Invited Paper

Coherent Scattering of an Electromagnetic Wave From, and its Transmission Through, a Slab of a Left-Handed Medium with a Randomly Rough Illuminated Surface

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ABSTRACT

Recently a physical medium was fabricated in which both the effective permittivity and the effective permeability are simultaneously negative over a restricted frequency range. Thus, in this frequency range such a medium is "left–handed", and is characterized by a negative refractive index. In this paper we study the scattering of $p-$ and $s-$polarized electromagnetic waves from, and their transmission through, a slab of a left–handed medium whose illuminated surface is a one–dimensional randomly rough surface. We assume that the surface profile function is a single-valued function of the coordinate in the mean plane of the surface that is normal to its grooves and ridges, and constitutes a zero-mean, stationary, Gaussian random process. In the frequency range we are interested in, the electric and magnetic excitations give rise to $p-$ and $s-$polarized surface polaritons, Brewster modes, and waveguide modes in the slab. The reflectivity and the transmissivity of such a slab as a function of the angle of incidence displays structure associated with the existence of a Brewster angle in both polarizations and the existence of a critical angle for total internal reflection in both polarizations. The presence of surface roughness leads to a shift of the Brewster angle, the sign of which depends on the existence or nonexistence of surface or guided waves at the frequency of the incident field.

1. INTRODUCTION

Recently a physical medium was fabricated[1] in which the effective dielectric permittivity $\epsilon(\omega)$ and the effective magnetic permeability $\mu(\omega)$ are simultaneously negative over a restricted frequency range. Thus, in this

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frequency range, such a medium is "left-handed," and is characterized by a negative refractive index.[2] A left-handed medium should therefore exhibit unusual phenomena associated with the propagation of electromagnetic waves through it and in the scattering of electromagnetic waves from it.

In a recent paper by the present authors[3] the coherent (specular) and incoherent (diffuse) scattering of $p-$ and $s-$polarized electromagnetic waves from the weakly rough one-dimensional randomly rough surface of a semi-infinite left-handed medium were studied perturbatively and by computer simulations. As a preface to these calculations it was shown that in contrast to nonmagnetic media with a negative dielectric function, which support surface electromagnetic waves of only p polarization, the planar surface of a left–handed medium in contact with vacuum can support both $p-$ and $s-$polarized electromagnetic waves in suitable frequency ranges. The reflectivity of such a surface as a function of the angle of incidence displays structure associated with the existence of a Brewster angle and a critical angle for total internal reflection for incident light of both polarizations. The shifts of both of these angles due to the random roughness of the surface were also studied in Ref. 3. The study of the incoherent scattering of electromagnetic waves from the randomly rough surface of the left-handed medium revealed that in a certain frequency range, determined by the parameters defining the effective dielectric permittivity and the effective magnetic permeability of the left-handed medium, enhanced backscattering[4] was predicted when the incident field was s polarized, but not when it was p polarized and *vice versa* . Thus, by changing the frequency and/or polarization of the incident field one can alter the optical response of a left-handed medium with a randomly rough surface in what can be desirable ways. In addition to the enhanced backscattering peak Yoneda bands[5] were also found in the angular dependence of the intensity of the incoherent component of the scattered field for both polarizations of the incident field.

In the present paper we extend the work reported in Ref. 3 by studying the coherent scattering of $p-$ and $s-$polarized electromagnetic fields from, and their transmission through, a slab of a left-handed material whose illuminated surface is a one-dimensional weakly rough random surface.

The interest in this system derives from the fact that in addition to the two surface electromagnetic waves of each polarization that can exist in suitable frequency ranges (one surface electromagnetic wave of each polarization localized to each surface), it supports guided waves of both polarizations in frequency ranges that do not overlap the ranges in which the surface waves exist, as well as Brewster modes. All of these surface and guided waves can participate in the scattering processes. Thus, the coherent scattering of an electromagnetic wave from, and its transmission through, a slab of a left-handed material with a randomly rough illuminated surface is expected to display a richer variety of features than is present in scattering from a semi-infinite left-handed medium bounded by a randomly rough surface.

This paper is organized as follows. In Section 2 we define the scattering system that we will be studying in this work. In Section 3 we calculate the reflectivity and transmissivity of the slab when both of its surfaces are planar and it is illuminated from vacuum by a $p-$ or $s-$polarized electromagnetic field, and we also obtain the dispersion relations for the surface and guided waves of p and s polarization supported by this slab. We then turn our attention to the case where the illuminated surface of the slab is a one-dimensional randomly rough surface. In Section 4 we study the coherent scattering of a $p-$ or $s-$polarized field from the surface of the slab and its coherent transmission through the slab, and the roughness-induced shift of the Brewster angle is determined. The conclusions drawn from the calculations carried out in this work are presented in Section 5.

