The Design of One-Dimensional Random Surfaces with Specified Scattering Properties

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ABSTRACT

We propose a method for designing a one-dimensional random perfectly conducting surface which, when illuminated normally by an s-polarized plane wave, scatters it with a prescribed angular distribution of intensity. The method is applied to the design of a surface that scatters light uniformly within a specified range of scattering angles, and produces no scattering outside this range; a surface that acts as a Lambertian diffuser; and a surface that suppresses single-scattering within a specified range of scattering angles. This method is tested by computer simulations, and a procedure for fabricating such surfaces on photoresist is described.

Keywords: random surfaces, scattering, band-limited uniform diffusers, Lambertian diffusers, computer simulations, Kirchhoff approximation

In a recent series of papers the present authors and their colleagues have presented a method for designing one-dimensional random surfaces that scatter[1–5] or transmit[6,7] light in a specified manner. This method is based on expressing the surface profile function of the random surface as a superposition of equally spaced identical trapezoidal grooves, whose amplitudes are assumed to be independent random deviates drawn from a probability density function (pdf) that is determined in such a way that the intensity of the light scattered from, or transmitted through, the resulting surface, has the specified angular distribution. It has been shown that such surfaces can be fabricated on photoresist,[1,5] and produce the specified angular distribution of the intensity of the scattered light.[5,8]

In this paper we present a method for designing a one-dimensional random surface that scatters light in a prescribed manner, that is simpler to implement than our original method, both theoretically and experimentally. Like the method of Refs. 1-7 it is based on the geometrical optics limit of the Kirchhoff approximation for the scattering of s-polarized light incident normally on a perfectly conducting surface. This method is illustrated by applying it to the determination of a surface that scatters light uniformly within a specified range of scattering angles, and produces no scattering outside this range (a band-limited uniform diffuser); a surface that produces a scattered intensity that is proportional to the cosine of the scattering angle (a Lambertian diffuser); and a surface that suppresses single-scattering processes within a specified range of scattering angles. It is tested by computer simulation calculations based on the Kirchhoff approximation, which proceed very quickly. Finally, we indicate how the kinds of surfaces generated by our approach can be fabricated on photoresist.

The physical system we consider in this work consists of vacuum in the region $x_3 > \zeta(x_1)$ and a perfect conductor in the region $x_3 < \zeta(x_1)$. The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1 that is differentiable, and constitutes a random process, but not necessarily a stationary one.

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The surface $x_3 = \zeta(x_1)$ is illuminated from the vacuum region by an s-polarized plane wave of frequency ω, whose plane of incidence is the x_1x_3-plane. The single nonzero component of the electric field can be written in the form

$$\begin{aligned} E_2(x_1, x_3|\omega) &= \exp[ikx_1 - i\alpha_0(k)x_3] \\ &+ \int_{-\infty}^{\infty} \frac{dq}{2\pi} R(q|k) \exp[iqx_1 + i\alpha_0(q)x_3], \end{aligned} \tag{1}$$

in the region $x_3 > \zeta(x_1)_{max}$, where $\alpha_0(q) = [(\omega/c)^2 - q^2]^{\frac{1}{2}}$, with $Re\alpha_0(q) > 0$, $Im\alpha_0(q) > 0$. In the Kirchhoff approximation, which we adopt here for simplicity, the scattering amplitude $R(q|k)$ is given by[1]

$$\begin{aligned} R(q|k) &= -\frac{(\omega/c)^2 + \alpha_0(q)\alpha_0(k) - qk}{\alpha_0(q)[\alpha_0(q) + \alpha_0(k)]} \\ &\times \int_{-\infty}^{\infty} dx_1 \exp\{-i(q-k)x_1 - i[\alpha_0(q) + \alpha_0(k)]\zeta(x_1)\}. \end{aligned} \tag{2}$$

We note that the result given by Eq. (2) with the opposite sign is the scattering amplitude in the Kirchhoff approximation for the scattering of a p-polarized plane wave from the perfectly conducting surface $x_3 = \zeta(x_1)$. Since, as we will see below, it is the squared modulus of $R(q|k)$ that is of interest, not $R(q|k)$ itself, the results obtained in this paper are valid for the scattering of p-polarized light as well as for the scattering of s-polarized light.

