A band-limited uniform diffuser in transmission

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ABSTRACT

In this work we consider a structure consisting of vacuum in the region $x_3 > \zeta(x_1)$; a dielectric film characterized by a real, positive, dielectric constant ϵ in the region $-D < x_3 < \zeta(x_1)$; and vacuum in the region $x_3 < -D$. The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1, that is differentiable, and constitutes a random process. This structure is illuminated from the region $x_3 > \zeta(x_1)$ by s-polarized light whose plane of incidence is the x_1x_3-plane. By the use of the geometrical optics limit of phase perturbation theory we show how to design the surface profile function $\zeta(x_1)$ in such a way that the mean differential transmission coefficient has a prescribed form within a specified range of the angle of transmission, and vanishes outside this range. In particular, we consider the case in which the transmitted intensity is constant within a specified range of the angle of transmission, and vanishes outside it. Rigorous numerical simulation calculations show that the transmitted intensity indeed has this property.

Keywords: transmissivity, surface roughness, dielectric film, band-limited uniform diffusers

1. Introduction

A band-limited uniform diffuser is defined as an optical element that scatters light uniformly within a specified range of scattering angles, and produces no scattering outside this range. In the existing theoretical studies of random surfaces that act as band-limited uniform diffusers(1–5) only their scattering properties have been investigated. However, the experimental demonstrations that surfaces fabricated according to the prescriptions given in Refs. 2–5 do act as band-limited uniform diffusers have been carried out for the transmission of light through dielectric films with a random surface of this type(3–5). It seemed desirable, therefore, to extend the theory developed in Refs. 1–5 for the scattering problem to the transmission problem. In this paper we consider the simplest version of this problem, namely the transmission of s-polarized light through a free-standing dielectric film, whose illuminated surface is a one-dimensional random surface whose generators are perpendicular to the plane of the incidence and whose back surface is planar. On the basis of the geometrical optics limit of phase perturbation theory the surface profile function of the illuminated random surface is determined in such a fashion that at normal incidence the angular dependence of the mean intensity of the light transmitted through the film is constant for the angle of transmission θ_t in the interval $(-\theta_m, \theta_m)$ and vanishes outside this interval. A rigorous computer simulation of the angular dependence of the mean intensity of the transmitted light confirms that the optical element designed in this way indeed possesses the properties specified for it.

The transmission properties of more complicated geometries, e.g. supported random dielectric films, will be investigated in a separate work, from the standpoint of designing them to possess specified mean intensity distributions.

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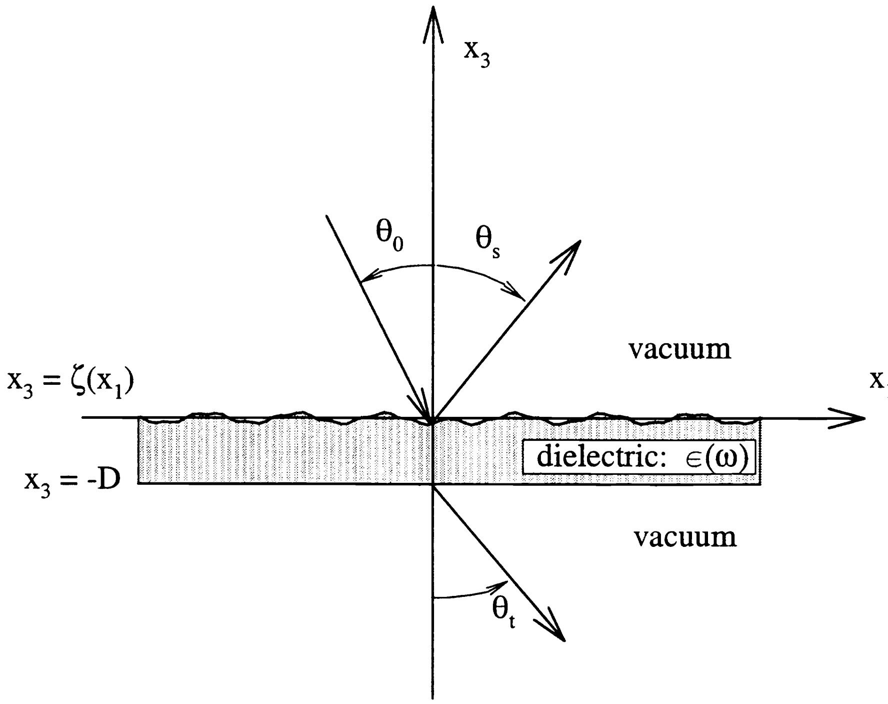

Figure 1. A sketch of the scattering system considered in the present work.

