q,k|q',q'-q+k)$, $C_0^{(10)}(q,k|q',q'+q-k)$, and $\Xi_0^{(10)}(q,k|q',q'+q-k)$ are the envelopes of the correlation functions $C^{(1)}$, $\Xi^{(1)}$, $C^{(10)}$, and $\Xi^{(10)}$, respectively, and are independent of the length of the rough surface. The envelopes are functions of θ_s' for fixed values of θ_0 and θ_s, while θ_0' is determined by the constraint of the δ−function entering the expression for the respective correlation functions. In Fig. 2(a) we plot $C_0^{(1)}(q,k|q',q'-q+k)$ (solid line) and $C_0^{(10)}(q,k|q',q'+q-k)$ (dashed line) when s−polarized light of wavelength $\lambda = 632.8nm$ is incident on a randomly rough dielectric surface from vacuum ($\epsilon_0 = 1$). The roughness parameters are $\delta = 20nm$ and $a = 100nm$. The dielectric constant of the scattering medium is $\epsilon = 2.69$. For the parameters of the scattering system assumed the envelopes of both $C^{(1)}$ and $C^{(10)}$ are structureless functions of θ_s' (the angles $\theta_s = 8^o$ and $\theta_0 = 3^o$ are fixed, while the angle θ_0' is determined by the constraint of the δ-function), and are of almost the same magnitude. The reason for this is that the scattering from a weakly rough dielectric surface is extremely weak, and no coherent effects can be observed for such a system. The results for the envelopes of the normalized

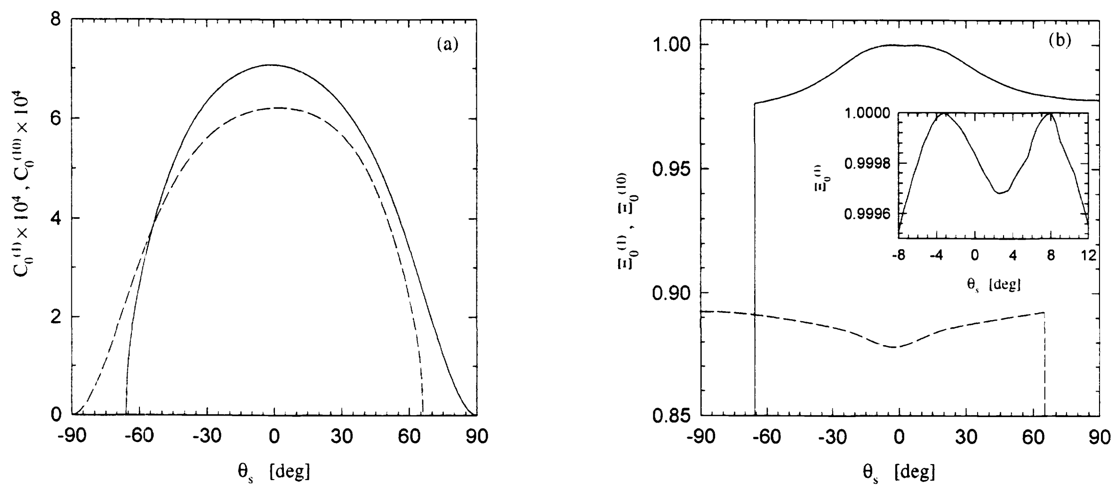
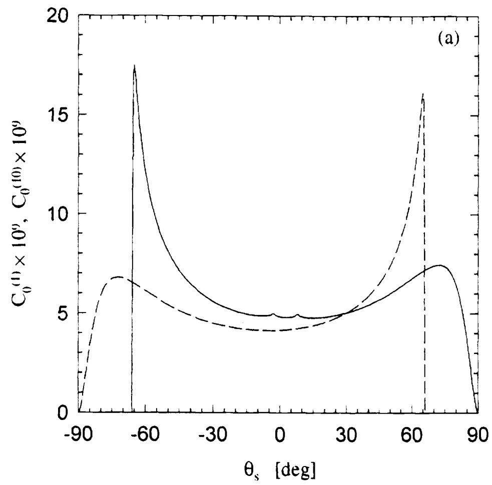