The differential reflection coefficient $\partial R/\partial\theta_s$ is defined in such a way that $(\partial R/\partial\theta_s)d\theta_s$ is the fraction of the total time-averaged incident flux that is scattered into the angular interval $(\theta_s, \theta_s + d\theta_s)$. It is given in terms of $R(q|k)$ by

$$\frac{\partial R}{\partial\theta_s} = \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_s}{\cos\theta_0}|R(q|k)|^2, \tag{3}$$

where L_1 is the length of the x_1-axis covered by the random surface, while θ_0 and θ_s are the angles of incidence and scattering, respectively, and are related to the wavenumbers k and q by

$$k = (\omega/c)\sin\theta_0, \qquad q = (\omega/c)\sin\theta_s. \tag{4}$$

As we are concerned with the scattering of light from a randomly rough surface, it is the mean differential reflection coefficient that we need to calculate. It is defined by

$$\left\langle\frac{\partial R}{\partial\theta_s}\right\rangle = \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{\cos^2\theta_s}{\cos\theta_0}\langle|R(q|k)|^2\rangle, \tag{5}$$

where the angle brackets denote an average over the ensemble of realizations of the surface profile function $\zeta(x_1)$.

The substitution of Eqs. (2) and (4) into Eq. (5) yields the latter in the form

$$\begin{aligned} \left\langle\frac{\partial R}{\partial\theta_s}\right\rangle &= \frac{1}{L_1}\frac{\omega}{2\pi c}\frac{1}{\cos\theta_0}\left[\frac{1+\cos(\theta_0+\theta_s)}{\cos\theta_0+\cos\theta_s}\right]^2 \\ &\times \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_1' \exp[-i(q-k)(x_1 - x_1')] \\ &\times\langle\exp[-ia(\zeta(x_1) - \zeta(x_1'))]\rangle \end{aligned} \tag{6}$$

where, to simplify the notation, we have defined $a = \alpha_0(q) + \alpha_0(k) = (\omega/c)(\cos\theta_0 + \cos\theta_s)$.

The geometrical optics limit of the Kirchhoff approximation is obtained by making the change of variable $x_1' = x_1 + u$ in Eq. (6), expanding the difference $\zeta(x_1) - \zeta(x_1 + u)$ in powers of u, and retaining only the leading nonzero term:

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{1}{\cos\theta_0} \left[\frac{1 + \cos(\theta_0 + \theta_s)}{\cos\theta_0 + \cos\theta_s} \right]^2 \times \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \, \exp[i(q-k)u] \langle \exp iau\zeta'(x_1) \rangle. \tag{7}$$

To evaluate the double integral in Eq. (7) we proceed as follows. We define a set of equally-spaced points along the x_1-axis by $x_n = nb$, where b is a characteristic length and $n = 0, \pm 1, \pm 2, \ldots$. Within the interval $nb \leq x_1 \leq (n+1)b$ the surface profile function $\zeta(x_1)$ is given by

$$\zeta(x_1) = a_n x_1 + b_n \qquad nb \leq x_1 \leq (n+1)b, \tag{8}$$

where the $\{a_n\}$ are independent random deviates. Therefore, the probability density function (pdf) of a_n,

$$f(\gamma) = \langle \delta(\gamma - a_n) \rangle, \tag{9}$$

is independent of n. The derivative of the surface profile function (8) is

$$\zeta'(x_1) = a_n \qquad nb < x_1 < (n+1)b. \tag{10}$$

In order that the surface be continuous at $x_1 = (n+1)b$, the relation

$$b_{n+1} = b_n - (n+1)(a_{n+1} - a_n)b \tag{11}$$

must be satisfied. From this recurrence relation the $\{b_n\}$ can be determined from a knowledge of the $\{a_n\}$, provided that an initial value, e.g. that of b_0, is specified. Alternatively, we can use Eq. (11) to relate b_n to b_0 according to

$$b_n = b_0 + (a_0 + a_1 + \cdots + a_{n-1} - na_n)b \qquad n \geq 1 \tag{12a}$$

and

$$b_{-n} = b_0 - (a_{-1} + a_{-2} + \cdots + a_{-(n-1)} - (n-1)a_{-n})b \qquad n \geq 1. \tag{12b}$$

For reasons that will become clear below, it is convenient to choose $b_0 = 0$, and we will do so in what follows.