2. THE SCATTERING SYSTEM

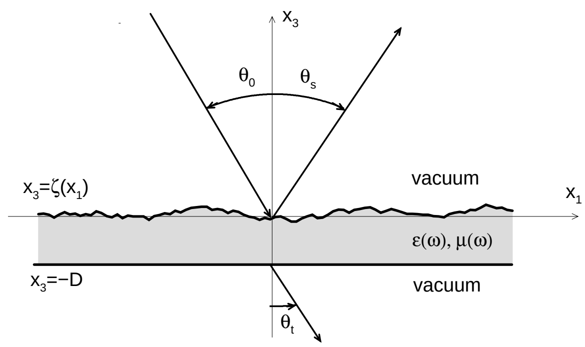

Figure 1: The system studied in this work.

The system we consider in this paper consists of vacuum in the region $x_3 > \zeta(x_1)$, a metamaterial of the kind constructed in Ref. [1] in the region $-D < x_3 < \zeta(x_1)$, and vacuum in the region $x_3 < -D$ (Fig. 1). The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1 that is differentiable as many times as is necessary, and constitutes a zero-mean, stationary, Gaussian random process defined by

$$\langle \zeta(x_1)\zeta(x_1')\rangle = \delta^2 W(|x_1 - x_1'|). \tag{2.1}$$

The angle brackets in this equation, and in all that follows, denote an average over the ensemble of realizations of $\zeta(x_1)$, and $\delta = \langle \zeta^2(x_1)\rangle^{\frac{1}{2}}$ is the rms height of the surface

It is convenient to introduce the Fourier representation of the surface profile function $\zeta(x_1)$,

$$\zeta(x_1) = \int_{-\infty}^{\infty} \frac{dk}{2\pi}\hat{\zeta}(k)\exp(ikx_1). \tag{2.2}$$

The Fourier coefficient $\hat{\zeta}(k)$ is also a zero-mean Gaussian random process, that is defined by

$$\langle \hat{\zeta}(k)\hat{\zeta}(k')\rangle = 2\pi\delta(k+k')\delta^2 g(|k|), \tag{2.3}$$

where $g(|k|)$, the power spectrum of the surface roughness, is given by

$$g(|k|) = \int_{-\infty}^{\infty} dx_1 W(|x_1|) \exp(-ikx_1). \tag{2.4}$$

In the present work we assume that the surface height autocorrelation function $W(|x_1|)$ has the Gaussian form

$$W(|x_1|) = \exp(-x_1^2/a^2), \tag{2.5}$$

where the characteristic length a is the transverse correlation length of the surface roughness, so that the power spectrum of the surface roughness $g(|k|)$ also has a Gaussian form,

$$g(|k|) = \sqrt{\pi} a \exp(-k^2 a^2/4). \tag{2.6}$$

It is assumed that in the frequency range of interest to us the metamaterial that occupies the region $-D < x_3 < \zeta(x_1)$ can be characterized by an isotropic, complex, frequency-dependent, dielectric function $\epsilon(\omega)$, and by an isotropic, complex, frequency-dependent magnetic permeability $\mu(\omega)$ of the forms [3]

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma_p)} \tag{2.7}$$

$$\mu(\omega) = 1 - \frac{F\omega_0^2}{\omega^2 - \omega_0^2 + 2i\omega\gamma_m}, \tag{2.8}$$

where ω_p and γ_p are the effective plasma frequency and the collision frequency of the electrons of the metamaterial, and F, ω_0, and γ_m are the effective "oscillator strength," resonant frequency, and damping constant, respectively, associated with the magnetic properties of the metamaterial.

From Eqs. (2.7) and (2.8) we find that in the lossless case ($\gamma_p = \gamma_m = 0$) $\epsilon(\omega)$ is negative for frequencies in the range $0 < \omega < \omega_p$, while $\mu(\omega)$ is negative in the range $\omega_0 < \omega < \omega_m$, where ω_m is the frequency at which $\mu(\omega_m) = 0$, and is given by $\omega_m = \omega_0(1 + F)^{1/2}$. In this paper we will restrict our attention to the case that $\omega_p > \omega_m$. Then, in the frequency range $\omega_0 < \omega < \omega_m$ both $\epsilon(\omega)$ and $\mu(\omega)$ are negative, and the metamaterial is left–handed.

3. REFLECTIVITIES, TRANSMISSIVITIES, AND SURFACE AND GUIDED WAVES

Before beginning a study of the possible effects of the "left-handedness" of the metamaterial on the scattering of electromagnetic waves from it, and on the transmission of electromagnetic waves through it, in the presence of the random roughness of the surface $x_3 = \zeta(x_1)$, we first analyze the case in which the random roughness is absent, $\zeta(x_1) \equiv 0$.