2. The Transmission Amplitude

The system we consider in this work consists of vacuum in the region $x_3 > \zeta(x_1)$; a dielectric film, characterized by a real, positive, dielectric constant ϵ, in the region $-D < x_3 < \zeta(x_1)$; and vacuum in the region $x_3 < -D$ (Fig. 1). The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1 that is differentiable, and constitutes a random process. This system is illuminated from the region $x_3 > \zeta(x_1)$ by an s-polarized plane wave, whose plane of incidence is the x_1x_3-plane. The single nonzero component of the electric field in this system is

$$E_2(x_1, x_3|\omega) = \exp[ikx_1 - i\alpha_0(k)x_3] + \int_{-\infty}^{\infty} \frac{dq}{2\pi} R(q|k) \exp[iqx_1 + i\alpha_0(q)x_3] \tag{2.1}$$

in the region $x_3 > \zeta(x_1)$,

$$E_2(x_1, x_3|\omega) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} \exp(iqx_1)[A(q|k) \exp[i\alpha(q)x_3] + B(q|k) \exp[-i\alpha(q)x_3] \tag{2.2}$$

in the region $-D < x_3 < \zeta(x_1)$, and

$$E_2(x_1, x_3|\omega) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} T(q|k) \exp[iqx_1 - i\alpha_0(q)(D + x_3)] \tag{2.3}$$

in the region $x_3 < -D$. In these expressions the functions $\alpha_0(q)$ and $\alpha(q)$ are defined by

$$\alpha_0(q) = \begin{cases} \left(\frac{\omega^2}{c^2} - q^2\right)^{\frac{1}{2}} & |q| < \frac{\omega}{c} \\ i\left(q^2 - \frac{\omega^2}{c^2}\right)^{\frac{1}{2}} & |q| > \frac{\omega}{c} \end{cases} \tag{2.4}$$

and

$$\alpha(q) = \begin{cases} \left(\epsilon\frac{\omega^2}{c^2} - q^2\right)^{\frac{1}{2}} & |q| < \sqrt{\epsilon}\frac{\omega}{c} \\ i\left(q^2 - \epsilon\frac{\omega^2}{c^2}\right)^{\frac{1}{2}} & |q| > \sqrt{\epsilon}\frac{\omega}{c}. \end{cases} \tag{2.5}$$

From the boundary conditions at the interface $x_3 = -D$, we obtain the relations

$$A(q|k) = \frac{1}{2}\left(1 - \frac{\alpha_0(q)}{\alpha(q)}\right) T(q|k)\exp[i\alpha(q)D] \tag{2.6a}$$

$$B(q|k) = \frac{1}{2}\left(1 + \frac{\alpha_0(q)}{\alpha(q)}\right) T(q|k)\exp[-i\alpha(q)D]. \tag{2.6b}$$

The boundary conditions at the interface $x_3 = \zeta(x_1)$ can be written in the forms

$$\int_{-\infty}^{\infty}\frac{dq}{2\pi}R(q|k)\exp[iqx_1 + i\alpha_0(q)\zeta(x_1)] = -\exp[ikx_1 - i\alpha_0(k)\zeta(x_1)] + \int_{-\infty}^{\infty}\frac{dq}{2\pi}\exp[iqx_1][A(q|k)\exp[i\alpha(q)\zeta(x_1)] + B(q|k)\exp[-i\alpha(q)\zeta(x_1)] \tag{2.7a}$$

$$\begin{aligned}\int_{-\infty}^{\infty}\frac{dq}{2\pi}[-q\zeta'(x_1) + \alpha_0(q)]R(q|k)\exp[iqx_1 + i\alpha_0(q)\zeta(x_1)] &= [k\zeta'(x_1) + \alpha_0(k)]\exp[ikx_1 - i\alpha_0(k)\zeta(x_1)] \\ &+ \int_{-\infty}^{\infty}\frac{dq}{2\pi}\exp[iqx_1]\{[-q\zeta'(x_1) + \alpha(q)]A(q|k)\exp[i\alpha(q)\zeta(x_1)] \\ &+[-q\zeta'(x_1) - \alpha(q)]B(q|k)\exp[-i\alpha(q)\zeta(x_1)]\}.\end{aligned} \tag{2.7b}$$

We eliminate $R(q|k)$ from this pair of equations by multiplying Eq. (2.7a) by $[p\zeta'(x_1)+\alpha_0(p)]\exp[-ipx_1+i\alpha_0(p)\zeta(x_1)]$ and integrating on x_1; multiplying Eq. (2.7b) by $\exp[-ipx_1 + i\alpha_0(p)\zeta(x_1)]$ and integrating on x_1; then subtracting the second equation from the first. The result takes the form

$$(1-\epsilon)\frac{\omega^2}{c^2}\int_{-\infty}^{\infty}\frac{dq}{2\pi}\left\{\frac{I(\alpha_0(p)+\alpha(q)|p-q)}{\alpha_0(p)+\alpha(q)}A(q|k) + \frac{I(\alpha_0(p)-\alpha(q)|p-q)}{\alpha_0(p)-\alpha(q)}B(q|k)\right\} = 2\pi\delta(p-k)2\alpha_0(k), \tag{2.8}$$

where

$$I(\gamma|Q) = \int_{-\infty}^{\infty}dx_1\exp[-iQx_1 + i\gamma\zeta(x_1)]. \tag{2.9}$$

The use of the relations (2.6) in Eq. (2.8) yields the reduced Rayleigh equation for the transmission amplitude $T(q|k)$:

$$\begin{aligned}\int_{-\infty}^{\infty}\frac{dq}{2\pi}\left\{\frac{I(\alpha_0(p)+\alpha(q)|p-q)}{\alpha_0(p)+\alpha(q)}\frac{1}{2}\left(1-\frac{\alpha_0(q)}{\alpha(q)}\right)\exp[i\alpha(q)D]\right. \\ \left. + \frac{I(\alpha_0(p)-\alpha(q)|p-q)}{\alpha_0(p)-\alpha(q)}\frac{1}{2}\left(1+\frac{\alpha_0(q)}{\alpha(q)}\right)\exp[-i\alpha(q)D]\right\}T(q|k) &= -2\pi\delta(p-k)\frac{2\alpha_0(k)}{(\epsilon-1)(\omega^2/c^2)}.\end{aligned} \tag{2.10}$$