The double integral in Eq. (7) can now be evaluated. We write it as

$$\begin{aligned}
&\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} du \, \exp[i(q-k)u] \langle \exp iau\zeta'(x_1) \rangle \\
&= \int_{-\infty}^{\infty} du \exp[i(q-k)u] \sum_{n=-N}^{N-1} \int_{nb}^{(n+1)b} dx_1 \langle \exp iau \, a_n \rangle \\
&= \int_{-\infty}^{\infty} du \exp[i(q-k)u] \sum_{n=-N}^{N-1} \int_{nb}^{(n+1)b} dx_1 \int_{-\infty}^{\infty} d\gamma f(\gamma) \exp iau\gamma \\
&= L_1 \int_{-\infty}^{\infty} du \exp[i(q-k)u] \int_{-\infty}^{\infty} d\gamma \, f(\gamma) \exp iau\gamma
\end{aligned}$$

$$= L_1 \int_{-\infty}^{\infty} d\gamma\, f(\gamma) 2\pi\delta(q - k + a\gamma)$$
$$= \frac{2\pi L_1}{a} f\left(\frac{k-q}{a}\right)$$
$$= \frac{2\pi L_1}{(\omega/c)(\cos\theta_0 + \cos\theta_s)} f\left(\frac{\sin\theta_0 - \sin\theta_s}{\cos\theta_0 + \cos\theta_s}\right). \tag{13}$$

In obtaining this result we have assumed that the length L_1 is equal to $2Nb$.

On combining Eqs. (7) and (13) we obtain

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{[1 + \cos(\theta_0 + \theta_s)]^2}{\cos\theta_0 (\cos\theta_0 + \cos\theta_s)^3} f\left(\frac{\sin\theta_0 - \sin\theta_s}{\cos\theta_0 + \cos\theta_s}\right). \tag{14}$$

Thus, we find that the mean differential reflection coefficient is expressed in terms of the pdf of the random deviate a_n.

The result given by Eq. (14) simplifies considerably in the case of normal incidence, $\theta_0 = 0$,

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{1 + \cos\theta_s} f\left(\frac{-\sin\theta_s}{1 + \cos\theta_s}\right), \tag{15}$$

and we will restrict ourselves to this case in what follows.

A useful result follows from Eq. (15) and the fact that $f(\gamma)$ is normalized to unity, namely that $\langle \partial R/\partial\theta_s \rangle$ is itself normalized to unity,

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\langle \frac{\partial R}{\partial \theta_s} \right\rangle d\theta_s = 1. \tag{16}$$

We now make the change of variable $\sin\theta_s/(1 + \cos\theta_s) = \tan(\theta_s/2) = \gamma$. Then Eq. (15) can be rewritten as

$$\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle (\gamma) = \frac{1}{2}(1 + \gamma^2) f(-\gamma), \tag{17}$$

whence it follows that

$$f(\gamma) = \frac{2}{1 + \gamma^2} \left\langle \frac{\partial R}{\partial \theta_s} \right\rangle (-\gamma). \tag{18}$$

It should be noted that this result does not depend explicitly on the wavelength of the incident light.

Thus, if we wish to design a perfectly conducting random surface which, when illuminated at normal incidence by s-polarized light, produces a specified form of $\langle \partial R/\partial\theta_s \rangle(\theta_s)$, we must first replace θ_s by γ in the latter function by the use of the relation $\gamma = \tan(\theta_s/2)$, and then determine $f(\gamma)$ from Eq. (18). From the result a long sequence of $\{a_n\}$ is determined, e.g. by the rejection method,[9] and the corresponding sequence of $\{b_n\}$ is obtained from Eqs. (11) or (12). The surface profile function $\zeta(x_1)$ is then constructed on the basis of Eq. (8).

The resulting surface profile functions possess the following general properties. The mean value of $\zeta(x_1)$ is given by

$$\langle \zeta(x_1) \rangle = \langle a_n x_1 + b_n \rangle \qquad nb \le x_1 \le (n+1)b. \tag{19}$$

The average of a_n is $\langle a \rangle$, independent of n. The average of b_n is therefore b_0, according to Eqs. (12), also independent of n. Consequently, we find that

$$\langle \zeta(x_1) \rangle = \langle a \rangle x_1 + b_0, \tag{20}$$

independent of n. This result means that the mean surface profile function will grow linearly with x_1 unless $\langle a\rangle = 0$. This requires that $\langle \partial R/\partial\theta_s\rangle(\gamma)$ be an even function of γ. This condition is satisfied for all the surfaces we will consider. It is then convenient to set $b_0 = 0$, so that the resulting surface profile function is a zero-mean random process.