The Fresnel amplitude reflection coefficient for $p-$polarized electromagnetic waves of frequency ω reflected from a slab of thickness D with $\mu(\omega) \neq 1$ is given by

$$r_p(k) = \frac{[\epsilon(\omega)\alpha_0(k) - \alpha(k)][\epsilon(\omega)\alpha_0(k) + \alpha(k)]}{[\epsilon(\omega)\alpha_0(k) + i\alpha(k)\cot\alpha(k)D/2][\epsilon(\omega)\alpha_0(k) - i\alpha(k)\tan\alpha(k)D/2]} \tag{3.1}$$

where $k = (\omega/c)\sin\theta_0$, θ_0 is the angle of incidence, and $\alpha_0(k) = [(\omega/c)^2 - k^2]^{\frac{1}{2}} = (\omega/c)\cos\theta_0$, while $\alpha(k) = [\epsilon(\omega)\mu(\omega)(\omega/c)^2 - k^2]^{\frac{1}{2}}$. The corresponding expression for the reflection coefficient $r_s(k)$ when the incident electromagnetic field is s polarized is obtained by interchanging $\epsilon(\omega)$ and $\mu(\omega)$ in Eq. (3.1). The reflectivities of the slab with planar surfaces are $R_{p,s}(\theta_0) = |r_{p,s}(k)|^2$. The function $\alpha(k)$ entering Eq. (3.1) is a double valued function of k. The correct branch of this function is the one that yields the result that $Im\alpha(k) > 0$. Then, in the case that both $\epsilon(\omega)$ and $\mu(\omega)$ are negative, i.e. when the metamaterial is left handed, the real part of $\alpha(k)$ is negative. This result reflects the fact that a left-handed medium is characterized by the negative refractive index $n(\omega) = -\sqrt{\epsilon(\omega)\mu(\omega)}$.

The Fresnel amplitude transmission coefficient for $p-$polarized electromagnetic waves of frequency ω transmitted through a slab of thickness D with $\mu(\omega) \neq 1$ is given by

$$t_p(k) = 2i\frac{\epsilon(\omega)\alpha_0(k)\alpha(k)}{\sin\alpha(k)D}\frac{1}{[\epsilon(\omega)\alpha_0(k) + i\alpha(k)\cot\alpha(k)D/2][\epsilon(\omega)\alpha_0(k) - i\alpha(k)\tan\alpha(k)D/2]}. \tag{3.2}$$

The corresponding expression for the transmission coefficient $t_s(k)$ when the incident electromagnetic field is s polarized is obtained by interchanging $\epsilon(\omega)$ and $\mu(\omega)$ in Eq. (3.2). The transmissivities of the slab with planar surfaces are $T_{p,s}(\theta_0) = |t_{p,s}(k)|^2$.

The wavenumbers at which the reflectivities vanish are the wavenumbers of the Brewster modes supported by the system being studied. In the lossless case they are given by

$$k_p^2(\omega) = \frac{\omega^2}{c^2}\epsilon(\omega)\frac{\epsilon(\omega) - \mu(\omega)}{\epsilon^2(\omega) - 1}, \qquad k_s^2(\omega) = \frac{\omega^2}{c^2}\mu(\omega)\frac{\mu(\omega) - \epsilon(\omega)}{\mu^2(\omega) - 1}. \tag{3.3}$$

A careful examination of Eq. (3.1) and its version for s polarization shows that $r_p(k)$ and $r_s(k)$ also vanish at wavenumbers for which $\sin\alpha(k)D = 0$, i.e. when

$$k_{p,s}^2 = \epsilon(\omega)\mu(\omega)\frac{\omega^2}{c^2} - \left(\frac{n\pi}{D}\right)^2, \qquad n = 0, 1, 2, \ldots \tag{3.4}$$

These wavenumbers are therefore also wavenumbers of Brewster modes supported by the system being studied.

The poles of the reflection amplitudes (and of the transmission amplitudes) give the dispersion relations for the surface and guided waves supported by the system. In the lossless case they are

$$\epsilon(\omega)\alpha_0(k) = -i\alpha(k)\cot[\alpha(k)D/2] \tag{3.5a}$$

$$\epsilon(\omega)\alpha_0(k) = i\alpha(k)\tan[\alpha(k)D/2] \tag{3.5b}$$

in p polarization, and

$$\mu(\omega)\alpha_0(k) = -i\alpha(k)\cot[\alpha(k)D/2] \tag{3.6a}$$

$$\mu(\omega)\alpha_0(k) = i\alpha(k)\tan[\alpha(k)D/2] \tag{3.6b}$$

in s polarization. In Eqs. (3.5) the first equation is the dispersion relation for modes whose single nonzero component of the magnetic vector is an odd function of x_3 with respect to the midplane of the slab, while the second equation is the dispersion relation for modes for which this component is an even function of x_3 with respect to the midplane. In Eq. (3.6) the first equation is the dispersion relation for modes whose single nonzero component of the electric vector is an even function of x_3 with respect to the midplane of the slab, while the second equation is the dispersion relation for modes for which this component is an odd function of x_3 with respect to the midplane.