We solve Eq. (2.10) as an expansion in powers of the surface profile function through terms linear in $\zeta(x_1)$, with the result that

$$T(q|k) = T_0(k)[2\pi\delta(q-k) + i(\epsilon-1)\frac{\omega^2}{c^2}\frac{\alpha(q)}{\alpha(k)}\frac{\Lambda(k)}{\mathcal{D}(q)}\hat{\zeta}(q-k) + O(\zeta^2)], \tag{2.11}$$

where $T_0(k)$ is the Fresnel transmission amplitude in the absence of the surface roughness,

$$T_0(k) = \frac{4\alpha_0(k)\alpha(k)}{\mathcal{D}(k)}, \tag{2.12}$$

and

$$\begin{aligned}
\Lambda(k) &= 2\alpha(k)\cos[\alpha(k)D] - i2\alpha_0(k)\sin[\alpha(k)D] \qquad (2.13)\\
\mathcal{D}(q) &= 4\alpha_0(q)\alpha(q)\cos[\alpha(q)D] - i2[\alpha_0^2(q)+\alpha^2(q)]\sin[\alpha(q)\mathcal{D}] \qquad (2.14)\\
\hat{\zeta}(q-k) &= \int_{-\infty}^{\infty} dx_1 \exp[-i(q-k)x_1]\zeta(x_1). \qquad (2.15)
\end{aligned}$$

The result given by Eq. (2.11) can be rewritten as

$$T(q|k) = T_0(k)\int_{-\infty}^{\infty} dx_1 \exp[-i(q-k)x_1]\left[1 + i(\epsilon-1)\frac{\omega^2}{c^2}\frac{\alpha(q)}{\alpha(k)}\frac{\Lambda(k)}{\mathcal{D}(q)}\zeta(x_1)\right]. \qquad (2.16)$$

The phase perturbation theory result for $T(q|k)$ is obtained by exponentiating the expression in brackets in the integrand. Thus, finally, we have the result that

$$T(q|k) = T_0(k)\int_{-\infty}^{\infty} dx_1 \exp[-i(q-k)x_1]\exp[i(\epsilon-1)\frac{\omega^2}{c^2}\frac{\alpha(q)}{\alpha(k)}\frac{\Lambda(k)}{\mathcal{D}(q)}\zeta(x_1)]. \qquad (2.17)$$

3. The Mean Differential Transmission Coefficient

The differential transmission coefficient is defined as the fraction of the total time-averaged incident flux that is transmitted into the angular interval $(\theta_t, \theta_t + d\theta_t)$, where θ_t is the angle of transmission (Fig. 1). The magnitude of the total time-averaged incident flux is given by

$$\begin{aligned}
p_{inc} &= -Re\int dx_1 \int dx_2 (S_3^c)_{inc}\\
&= L_1 L_2 \frac{c^2}{8\pi\omega}\alpha_0(k), \qquad (3.1)
\end{aligned}$$

where S_3^c is the 3-component of the complex Poynting vector, and the minus sign compensates for the fact that the flux is in the $-x_3$-direction. L_1 and L_2 are the lengths of the surface along the x_1-and x_2-axes. The magnitude of the total time-averaged transmitted flux is given by

$$\begin{aligned}
p_{tr} &= -Re\int dx_1 \int dx_2 (S_3^c)_{tr}\\
&= L_2 \frac{c^2}{8\pi\omega}\int_{-\frac{\omega}{c}}^{\frac{\omega}{c}} \frac{dq}{2\pi}\alpha_0(q)|T(q|k)|^2. \qquad (3.2)
\end{aligned}$$

We introduce the angles of incidence and transmission, θ_0 and θ_t, respectively, by (Fig. 1)

$$k = \frac{\omega}{c}\sin\theta_0, \qquad q = \frac{\omega}{c}\sin\theta_t. \qquad (3.3)$$

It follows that the incident and scattered fluxes are given by

$$\begin{aligned}
p_{inc} &= L_1 L_2 \frac{c}{8\pi}\cos\theta_0 \qquad (3.4)\\
p_{tr} &= L_2 \frac{\omega}{16\pi^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta_t \cos^2\theta_t |T(q|k)|^2. \qquad (3.5)
\end{aligned}$$

The differential transmission coefficient by definition is then given by

$$\frac{\partial T}{\partial\theta_t} = \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_t}{\cos\theta_0}|T(q|k)|^2. \qquad (3.6)$$