The mean square height of the surface is given by

$$\begin{aligned}\langle\zeta^2(x_1)\rangle &= \langle(a_n x_1 + b_n)^2\rangle \qquad nb \le x_1 \le (n+1)b \\ &= \langle a_n^2\rangle x_1^2 + 2\langle a_n b_n\rangle x_1 + \langle b_n^2\rangle \quad nb \le x_1 \le (n+1)b.\end{aligned} \tag{21}$$

The value of $\langle a_n^2\rangle$ is $\langle a^2\rangle$, independent of n. From Eqs. (12) we find that $\langle a_n b_n\rangle = -n\langle a^2\rangle b$ for $n \ge 0$, while $\langle b_n^2\rangle = n(n+1)\langle a^2\rangle b^2$ for $n \ge 0$. Thus, we find that

$$\langle\zeta^2(x_1)\rangle = \langle a^2\rangle[x_1^2 - 2nbx_1 + n(n+1)b^2] \quad nb \le x_1 \le (n+1)b, \tag{22a}$$

for $n \ge 0$. A similar calculation yields the result that

$$\langle\zeta^2(x_1)\rangle = \langle a^2\rangle[x_1^2 + 2nbx_1 + n(n+1)b^2] \quad -(n+1)b \le x_1 \le -nb, \tag{22b}$$

for $n \ge 0$. These results show that $\zeta(x_1)$ is not a stationary function of x_1. In addition, they show that $\langle\zeta^2(x_1)\rangle$ is an even function of x_1.

We first seek to design a random surface that gives rise to a mean differential reflection coefficient that is a constant in the angular interval $|\theta_s| < \theta_m < \pi/2$, and vanishes for $|\theta_s| > \theta_m$ (a band limited uniform diffuser),

$$\begin{aligned}\left\langle\frac{\partial R}{\partial\theta_s}\right\rangle &= \frac{\theta(\theta_m - |\theta_s|)}{2\theta_m} \\ &= \frac{\theta(\tan(\theta_m/2) - |\tan(\theta_s/2)|)}{2\theta_m} \\ &= \frac{\theta(\gamma_m - |\gamma|)}{4\tan^{-1}\gamma_m},\end{aligned} \tag{23}$$

where $\gamma_m = \tan(\theta_m/2)$. We find from Eq. (18) that the pdf of a_n is given by

$$f(\gamma) = \frac{1}{2\tan^{-1}\gamma_m}\frac{\theta(\gamma - |\gamma|)}{1+\gamma^2}. \tag{24}$$

Similarly, if we wish to design a random surface that produces a mean differential reflection coefficient that is proportional to the cosine of the scattering angle (a Lambertian diffuser),

$$\begin{aligned}\left\langle\frac{\partial R}{\partial\theta_s}\right\rangle &= \frac{1}{2}\cos\theta_s \qquad -\pi/2 \le \theta_s \le \pi/2, \\ &= \frac{1}{2}\frac{1-\gamma^2}{1+\gamma^2} \qquad -1 \le \gamma \le 1,\end{aligned} \tag{25}$$

then we find from Eq. (18) that the pdf of a_n is given by

$$f(\gamma) = \frac{1-\gamma^2}{(1+\gamma^2)^2}\theta(1-|\gamma|). \tag{26}$$

Finally, if we wish to design a random surface that suppresses single-scattering processes within the angular interval $|\theta|_s < \theta_m < \pi/2$, the arguments put forward in Ref. 2 indicate that we wish to produce a mean differential reflection coefficient of the form

$$\begin{aligned}\left\langle\frac{\partial R}{\partial\theta_s}\right\rangle &= 0 \qquad |\theta_s| < \theta_m < \pi/2 \\ &= \frac{1}{2}\frac{\cos\theta_s}{1-\sin\theta_m} \qquad \theta_m < |\theta_s| < \pi/2 \\ &= \frac{1+\gamma_m^2}{2(1-\gamma_m)^2}\frac{1-\gamma^2}{1+\gamma^2} \qquad \gamma_m < |\gamma| < 1,\end{aligned} \tag{27}$$

where $\gamma_m = \tan(\theta_m/2)$. From Eq. (18) we find that the pdf of a_n is

$$f(\gamma) = \frac{1+\gamma_m^2}{(1-\gamma_m)^2}\frac{1-\gamma^2}{(1+\gamma^2)^2}\theta(|\gamma|-\gamma_m)\theta(1-|\gamma|). \tag{28}$$