Unlike the situation where the left-handed material is semi-infinite in extent, where a frequency range exists within which a critical angle for total internal reflection exists, no such angle exists in the case where the material is a slab of finite thickness. However, if we set $\alpha(k) = 0$ in the expressions for the reflection coefficients, which occurs at an angle of incidence given by

$$\theta_0 = \arcsin(|n(\omega)|), \tag{3.7}$$

we find that the amplitude reflection coefficients take the forms

$$r_p(k) = \frac{1}{1 + i\frac{2}{\epsilon(\omega)\alpha_0(k)D}}, \qquad r_s(k) = \frac{1}{1 + i\frac{2}{\mu(\omega)\alpha_0(k)D}}. \tag{3.8}$$

Thus, we see that if D is large enough, then, at the angle of incidence given by Eq. (3.7), $2/[\epsilon(\omega)\alpha_0(k)D]$ will be negligible compared to unity, and both reflection amplitudes, and hence the corresponding reflectivities, become sensibly equal to unity. If the angle of incidence θ_0 is increased beyond the value given by Eq. (3.7), $\alpha(k)$ becomes imaginary, and $|r_p(k)|^2$ and $|r_s(k)|^2$ remain close to unity as θ_0 approaches 90°. Thus, while there is no true critical angle for total internal reflection in the system we are considering, there is a kind of pseudo-total internal reflection effect, if the slab is sufficiently thick.

It is convenient to introduce the frequency ω_v at which $\epsilon(\omega_v)\mu(\omega_v) = 1$, and the frequency ω_s at which $\mu(\omega_s) = -1$, which is the frequency of a surface magnon in the limit as $k \to \infty$. These frequencies are given by

$$\omega_v = \omega_p\omega_0 \left(\frac{1+F}{\omega_p^2 + F\omega_0^2}\right)^{1/2} \quad \omega_s = \omega_0 \left(\frac{2+F}{2}\right)^{1/2}. \tag{3.9}$$

In Figs. 2(a) and 2(b) the solutions of the dispersion relations (3.5) and (3.6), respectively, are presented for the set of material parameters given by $\omega_p = 10$GHz, $\omega_0 =$ GHz, $F = 0.56$. In this case $\omega_m = 4.996$GHz, $\omega_v = 4.786$GHz, and $\omega_s = 4.5255$GHz. The thickness of the slab is $D = 10$cm. Thus the metamaterial is

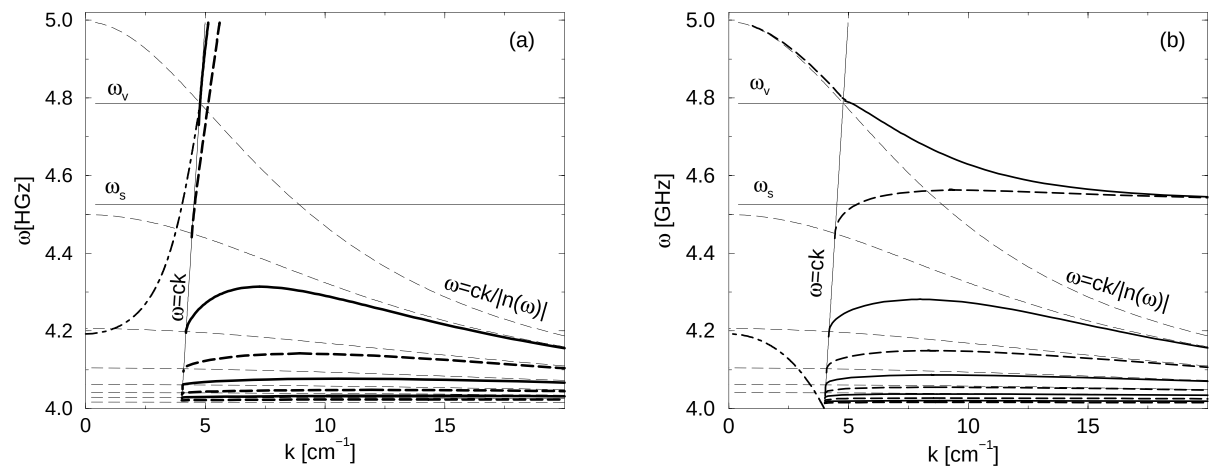

Figure 2. Dispersion curves for the Brewster modes, waveguide modes, and surface waves supported by a slab of left-handed metamaterial bounded by plane parallel surfaces. The metamaterial is left-handed in the frequency range, 4GHz $< \omega <$ 4.996GHz. (a) p polarization; (b) s polarization.

left–handed in the frequency range 4GHz $< \omega <$ 4.996GHz. It is in this frequency range that the dispersion curves are plotted in Fig. 2. In plotting these curves we have represented the solutions of Eqs. (3.5a) and (3.6a) by heavy solid curves, and the solutions of Eqs. (3.5b) and (3.6b) by heavy long–dashed curves.

Surface polaritons of both polarizations exist in the region of the $(\omega, k)-$plane to the right of both the curves $\omega = ck$ (the vacuum light line) and $\omega = ck/|n(\omega)|$ "the light line" in the left–handed medium. The latter curve is obtained from Eq. (3.4) with $n = 0$. In this region both $\alpha_0(k)$ and $\alpha(k)$ are purely imaginary, so that the electromagnetic field decays exponentially with increasing distance from each interface into both the vacuum and the left–handed material. In the case of $p-$polarized modes (Fig. 2(a)) dispersion curves of two surface polaritons are seen, one associated with each vacuum - metamaterial interface. These dispersion curves are close to the vacuum light line in the frequency range considered. As k tends to infinity, both curves tend to a limiting frequency given by $\omega_p/\sqrt{2}$, which is outside the frequency range within which the metamaterial is left–handed. In the case of $s-$polarized modes dispersion curves of two surface polaritons are also seen, one associated with each vacuum - metamaterial interface. With increasing k both approach a limiting frequency given by ω_s.