Since we are studying transmission through a random surface, it is the mean differential transmission coefficient that is of interest to us. It is given by

$$\left\langle \frac{\partial T}{\partial \theta_t} \right\rangle = \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_t}{\cos\theta_0}\langle |T(q|k)|^2\rangle, \tag{3.7}$$

where the angle brackets denote an average over the ensemble of realizations of the surface profile function $\zeta(x_1)$. On combining Eqs. (2.17) and (3.7) we find that in phase perturbation theory

$$\begin{aligned}\left\langle \frac{\partial T}{\partial \theta_t} \right\rangle &= \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_t}{\cos\theta_0}|T_0(k)|^2 \\ &\times \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_1' \exp[-i(q-k)(x_1-x_1')]\langle \exp\{i(\epsilon-1)\frac{\omega^2}{c^2}\frac{\alpha(q)}{\alpha(k)}[\frac{\Lambda(k)}{\mathcal{D}(q)}\zeta(x_1) - \frac{\Lambda^*(k)}{\mathcal{D}^*(q)}\zeta(x_1')]\}\rangle. \end{aligned} \tag{3.8}$$

In what follows, we specialize to the case of normal incidence: $\theta_0 = 0$, so that $k = 0$. In this case Eq. (3.8) becomes

$$\begin{aligned}\left\langle \frac{\partial T}{\partial \theta_t} \right\rangle &= \frac{1}{L_1}\frac{\omega}{2\pi c}\cos^2\theta_t |T_0(0)|^2 \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_1' \exp[-iq(x_1-x_1')] \\ &\times\langle \exp\{i\frac{(\epsilon-1)}{\sqrt{\epsilon}}\frac{\omega}{c}\alpha(q)[\frac{\Lambda(0)}{\mathcal{D}(q)}\zeta(x_1) - \frac{\Lambda^*(0)}{\mathcal{D}^*(q)}\zeta(x_1')]\}\rangle, \end{aligned} \tag{3.9}$$

where

$$|T_0(0)|^2 = \frac{16\epsilon}{16\epsilon\cos^2(\sqrt{\epsilon}\frac{\omega}{c}D) + 4(\epsilon+1)^2\sin^2(\sqrt{\epsilon}\frac{\omega}{c}D)}. \tag{3.10}$$

In order to pass to the geometrical optics limit of phase perturbation theory, we have to work in a parameter range where the coefficients of $\zeta(x_1)$ and $\zeta(x_1')$ in the exponent in the integrand in Eq. (3.9) are the same and real. This requires that the inequality

$$Im\left[\frac{\Lambda(0)}{\mathcal{D}(q)}\right] \ll Re\left[\frac{\Lambda(0)}{\mathcal{D}(q)}\right] \tag{3.11}$$

be satisfied. For specified values of the wavelength of the incident light λ, the mean thickness of the film D, and the dielectric constant of the film ϵ, this inequality defines the range of q values, or equivalently of θ_t values, for which the geometrical optics limit of the phase perturbation theory is valid.

An indication of the conditions under which the inequality (3.11) is satisfied can be obtained by considering the limit where $q = 0$. In this limit the inequality (3.11) can be expressed as

$$\sin\left(2\sqrt{\epsilon}\frac{\omega}{c}D\right) \ll \frac{2}{\sqrt{\epsilon}}\left[\frac{\epsilon+1}{\epsilon-1} + \cos^2\left(\sqrt{\epsilon}\frac{\omega}{c}D\right)\right]. \tag{3.12}$$

For the value of ϵ we assume in the present work ($\epsilon = 2.69$), the right hand side of this inequality is greater than unity for all values of $\sqrt{\epsilon}(\omega/c)D$. Consequently, this inequality will be satisfied whenever $2\sqrt{\epsilon}(\omega/c)D$ is close to $n\pi$, where n is a postive integer. In what follows we will assume that this is the case.

The expression (3.9) for $\langle \partial T/\partial\theta_t\rangle$ now takes the form

$$\left\langle \frac{\partial T}{\partial \theta_t} \right\rangle = \frac{1}{L_1}\frac{\omega}{2\pi c}\cos^2\theta_t |T_0(0)|^2 \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_1' \exp[-iq(x_1-x_1')]\langle \exp[ia(\zeta(x_1)-\zeta(x_1'))]\rangle, \tag{3.13}$$

where

$$a = \frac{\epsilon-1}{\sqrt{\epsilon}}\frac{\omega}{c}\alpha(q)Re\left[\frac{\Lambda(0)}{\mathcal{D}(q)}\right]. \tag{3.14}$$

We now make the change of variable $x_1' = x_1 + u$, and obtain

$$\left\langle \frac{\partial T}{\partial \theta_t} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \cos^2 \theta_t |T_0(0)|^2 \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \; \exp[iqu] \langle \exp[ia(\zeta(x_1) - \zeta(x_1 + u))] \rangle. \tag{3.15}$$

To obtain the geometrical optics limit of this expression, we expand the difference $\zeta(x_1) - \zeta(x_1 + u)$ in powers of u, and retain only the term linear in u. Thus, we have that in this limit

$$\left\langle \frac{\partial T}{\partial \theta_t} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \cos^2 \theta_t |T_0(0)|^2 \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \exp[iqu] \langle \exp[-iau\zeta'(x_1)] \rangle. \tag{3.16}$$

This expression is the starting point for the determination of a surface profile function $\zeta(x_1)$ that yields a specified form for $\langle \partial T / \partial \theta_t \rangle$.