A result for the mean differential reflection coefficient is readily obtained in the Kirchhoff approximation, i.e. without passing to the geometrical optics limit of this approximation. From Eq. (2) we find that at normal incidence ($\theta_0 = 0, k = 0$) the scattering amplitude is given by

$$R(q|0) = -\frac{1}{\cos\theta_s}\int_{-\infty}^{\infty} dx_1 \exp\left(-iqx_1 - ia\zeta(x_1))\right), \tag{29}$$

where now $a = (\omega/c)(1+\cos\theta_s)$. We can rewrite this expression in the form

$$R(q|0) = \frac{-1}{\cos\theta_s}\sum_{n=-N}^{N-1}\int_{nb}^{(n+1)b} dx_1 \exp[-iqx_1 - ia(a_n x_1 + b_n)], \tag{30}$$

so that we are assuming that $L_1 = 2Nb$. The integral is readily evaluated with the result that

$$R(q|0) = -\frac{br(q)}{\cos\theta_s}, \tag{31}$$

where

$$r(q) \quad = \quad \sum_{n=-N}^{N-1} \exp[-iab_n - i(q+aa_n)(n+\frac{1}{2})b]\mathrm{sinc}\frac{1}{2}(q+aa_n)b, \tag{32}$$

and $\mathrm{sinc}x = \sin x/x$. We thus have the result that

$$\left\langle\frac{\partial R}{\partial\theta_s}\right\rangle = \frac{b}{2N}\frac{\omega}{2\pi c}\langle|r(q)|^2\rangle. \tag{33}$$

The procedure now is to generate a large number N_p of realizations of the random surface, and for each realization to calculate the corresponding value of $|r(q)|^2$, recalling that $q = (\omega/c)\sin\theta_s$ and $a = (\omega/c)(1+\cos\theta_s)$. The N_p values of $|r(q)|^2$ so obtained are summed and then divided by N_p to yield the average indicated in Eq. (33). It should be noted that $\langle|r(q)|^2\rangle$ must increase linearly with N if the result for $\langle\partial R/\partial\theta_s\rangle$ is to be independent of N. In all of our calculations this is found to be the case.

In this paper the calculations of the mean differential reflection coefficient $\langle\partial R/\partial\theta_s\rangle$ will be carried out on the basis of Eqs. (32) and (33), which have been obtained on the assumption that the scattering medium is a perfect conductor that is illuminated at normal incidence by an s-polarized plane wave. Results obtained by a rigorous computer simulation approach[10] that takes multiple scattering proceses of all orders into account, for p-polarized incident light as well as for s-polarized incident light, and in the case where the perfectly conducting scattering medium is replaced by a finitely conducting metal, will be reported elsewhere.

A segment of the surface profile function $\zeta(x_1)$ and its derivative calculated by the approach proposed here is plotted in Figs. 1(a) and 1(b), respectively, for the case of a band-limited uniform diffuser, for which $f(\gamma)$ is given by Eq. (24). The parameters used in generating these functions were $\theta_m = 20°$, $b = 22\mu m$, and $N = 1000$. In addition, we set $b_0 = 0$ in Eqs. (12).

In Fig. 2 we present plots of $\langle\partial R/\partial\theta_s\rangle$ as a function of θ_s for the band-limited uniform diffuser defined by Eqs. (23) and (24). The parameters used in obtaining the result plotted in Fig. 2(a) were $\theta_m = 20°$, $b = 22\mu m$, $\lambda = 0.6328\mu m$, $N = 1000$, and $b_0 = 0$. The parameters used in obtaining the result plotted in Fig. 2(b) were $\theta_m = 20°$, $b = 220\mu m$, $\lambda = 0.6328\mu m$, $N = 100$, and $b_0 = 0$. Thus, although the value of the characteristic

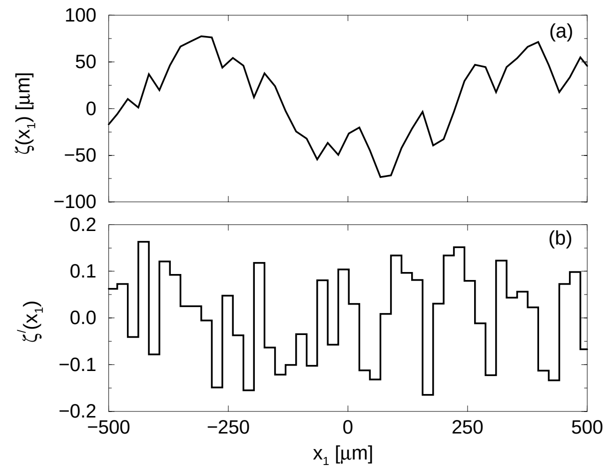

Figure 1. (a) A segment of the surface profile function $\zeta(x_1)$ for the case of a band-limited uniform diffuser, for which $f(\gamma)$ is given by Eq. (24). The parameters used in obtaining this surface are $\theta_m = 20°$, $b = 22\mu m$, and $b_0 = 0$. (b) The derivative $\zeta'(x_1)$ of this surface profile function.