Guided waves of both polarizations exist in the region of the $(\omega, k)-$plane to the right of the vacuum light line $\omega = ck$ and to the left of the metamaterial light line $\omega = ck/|n(\omega)|$. In this case $\alpha_0(k)$ is purely imaginary, so that the electromagnetic field decays exponentially into the vacuum with increasing distance from each interface. However, $\alpha(k)$ is purely real, so that the electromagnetic field inside the metamaterial has the nature of standing waves. The dispersion curves corresponding to the solutions of Eqs. (3.5a) and (3.5b) interleave each other, as do the curves corresponding to the solutions of Eqs. (3.6a) and (3.6b). For a given set of values of ω_p, ω_0, F and D, the number of guided waves is infinite. We have plotted the dispersion curves of only several of the highest frequency modes occuring in the frequency region $\omega_0 < \omega < \omega_m$ where the medium is left–handed. Because the dispersion curves of the guided waves lie to the right of the vacuum light line, the incident electromagnetic field cannot couple into them in the case that the vacuum - metamateral interfaces are planar. However, they can be excited by the incident field when the illuminated surface is randomly rough.

The Brewster modes exist in the radiative region of the $(\omega, k)-$plane, i.e. to the left of the vacuum light line. They are seen to be of two types. Those depicted by heavy short dashed–long dashed curves are the solutions of Eqs. (3.3). Those that are depicted by light dashed curves are given by the solutions of Eq. (3.4), and correspond to values of $n = 0, 1, 2, , \ldots$ in the order of decreasing frequency. We extend the latter dispersion curves into the region of the $(\omega, k)-$plane to the right of the vacuum light line $\omega = ck$, where they are the asymptotes that the dispersion curves of the guided modes obtained from Eqs. (3.5) and (3.6) approach as $k \to \infty$. However, the incident light cannot couple into these modes in the absence of the surface roughness.

The dispersion curves of the higher frequency surface polaritons of p and s polarizations cross the vacuum light line and merge into the dispersion curves of the Brewster modes, while the dispersion curves of the lower frequency surface polaritons of both polarizations after crossing the metamaterial light line become the dispersion curves of the lowest index guided modes. These dispersion curves merge with the vacuum light line at the frequency of the Brewster mode associated with the solution of Eq. (3.4) with $n = 1$.

In concluding this section we note that dispersion curves for surface polaritons of both p and s polarizations in a slab of a left–handed medium were calculated by Ruppin,[6] and for surface polaritons of s polarization in such a slab by Bespyatykh *et al.*[7] Dispersion curves for surface polaritons and guided waves of s polarization in a slab of a left–handed medium were calculated by Shadrivov *et al.*[8] However, all of these studies were based on an expression for the magnetic permeability $\mu(\omega)$ different from the one given by Eq. (2.6).

4. COHERENT SCATTERING AND TRANSMISSION

In the case where the surface is weakly rough we use the self-energy perturbation approach[9] to calculate the reflectivity and transmittivity. In the self-energy perturbation theory[9] the averaged reflection coefficient in $p-$polarization has the form

$$r_p(k) = \frac{[\epsilon(\omega)\alpha_0(k) - \alpha(k)](1 - \exp[2i\alpha(k)D]) - i\epsilon(\omega)M(k)(1 - r(k))}{[\epsilon(\omega)\alpha_0(k) + \alpha(k)] - [\epsilon(\omega)\alpha_0(k) - \alpha(k)]r(k) - i\epsilon(\omega)M(k)(1 - r(k))}. \tag{4.1}$$

In Eq. (4.1) $M(k)$ is the averaged proper self-energy,[9] which can be calculated in a standard way that is described, for example in Ref. 9. Through terms of second order in the surface profile function $\zeta(x_1)$ the self–energy $M_p(k)$ in the case of p polarization is given by

$$M_p(k) = -i\delta^2 \left\{ \left(1 - \frac{1}{\epsilon^2(\omega)}\right) k^2 - \left(1 - \frac{\mu(\omega)}{\epsilon(\omega)}\right) \frac{\omega^2}{c^2} \right\} \frac{1+r(k)}{1-r(k)}$$

$$+i\delta^2 \left(1 - \frac{1}{\epsilon(\omega)}\right) \alpha(k) \frac{1+r(k)}{1-r(k)} \int_{-\infty}^{\infty} \frac{dp}{2\pi} g(|k-p|) v_1(p|k) + \delta^2 \int_{-\infty}^{\infty} \frac{dp}{2\pi} g(|k-p|) G_0(p) v_1(p|k)^2, \quad (4.2a)$$

where

$$v_1(p|k) = \left(1 - \frac{1}{\epsilon(\omega)}\right) pk + (\mu(\omega) - 1)\frac{\omega^2}{c^2} - \frac{\alpha(p)\alpha(k)}{\epsilon(\omega)} \left(1 - \frac{1}{\epsilon(\omega)}\right) \frac{1+r(p)}{1-r(p)} \frac{1+r(k)}{1-r(k)}, \quad (4.2b)$$

and

$$G_0(p) = \frac{i\epsilon(\omega)(1-r(p))}{\epsilon(\omega)\alpha_0(p) + \alpha(p) - r(p)(\epsilon(\omega)\alpha_0(p) - \alpha(p))} \quad (4.2c)$$

$$r(p) = \frac{\epsilon(\omega)\alpha_0(p) - \alpha(p)}{\epsilon(\omega)\alpha_0(p) + \alpha(p)} \exp[2i\alpha(p)D]. \quad (4.2d)$$