4. The Surface Profile Function $\zeta(x_1)$

We assume that the surface profile function $\zeta(x_1)$ is written in the form[2]

$$\zeta(x_1) = \sum_{\ell=-\infty}^{\infty} c_\ell s(x_1 - \ell 2b), \tag{4.1}$$

where the $\{c_\ell\}$ are independent, positive, random deviates, b is a characteristic length, and the function $s(x_1)$ is defined by

$$s(x_1) = \begin{cases} 0 & x_1 < -(m+1)b \\ -(m+1)bh - hx_1 & -(m+1)b < x_1 < -mb \\ -bh & -mb < x_1 < mb \\ -(m+1)bh + hx_1 & mb < x_1 < (m+1)b \\ 0 & (m+1)b < x_1, \end{cases} \tag{4.2}$$

where m is a positive integer. Due to the positivity of the coefficient c_ℓ, its probability density function (pdf) $f(\gamma) = \langle \delta(\gamma - c_\ell) \rangle$ is nonzero only for $\gamma > 0$.

It has been shown[2] that for the random surfaces defined by Eqs. (4.1) and (4.2),

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \exp[iqu] \langle \exp[-iau\zeta'(x_1)] \rangle = \frac{\pi L_1}{ah} \left[f\left(\frac{q}{ah}\right) + f\left(\frac{-q}{ah}\right) \right]. \tag{4.3}$$

On combining Eqs. (3.16) and (4.3) we find that the mean differential transmission coefficient is given in terms of the pdf of c_ℓ by

$$\left\langle \frac{\partial T}{\partial \theta_t} \right\rangle = \frac{\omega}{2c} \cos^2 \theta_t |T_0(0)|^2 \frac{1}{ah} \left[f\left(\frac{q}{ah}\right) + f\left(-\frac{q}{ah}\right) \right]. \tag{4.4}$$

We now make the change of variable $(q/ah) = \gamma$, so that

$$\gamma h = \frac{q}{a} = \frac{\sqrt{\epsilon}}{\epsilon - 1} \frac{\sin \theta_t}{(\epsilon - \sin^2 \theta_t)^{\frac{1}{2}}} \frac{1}{(\omega/c) Re[\Lambda(0)/\mathcal{D}((\omega/c)\sin\theta_t)]}. \tag{4.5}$$

We need to invert this expression to obtain $\sin \theta_t$ and $\cos \theta_t$ as functions of γ. This can be done analytically if we replace $(\omega/c) Re[\Lambda(0)/\mathcal{D}(\frac{\omega}{c} \sin \theta_t)]$ by its value for $\theta_t = 0$, which is valid for small angles of transmission. In this limit we have that

$$\begin{aligned} \frac{\omega}{c} Re\left[\frac{\Lambda(0)}{\mathcal{D}(0)}\right] &= \frac{2\epsilon \cos^2(\sqrt{\epsilon}\frac{\omega}{c}D) + (\epsilon + 1)\sin^2(\sqrt{\epsilon}\frac{\omega}{c}D)}{4\epsilon \cos^2(\sqrt{\epsilon}\frac{\omega}{c}D) + (\epsilon+1)^2 \sin^2(\sqrt{\epsilon}\frac{\omega}{c}D)}. \\ &\equiv f. \end{aligned} \tag{4.6}$$

It then follows from Eq. (4.5) that

$$\sin\theta_t = \left[\frac{\epsilon(\epsilon-1)^2 f^2\gamma^2 h^2}{\epsilon+(\epsilon-1)^2 f^2\gamma^2 h^2}\right]^{\frac{1}{2}} \tag{4.7a}$$

$$\cos\theta_t = \left[\frac{\epsilon-(\epsilon-1)^3 f^2\gamma^2 h^2}{\epsilon+(\epsilon-1)^2 f^2\gamma^2 h^2}\right]^{\frac{1}{2}}. \tag{4.7b}$$

On inverting Eq. (4.4) with the aid of these results we find that

$$f(\gamma)+f(-\gamma) = 2fh\frac{\sqrt{\epsilon}(\epsilon-1)}{|T_0(0)|^2}\frac{[\epsilon+(\epsilon-1)^2 f^2\gamma^2 h^2]^{\frac{1}{2}}}{\epsilon-(\epsilon-1)^3 f^2\gamma^2 h^2}\left\langle\frac{\partial T}{\partial\theta_t}\right\rangle(\gamma). \tag{4.8}$$

Thus, if we wish to design a surface for which $\langle \partial T/\partial\theta_t\rangle$ has the form

$$\left\langle\frac{\partial T}{\partial\theta_t}\right\rangle = A\theta(\theta_m - |\theta_t|), \tag{4.9a}$$

$$= A\theta(\sin^2\theta_m - \sin^2\theta_t), \tag{4.9b}$$

where A is a constant and $\theta(x)$ is the Heaviside unit step function, we can use the fact that the expression for $\sin\theta_t$ given by Eq. (4.7a) is a monotonically increasing function of γ, for the value of ϵ we assume here, to rewrite Eq. (4.9) as

$$\left\langle\frac{\partial T}{\partial\theta_t}\right\rangle = A\theta(\gamma_m - |\gamma|), \tag{4.10}$$

where γ_m is related to θ_m by

$$\gamma_m = \frac{1}{fh}\frac{\sqrt{\epsilon}}{\epsilon-1}\frac{\sin\theta_m}{[\epsilon-\sin^2\theta_m]^{\frac{1}{2}}}. \tag{4.11}$$