Fig. 2. The envelopes of the $C^{(1)}$ (solid line) and $C^{(10)}$ (dashed line) correlation functions (a) and $\Xi^{(1)}$ (solid line) and $\Xi^{(10)}$ (dashed line) correlation functions (b) as functions of θ_s' when $s-$polarized light is scattered from a randomly rough surface of a dielectric medium with $\epsilon = 2.69$.

correlation functions $\Xi^{(1)}$ (solid line) and $\Xi^{(10)}$ (dashed line) obtained for the same parameters are plotted in Fig. 2(b). The function $\Xi^{(1)}$ shows perfect correlations in the vicinity of the memory and reciprocal memory effect peaks, although the envelope of $C^{(1)}$ does not display any peaks. The minimum in the plot of $\Xi_0^{(10)}(q, k|q', q' + q - k)$ occurs in the vicinity of the backscattering direction and is due to its normalization.

As is known [16], when light is scattered by a rough interface between two dielectric media with low dielectric constrast, although the scattering is weak, nevertheless, an enhanced backscattering peak can be formed in the angular dependence of the intensity of the light scattered incoherently, due to the coherent interference of the direct and reciprocal paths traversed by multiply scattered lateral waves. When such an enhanced backscattering peak occurs one can expect peaks associated with the memory and reciprocal meanory effects to appear in $C_0^{(1)}(q, k|q', q' - q + k)$. This is indeed what is found. In Fig. 3(a) the envelopes of the correlation functions $C^{(1)}$ (solid line) and $C^{(10)}$ (dashed line) are plotted as functions of θ_s' when $s-$polarized light of (vacuum) wavelength $\lambda = 632.8nm$ is incident on the rough interface between two dielectrics characterized by the dielectric constants $\epsilon_0 = 2.6$ and $\epsilon = 2.69$. The roughness parameters are the same ones assumed in obtaining the results plotted in Fig. 2. The envelope of $C^{(1)}$ displays peaks at $q' = q$ (memory effect) and at $q' = -k$ (reciprocal memory effect), while the envelope of $C^{(10)}$ remains structureless, but smaller in its amplitude. The envelopes of the normalized correlation functions $\Xi^{(1)}$ (Fig. 3(b), solid line) and $\Xi^{(10)}$ (Fig. 3(b), dashed line) show that perfect correlations occur in the vicinity of the memory and reciprocal memory effect peaks.



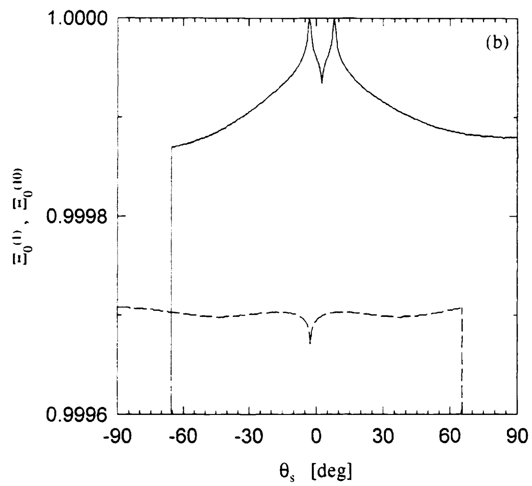


Fig. 3. The envelopes of the $C^{(1)}$ (solid line) and $C^{(10)}$ (dashed line) correlation functions (a) and $\Xi^{(1)}$ (solid line) and $\Xi^{(10)}$ (dashed line) correlation functions (b) as functions of θ_s' when $s-$polarized light is scattered from a randomly rough interface between two dielectric media with $\epsilon_0 = 2.6$ and $\epsilon = 2.69$.