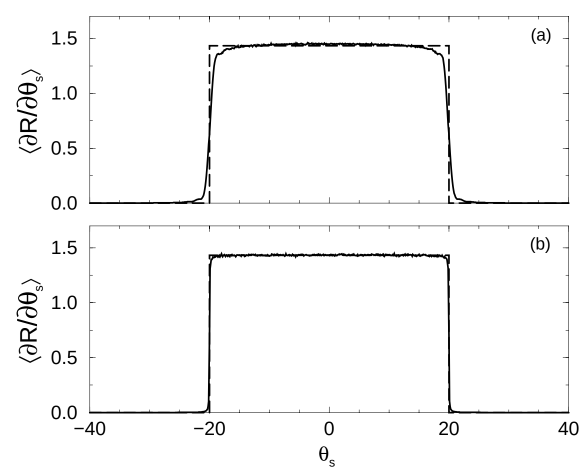

Figure 2. The mean differential reflection coefficient $\langle \partial R/\partial\theta_s \rangle$ estimated from $N_p = 100,000$ realizations of the surface profile function for the case of a band-limited uniform diffuser. The parameters employed are (a) $\theta_m = 20°$, $b = 22\mu m$, $\lambda = 0.6328\mu m$, $N = 1000$, and $b_0 = 0$; (b) $\theta_m = 20°$, $b = 220\mu m$, $\lambda = 0.6328\mu m$, $N = 100$, and $b_0 = 0$. The dashed curve presents the ideal distribution of scattered intensity given by Eq. (23).

length b was significantly larger in the calculations that led to Fig. 2(b) than it was in obtaining Fig. 2(a), the overall length of the surface, $L_1 = 2Nb$, was the same in both cases. The results obtained for $N_p = 100,000$ realizations of the surface profile function were used in calculating the average appearing in Eq. (33). This large number of realizations could be used because the calculations of $\langle \partial R/\partial\theta_s \rangle$ on the basis of Eqs. (32)-(33) are very fast (1200 realizations/minute on a Compaq XP100 workstation). We see that the random surface generated by our approach indeed acts as a band-limited uniform diffuser: there is virtually no scattered intensity for scattering angles larger than 20° in magnitude, and for values of $|\theta_s|$ smaller than 20° the scattered intensity is very closely uniform. We also see that the larger the characteristic length b is, the sharper are the corners of $\langle \partial R/\partial \theta_s \rangle$ and the more uniform is the scattered intensity in the region $|\theta_s| < 20°$. This appears to be a general result: The larger is b, the closer to the ideal $\langle \partial R/\partial \theta_s \rangle$ is the calculated one. Additional calculations, whose results are not shown, suggest that, for a given value of b, increasing the length of the surface L_1 also brings the calculated $\langle \partial R/\partial \theta_s \rangle$ closer to the ideal one.

In Figs. 3 and 4 we present the analogous results for the case of a Lambertian diffuser defined by Eqs. (25) and (26). The values of the parameters used in generating the segment of the surface profile function

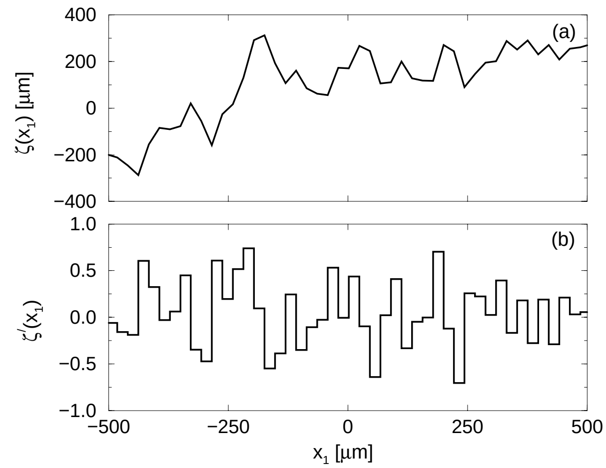

Figure 3. The same as Fig. 1, except for a Lambertian diffuser, for which $f(\gamma)$ is given by Eq. (26).