If we then substitute Eq. (4.10) into Eq. (4.8) and assume that γ is positive, we obtain finally that the pdf of c_ℓ is given by

$$f(\gamma) = 2fh\frac{\sqrt{\epsilon}(\epsilon-1)}{|T_0(0)|^2}\frac{[\epsilon+(\epsilon-1)^2 f^2 h^2\gamma^2]^{\frac{1}{2}}}{\epsilon-(\epsilon-1)^3 f^2 h^2\gamma^2}A\theta(\gamma_m-\gamma)\theta(\gamma). \tag{4.12}$$

The constant A is then obtained from the normalization condition

$$2fh\frac{\sqrt{\epsilon}(\epsilon-1)}{|T_0(0)|^2}A\int_0^{\gamma_m} d\gamma\,\frac{[\epsilon+(\epsilon-1)^2 f^2 h^2\gamma^2]^{\frac{1}{2}}}{\epsilon-(\epsilon-1)^3 f^2 h^2\gamma^2} = 1. \tag{4.13}$$

From this form for $f(\gamma)$ a long sequence of $\{c_\ell\}$ can be generated, e.g. by the rejection method(6), and the surface profile function $\zeta(x_1)$ generated by the use of Eqs. (4.1) and (4.2).

5. Numerical Results and Discussions

The pdf $f(\gamma)$ given by Eq. (4.12), was obtained by taking the geometrical limit of phase perturbation theory, a single-scattering theory. It is therefore not known *a priori* if random surfaces generated from this pdf will give rise to a band-limited, uniform mean differential transmission coefficient (DTC) for the light transmitted through the dielectric film when multiple-scattering effects are taken into account.

In order to see how well the geometrical optics limit is able to represent the full solution to the transmission problem we have to resort to numerical simulations(7). Such simulations are based on the use of Green's second integral identity in the plane(8) to derive a set of four coupled inhomogeneous integral equations for the field and its

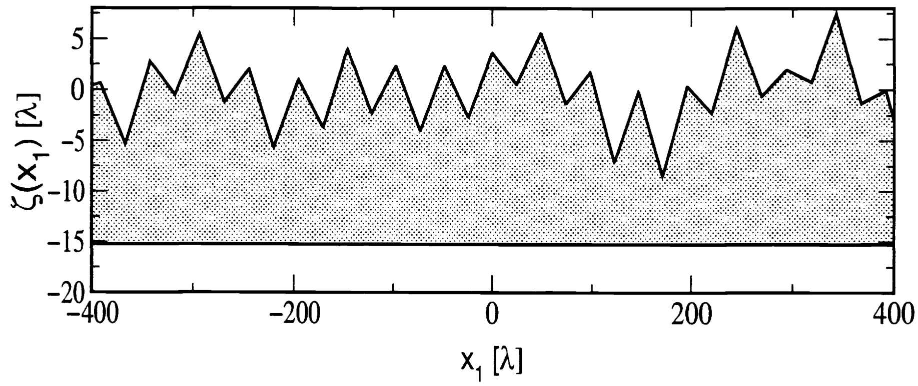

Figure 2. A particular example of the film geometry that we are considering in the present study. The parameters used to generate the rough upper surface were $b = 15\mu$m, $h = 0.01$, and $m = 1$. The mean thickness of the film was $D \simeq 15.24\lambda$, where λ is a the wavelength of the incident light.

normal derivative on both surfaces, from which the transmitted field can be calculated. This procedure is described in detail in Ref. 9. With this approach the transmitted field can be calculated in a rigorous way.

The rough illuminated upper surface of the dielectric film was generated according to Eq. (4.1) with the independent random deviates $\{c_\ell\}$ drawn from the pdf $f(\gamma)$, Eq. (4.12), by the use of the rejection method [6]. Notice that the profiles generated according to Eq. (4.1) do not give rise to surfaces of vanishing mean. Hence, for each realization of the surface used in the numerical calculation, its mean was adjusted to zero in order to have a well-defined mean film-thickness, independent of the parameters used to generate the surface profile function $\zeta(x_1)$. A plot of one particular realization of the rough profile generated in this way is presented as the upper surface in Fig. 2, where we also have included the planar lower surface.

The free standing dielectric film that we considered in this work was characterized by a dielectric constant $\epsilon = 2.69$ and a mean thickness $D/\lambda \simeq n/(4\sqrt{\epsilon})$ (see Eq. (3.12)) where n is a positive integer. S-polarized light of wavelength $\lambda = 612.7$nm was incident normally on the rough surface of the film, which was characterized by the parameters $b = 15\mu$m, $h = 0.01$, $m = 1$, and $\theta_m = 20°$. Furthermore, in order to guarantee that the film for these parameters is continuous, *i.e.* without holes, the value $n = 100$ was used to define its mean thickness, which in the present case was $D \simeq 15.24\lambda \simeq 9.34\mu$m. For these parameters $Im\left[\Lambda(0)/\mathcal{D}(q)\right]/Re\left[\Lambda(0)/\mathcal{D}(q)\right] \sim 10^{-4}$.