When the incident electromagnetic field is s polarized the expression for $r_s(k)$ is obtained by interchanging $\epsilon(\omega)$ and $\mu(\omega)$ in Eqs. (4.1) - (4.2).

The reflectivities $R_{p,s}(\theta_0)$ are then given by

$$R_{p,s}(\theta_0) = |r_{p,s}(k)|^2 \quad (4.3)$$

with $k = (\omega/c)\sin\theta_0$.

A self-energy perturbation theory for the transmission amplitude $T(q|k)$ can be constructed in a way analogous to that described in Ref. 9. The result is that the transmission coefficient in p polarization has the form

$$t_p(k) = \frac{2\alpha_0(k)t(k)}{[\epsilon(\omega)\alpha_0(k) + \alpha(k)] - [\epsilon(\omega)\alpha_0(k) - \alpha(k)]r(k) - i\epsilon(\omega)M_t(k)(1-r(k))}, \quad (4.4)$$

where

$$t(k) = \frac{2\alpha(k)\exp[i\alpha(k)D]}{\epsilon(\omega)\alpha_0(k) + \alpha(k)}. \quad (4.5)$$

The averaged proper self-energy for the transmission problem $M_t(k)$, can also be calculated in a standard way described, for example in Ref. 9. Through terms of second order in the surface profile function $\zeta(x_1)$ it is given by

$$M_t(k) = i\frac{\delta^2}{2} \left\{ \alpha_0(k) \left[(\mu(\omega)-1)\frac{\omega^2}{c^2} + \left(1 - \frac{1}{\epsilon(\omega)}\right) \left(2k^2 - \epsilon(\omega)\mu(\omega)\frac{\omega^2}{c^2}\right) \right] \right.$$

$$\left. + \alpha(k) \left[(\mu(\omega)-1)\frac{\omega^2}{c^2} + \left(1 - \frac{1}{\epsilon(\omega)}\right) \left(2k^2 - \frac{\omega^2}{c^2}\right) \right] \frac{1+r(k)}{1-r(k)} \right\} \quad (4.6a)$$

$$+ \delta^2 \int_{-\infty}^{\infty} \frac{dp}{2\pi} v_t(k|p) G_0(p) v_t(p|k) g(|k-p|), \quad (4.7a)$$

where

$$v_t(k|p) = (\mu(\omega) - 1)\frac{\omega^2}{c^2} + \left(1 - \frac{1}{\epsilon(\omega)}\right) kp + \left(1 - \frac{1}{\epsilon(\omega)}\right) \alpha_0(k)\alpha(p)\frac{1 + r(p)}{1 - r(p)}, \tag{4.7b}$$

and $G_0(p)$ is given by Eq. (4.2c). The expression for $t_s(k)$ in s polarization is obtained by interchanging $\epsilon(\omega)$ and $\mu(\omega)$ in Eqs. (4.4) - (4.5). The transmissivities $T_{p,s}(\theta_0)$ are given by

$$T_{p,s}(\theta_0) = |t_{p,s}(k)|^2, \tag{4.8}$$

with $k = (\omega/c)\sin\theta_0$.

In Figs. 3-7 we present numerical results for the reflectivities $R_p(\theta_0)$ and $R_s(\theta_0)$, and the transmissivities $T_p(\theta_0)$ and $T_s(\theta_0)$ as functions of the angle of incidence θ_0. These results have been obtained on the basis of the self-energy perturbation theory expressions given by Eqs. (4.1), (4.2) and (4.4), (4.5). In the numerical calculations we have assumed that the roughness of the illuminated surface of the left-handed slab is characterized by the Gaussian power spectrum (2.6), with a transverse correlation length $a = 4$ cm, and an rms height $\delta = 0.4$ cm. The damping constants γ_p and γ_m entering the expressions for $\epsilon(\omega)$ and $\mu(\omega)$ are $\gamma_p = 0.001\ \omega_p$ and $\gamma_m = 0$. The remaining material parameters are the same as those used in obtaining the dispersion curves plotted in Fig. 2.