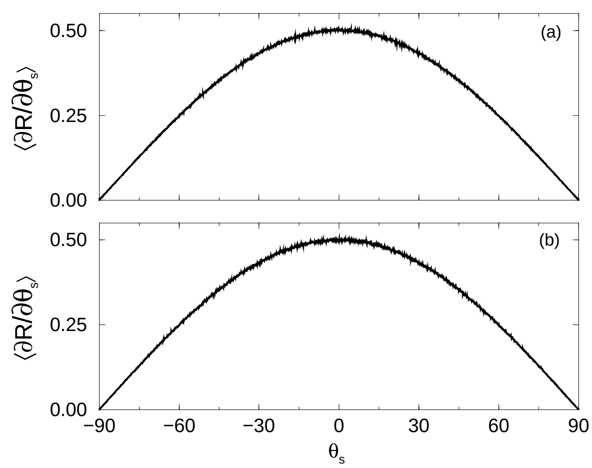

Figure 4. The same as Fig. 2, except for a Lambertian diffuser.

and its derivative in Fig. 3 were the same as those assumed in obtaining the results plotted in Fig. 1, while the parameters used in calculating the $\langle \partial R/\partial \theta_s \rangle$ plotted in Figs. 4(a) and 4(b) were the same as were used in obtaining the results plotted in Figs 3(a) and 3(b), respectively. Results obtained for $N_p = 100,000$ realizations of the surface profile function were used in calculating the average in Eq. (33). From the results presented in Figs. 4(a) and 4(b) we see that the calculated mean differential reflection coefficient $\langle \partial R/\partial \theta_s \rangle$ follows the cosine law (25) very closely. In this case increasing the value of the characteristic length b does not bring the calculated $\langle \partial R/\partial \theta_s \rangle$ signficantly closer to the ideal one, but it makes it a little less noisy.

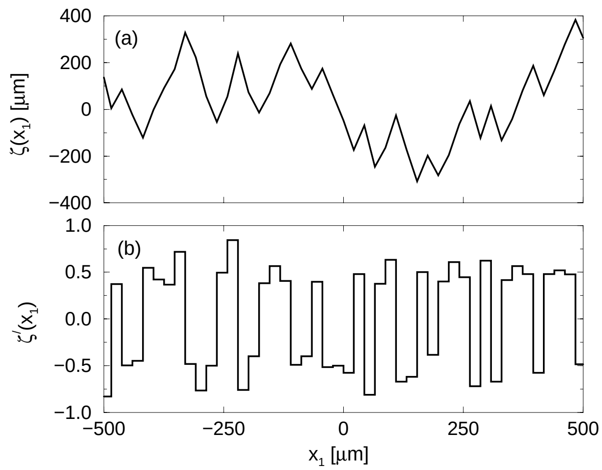

Figure 5. The same as Fig. 1, except for a surface that suppresses single scattering, for which $f(\gamma)$ is given by Eq. (28), with $\theta_m = 40°$.

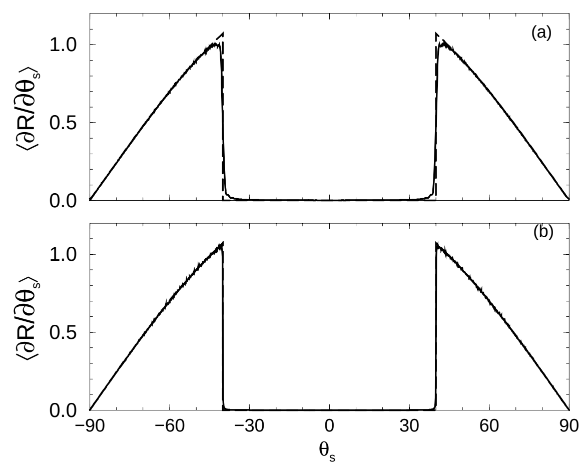

Figure 6. The same as Fig. 2, except for a surface that suppresses single scattering.

The corresponding results for a surface that supresses single scattering for θ_s in the interval $(-40°, 40°)$, for which the ideal $\langle \partial R/\partial \theta_s \rangle$ is given by Eq. (27) and the corresponding $f(\gamma)$ by Eq. (28), are presented in Figs. 5 and 6. The parameters used in generating the surface profile and its derivative were $\theta_m = 40°$,

$b = 22\mu m$, $N = 1000$, and $b_0 = 0$. In obtaining the mean differential reflection coefficients plotted in Fig. 6 the same values of θ_m and b_0 were assumed, and the results obtained for $N_p = 100,000$ realizations were used in calculating the average appearing in Eq. (33). The values $b = 22\mu m$, $N = 1000$, were used in obtaining the results plotted in Fig. 6(a), while the values $b = 220\mu m$, $N = 100$, were used in obtaining Fig. 6(b). It is seen from the results presented in Figs. 6(a) and 6(b) that $\langle\partial R/\partial\theta_s\rangle$ vanishes for θ_s in the interval $(-40°, 40°)$ for both values of b assumed. However, the use of the larger value of b sharpens the corners of the mean differential reflection coefficient and brings the calculated $\langle\partial R/\partial\theta_s\rangle$ into closer agreement with the ideal one. Since the results plotted in Fig. 6 were calculated on the basis of a single-scattering approximation, the vanishing of $\langle\partial R/\partial\theta_s\rangle$ for $|\theta_s| < 40°$ demonstrates that the surfaces used in obtaining this result indeed suppress single scattering for θ_s in this interval