8. Conclusions

In this paper we have presented a theoretical study of the angular intensity correlation functions $C^{(1)}$ and $C^{(10)}$ of s-polarized light scattered from a one-dimensional random interface between two semi-infinite dielectric media. The angular intensity correlation functions are calculated by means of an approach that explicitly separates out the $C^{(1)}$ and $C^{(10)}$ contributions to it. We have shown that in the case of a random vacuum-dielectric interface the envelopes of $C^{(1)}$ and $C^{(10)}$ are structureless functions of the scattering angle θ_s' of almost the same magnitude, nevertheless, the envelope of the normalized correlation function $\Xi^{(1)}$ shows perfect correlations at the positions of the peaks of the memory and reciprocal memory effects. The envelope of the normalized correlation function $\Xi^{(10)}$ displays a minimum at the position of the enhanced backscattering peak, which is due to its normalization.

It is also shown that in the case of a random interface between two dielectrics with a low dielectric contrast the envelope of $C^{(1)}$ displays the peaks of the memory and reciprocal memory effects.

ACKNOWLEDGEMENTS

The work of T .A. L. and A. A. M. was supported in part by Army Research Office Grant No. DAAG 55-98-C-0034. I. S. would like to thank the Research Council of Norway (Contract No. 32690/213) and Norsk Hydro ASA for financial support.

REFERENCES

1. V. Malyshkin, A.R. McGurn, T.A. Leskova, A.A. Maradudin, and M. Nieto–Vesperinas, Opt. Lett. **22**, 946 (1997).
2. V. Malyshkin, A.R. McGurn, T.A. Leskova, A.A. Maradudin, and M. Nieto–Vesperinas, Waves in Random Media **7**, 479 (1997).
3. S. Feng, C. Kane, P.A. Lee, and A.D. Stone, Phys. Rev. Lett. **61**, 834 (1988).
4. I. Freund, M. Rosenbluh, and S. Feng, Phys. Rev. Lett. **61**, 2328 (1988); I. Freund, M. Rosenbluh, and R. Berkovits, Phys. Rev. B**39**, 12403 (1989); I. Freund and R. Berkovits, Phys. Rev. B**41**, 496 (1990) (erratum **41**, 9540 (1990)); I. Freund and M. Rosenbluh, Opt. Commun. **82**, 362 (1991).
5. N. Garcia and A. Z. Genack, Phys. Rev. Lett. **63**, 1678 (1989); M. P. van Albada, J. F. de Boer, and A. Lagendijk, Phys. Rev. Lett. **64**, 2787 (1990).
6. F. Scheffold and G. Maret, Phys. Rev. Lett. **81**, 5800 (1998).
7. A. Arsenieva and S. Feng, Phys. Rev. B**47**, 13047 (1993).
8. B. Shapiro, Phys. Rev. Lett. **57**, 2168 (1986).
9. C. S. West and K. A. O'Donnell, Phys. Rev. B**59**, 2393 (1999).
10. Jun Q. Lu and Zu-Han Gu, Appl. Opt. **36**, 4562 (1997); Zu-Han Gu and Jun Q. Lu, SPIE **3141**, 269 (1997).
11. T. Tamir, in: *Electromagnetic Surface Modes*, ed. A. D. Boardman (J.Wiley & Sons, New York, 1992), p. 521.
12. A. A. Maradudin, R. E. Luna, and E. R. Méndez, Waves in Random Media **3**, 51 (1993).
13. F. Toigo, A. Marvin, V. Celli, and N. R. Hill, Phys. Rev. B**15**, 5618 (1977).
14. G. C. Brown, V. Celli, M. Haller, A.A. Maradudin, and A. Marvin, Phys. Rev. B**31**, 4993 (1985).
15. R. Kubo, J. Phys. Soc. Japan **17**, 1100 (1962).
16. T. Kawanishi, H. Ogura, and Z. L. Wang, Waves in Random Media **7**, 351 (1997).