In Fig. 3 we present rigorous numerical simulation results (solid curve) for the mean differential transmission coefficient, $\langle \partial T/\partial\theta_t \rangle$, for the scattering system defined above. The dashed curve in the same figure represents the results obtained in the geometrical optics limit of phase perturbation theory as defined by Eq. (4.4), where $f(\gamma)$ is given by Eq. (4.12). The length of the surface used in these simulations was $L_1 = 100\lambda$ and the sampling rate used was $\Delta x_1 = 0.1\lambda$, corresponding to $N = 1000$ sampling points along the surface. The number of surface realizations used to obtain these results was $N_\zeta = 2000$. As can be seen from the numerical simulation results (solid curve) of Fig. 3, the transmitted light is nicely restricted to a well-defined angular interval, viz it is band-limited. Furthermore, the mean DTC is quite uniform within the band-limited region. However, we observe from Fig. 3 a minor disagreement between the rigorous result (solid curve) and the result of the geometrical optics limit of the phase perturbation theory (dashed line) for the angular interval over which the transmitted light is expected to be band-limited: the geometrical optics limit seems to overestimate this region. This disagreement, we believe, is a sign of the inadequacy of the geometrical optics limit of phase perturbation theory to fully describe the transmitted light. It worth noting that even if the observed θ_m's from the rigorous and geometrical optics limit are somewhat different, the total transmitted power is the same (within the noise level) for the two cases.

Furthermore, if we want the transmitted light to be limited to $\pm\theta_m$, we have found (results not shown) that we can use a slightly larger "input" θ_m to obtain this. A good choice for the new "input" θ_m is empirically found to be given by $\theta_m \to \eta\theta_m$, where $\eta = \theta_m^{geo}/\theta_m^{rig}$ is the ratio between the upper limit in the geometrical optics calculation (θ_m^{geo}) and that of the rigorous simulations (θ_m^{rig}).

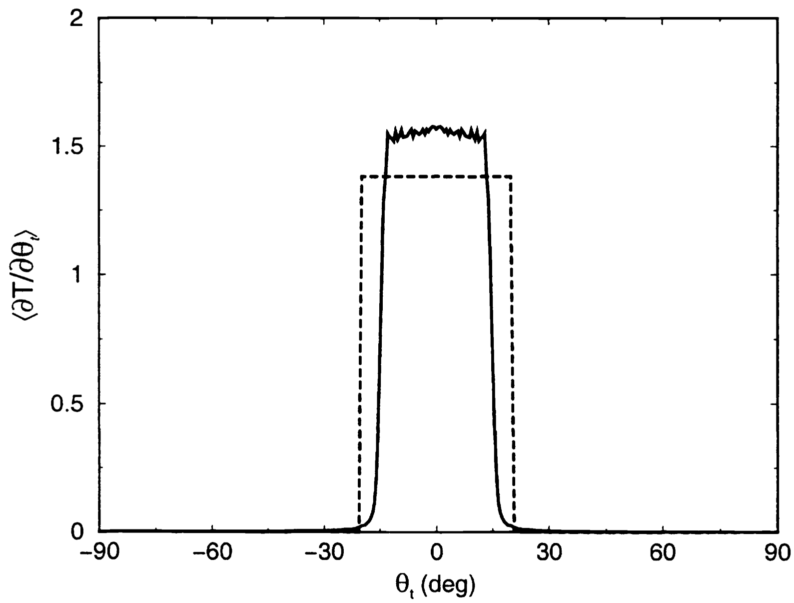

Figure 3. The mean differential transmission coefficient for s-polarized light of wavelength $\lambda = 612.7$nm transmitted through a free standing dielectric film of mean thickness $D \simeq 15.24\lambda \simeq 9.34\mu$m and characterized by a dielectric constant $\epsilon = 2.69$. The solid curve is the result of rigorous Monte Carlo simulations, while the dashed curve is the result obtained in the geometrical optics limit of phase perturbation theory, Eq. (4.4). The length of the surface used in the simulations was $L_1 = 100\lambda$, and the number of surface realization over which $\langle \partial T/\partial\theta_t \rangle$ was averaged was $N_\zeta = 2000$. The number of sampling point used was $N = 1000$ ($\Delta x_1 = 0.1\lambda$). The illuminated rough surface was characterized by the parameters $b = 15\mu$m, $h = 0.01$, $m = 1$, and $\theta_m = 20°$.

It should be mentioned that in order to obtain the numerical results shown in Fig. 3 we had to shift the lower limit of $f(\gamma)$ slightly away from zero in order to avoid a small specular peak that is due to the tails of the two distributions in Eq. (4.4) caused by diffraction effects. This procedure has been described in detail in Ref. 2, and the arguments for doing so will not be repeated here.

It has earlier been demonstrated[3] that for surfaces that are designed to act as band-limited uniform diffusers in reflection this property is not very sensitive to the wavelength of the incident light over a substantial range. To see how robust the property of band-limited uniform diffusers in transmission is when the wavelength is changed, in Fig. 4 we present rigorous simulation results for the case where the wavelength is $\lambda = 582.1$nm and the remaining parameters have the values given above. We observe from this figure (solid curve) that the transmitted light is nicely band-limited and is also rather uniform, except for the weak peak in the specular direction. This peak is due to the overlapping of the tails of the two distributions in Eq. (4.4) caused by diffraction effects. In calculating the result presented in Fig. 4 we used the same cut-off used to obtain the results shown in Fig. 3. By using another value for the lower cut-off of $f(\gamma)$ we can make this peak vanish. This is shown in Fig. 5, where the angular distribution of the transmitted light is uniform except for some noise, and is band-limited. That the results shown in Figs. 4 and 5 are so close to being band-limited and uniform we in fact find rather surprising since the assumption (3.11), on which the derivation of the pdf $f(\gamma)$ relies, is not even close to being satisfied in this case ($Im\left[\Lambda(0)/\mathcal{D}(q)\right]/Re\left[\Lambda(0)/\mathcal{D}(q)\right] \sim 1$). This shows that the prediction of the geometrical optics limit of phase perturbation theory is rather robust.