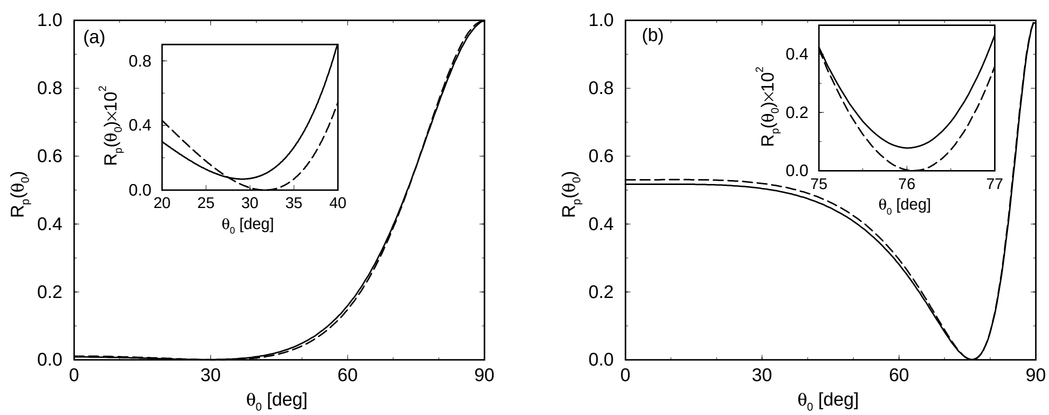

Figure 3. The reflectivity $R_p(\theta_0)$ as a function of the angle of incidence θ_0 in the case of a randomly rough illuminated surface (solid line) and in the case of a planar illuminated surface (dashed line). (a) $\omega = 4.25$ GHz; (b) $\omega = 4.7$ GHz.

Our choice of $\gamma_m = 0$ is due to the fact that when $\gamma_m \neq 0$, as the frequency ω approaches ω_0 damping in the medium becomes stronger and stronger, and the Brewster modes and waveguide modes whose frequencies lie just above ω_0 are strongly attenuated as a result. Consequently, the features in reflectivities and transmissivities, to which these modes give rise, tend to be washed out. This emphasizes the need for fabricating left-handed metamaterials with the smallest possible values of γ_m if these features are to be seen experimentally.

In Figs. 3(a)-3(b) we present plots of $R_p(\theta_0)$ for the randomly rough illuminated surface (solid curves) as functions of θ_0 for two different frequencies of the incident electromagnetic field, namely $\omega = 4.25$ GHz and 4.7 GHz, respectively. We also plot the corresponding results obtained in the absence of the surface roughness (dashed curves). In Fig. 3(a) we have plotted $R_p(\theta_0)$ for the frequency $\omega = 4.25$ GHz. From Fig. 2(a) we see that at this frequency there is a single Brewster mode in the radiative region $k < \omega/c$, whose dispersion curve is given by Eq. (3.5), which gives rise to the minimum in $R_p(\theta_0)$ at $\theta_0 \cong 29°$. From the inset it is seen that the Brewster angle is shifted to a smaller value by the surface roughness. When the frequency of the incident electromagnetic field is increased to 4.7 GHz, there is still a single Brewster mode in the nonradiative region, whose dispersion curve is given by Eq. (3.5), which gives rise to the minimum in $R_p(\theta_0)$ at $\theta_0 \cong 76°$. Again, the surface roughness shifts the Brewster angle to smaller values.

In Figs. 4(a) and (4b) we present plots of $T_p(\theta_0)$ for the randomly rough surface (solid curves) and for a planar surface (dashed curves) as functions of θ_0, for the same frequencies that were used in obtaining the results plotted in Fig. 3. All other parameters also have the same values as those used in obtaining Fig. 3. Qualitatively and semi-quantitatively these curves mimic the curves of $1 - R_p(\theta_0)$, and display maxima at values of θ_0 at which the Brewster effect occurs in $R_p(\theta_0)$. The quantitative difference between $T_p(\theta_0)$ and $1 - R_p(\theta_0)$ is due, of course, to the existence of incoherent scattering and absorption.

The reflectivities $R_s(\theta_0)$ for the randomly rough illuminated surface (solid curves) are plotted as functions of θ_0 for the two frequencies of the incident electromagnetic field given by $\omega = 4.10$ GHz and 4.96 GHz in Figs. 5(a) and 5(b), respectively. The corresponding results obtained in the absence of the surface roughness are also presented (dashed curves). The remaining experimental and material parameters are those assumed in obtaining the results plotted in Fig. 3. The result plotted in Fig. 5(a) displays a shallow minimum at an