We conclude this paper by describing the manner in which a one-dimensional surface of the kind defined by Eqs. (8) and (12) can be fabricated on photoresist. First, a single realization of such a surface profile function $\zeta_0(x)$ is used to fabricate a slit of variable width in the manner shown in Fig. 7. A good quality optical system

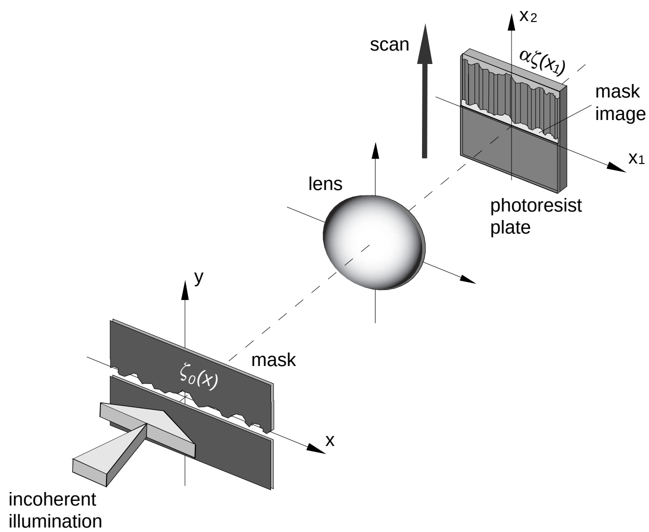

Figure 7. Schematic diagram of the proposed experimental arrangement for the fabrication of a surface with $f(\gamma)$ given by Eq. (18). The mask is imaged on the photoresist plate, which is then scanned along x_2.

is used to form an incoherent, demagnified image of the slit on the photoresist plate. It is assumed that the object is resolvable and, thus, that the intensity image is

$$I(x_1, x_2) = I_0\theta(x_2 + d)\theta(\zeta(x_1) - x_2), \tag{34}$$

where I_0 is a constant, the coordinates x_1 and x_2 are fixed on the plate, and $\zeta(x_1)$ is a scaled version of the mask profile $\zeta_0(x)$.

The photoresist plate is then scanned in the x_2-direction. The exposure of the plate due to this distribution and the motion of the plate, of length $L_2(> d_{max})$ and time T, may be written as

$$E(x_1) = K\int_{-T/2}^{T/2} I(x_1, x_2 + vt)dt, \tag{35}$$

where $v = L_2/T$ is the speed of the scan and K is a constant related to the sensitivity of the photoresist. Substitution of Eq. (34) into Eq. (35) gives

$$E(x_1) \quad = \quad KI_0\left[\frac{\zeta(x_1)}{v} + \frac{d}{v}\right], \tag{36}$$

which can be put in the form

$$E(x_1) = E_0 + \alpha\zeta(x_1), \tag{37}$$

where $E_0 = KI_0d/v$ and $\alpha = KI_0/v$. This expression shows that the exposure has a linear dependence on the generated surface profile function. Assuming that the relation between exposure and the resulting height on the surface is linear, the developed surface will have the desired properties. Moreover, characterized nonlinearities of the photoresist response may be taken into account by prescribed deformations of the mask illustrated in Fig. 7.

Thus, in this paper we have presented a method for generating a one-dimensional random perfectly conducting surface that scatters s-polarized light incident normally on it in such a way that the angular dependence of the intensity of the scattered light has a prescribed form. This method is simpler to use than the one used in our earlier studies of the same problem.[1–7] We have shown by numerical simulations based on the Kirchhoff approximation that the surfaces generated by this approach yield the prescribed scattering pattern. One of the advantages of this method over the one described in Refs. 1-7 is that the mean differential reflection coefficient is proportional to the pdf $f(\gamma)$ rather than to the sum $f(\gamma) + f(-\gamma)$, as in the earlier approach. This has the consequence that the small peak occurring in the mean differential reflection coefficient in the specular direction and caused by the overlap of the tails of the two distributions proportional to $f(\gamma)$ and $f(-\gamma)$,[1] is not present in the results obtained by the present approach. Finally, we have described a method for fabricating such surfaces on photoresist.

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