6. Conclusions

In this paper we have developed the geometrical optics limit of phase perturbation theory for the transmission of light through a free-standing dielectric film whose illuminated surface is a one-dimensional random surface while its back surface is planar. The result for the mean differential transmission coefficient in the case where the film

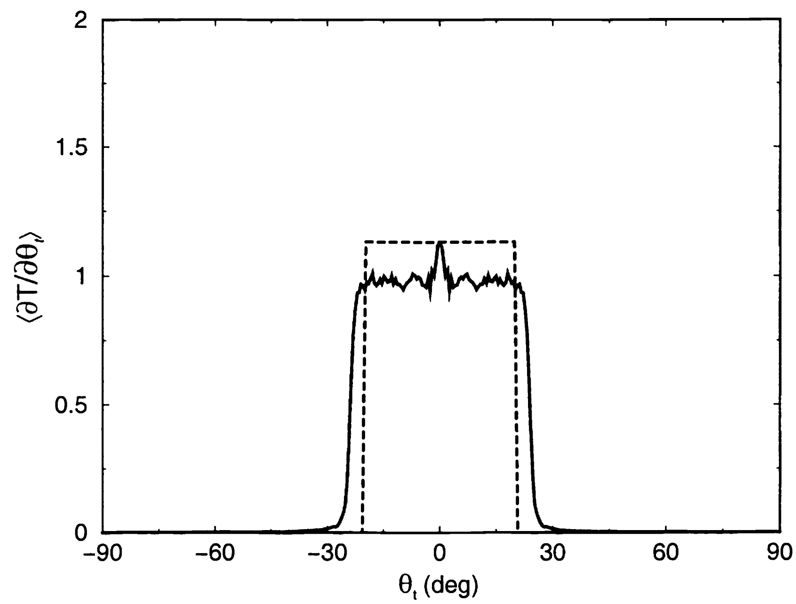

Figure 4. The same as Fig. 3, but now the wavelength of the incident light is $\lambda = 582.1$nm.

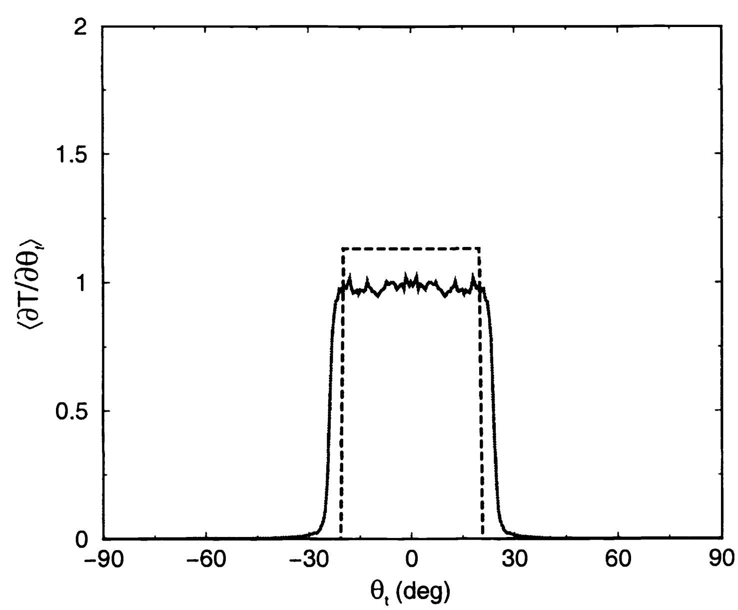

Figure 5. The same as Fig. 4, but with a slightly different value for the lower cut-off for the pdf.

is illuminated by s-polarized light whose plane of incidence is normal to the generators of the random surface is used to design that surface in such a fashion that the film acts as a band-limited uniform diffuser. That is, the angular dependence of the intensity of the transmitted light is constant within a specified region of the angle of transmission, and vanishes outside that region. The results of rigorous numerical simulations of the transmission of s-polarized light through a film whose random surface has been defined in this way show that it indeed acts as a band-limited uniform diffuser, although there are some small quantitative differences between the rigorous result for the mean differential transmission coefficient and the result obtained by the use of the geometrical optics limit of phase perturbation theory. These are believed to be a reflection of the limitations of the geometrical optics limit of phase perturbation theory in describing the transmission of light through the film system studied here. Moreover, in contrast with the results obtained in reflection, where the mean differential reflection coefficient proved to be independent of the wavelength of the incident light, the mean differential transmission coefficient depends on the wavelength, primarily due to the finite mean thickness of the dielectric film. Nevertheless, the results obtained here demonstrate that the approach used in our earlier papers to generate one-dimensional random surfaces that act as band-limited uniform diffusers in reflection can be applied to solve the same problem in transmission as well.

Acknowledgments

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