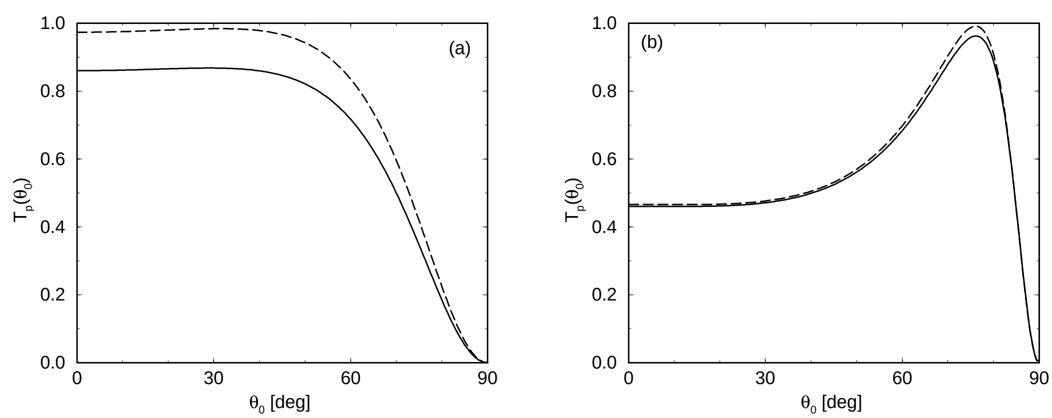

Figure 4. The transmissivity $T_p(\theta_0)$ as a function of the angle of incidence θ_0 in the case of a randomly rough illuminated surface (solid line) and in the case of a planar illuminated surface (dashed line). (a) $\omega = 4.25$ GHz; (b) $\omega = 4.7$ GHz.

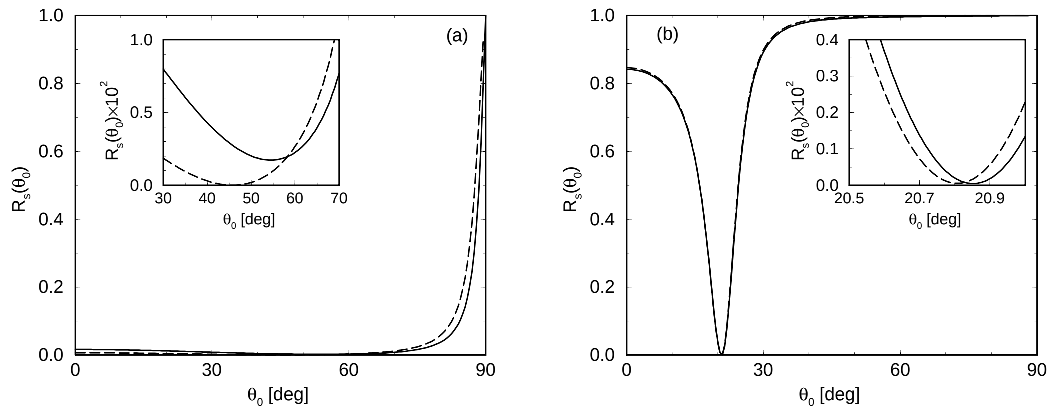

Figure 5. The reflectivity $R_s(\theta_0)$ as a function of the angle of incidence θ_0 in the case of a randomly rough illuminated surface (solid line) and in the case of a planar illuminated surface (dashed line). (a) $\omega = 4.10$ GHz; (b) $\omega = 4.96$ GHz.

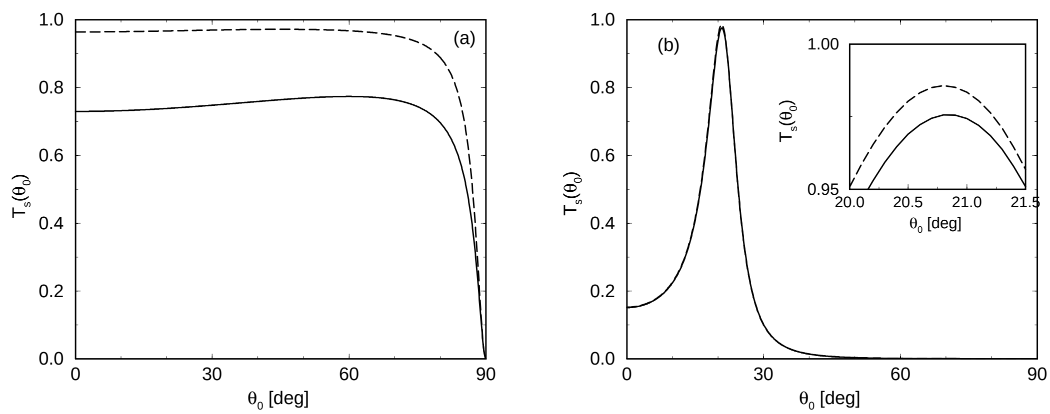

Figure 6. The transmissivity $T_s(\theta_0)$ as a function of the angle of incidence θ_0 in the case of a randomly rough illuminated surface (solid line) and in the case of a planar illuminated surface (dashed line). (a) $\omega = 4.10$ GHz; (b) $\omega = 4.96$ GHz.

angle of incidence $\theta_0 = 55°$. From Fig. 2(b) we see that there exists a single Brewster mode at this frequency, whose dispersion relation is Eq. (3.6), that is responsible for this minimum. We also see that, in contrast with the reflectivity obtained for a p-polarized incident field, the Brewster angle in the present case is shifted to a larger value by the surface roughness. When the frequency of the incident field is increased to $\omega = 4.96$ GHz (Fig. 5(b)) $R_s(\theta_0)$ displays a sharp dip at $\theta_0 \cong 20.85°$ due to the Brewster mode whose dispersion relation is Eq. (3.7) with $n = 0$. (This is also the angle at which the pseudo-total internal reflection effect would occur in a much thicker slab.) The roughness-induced shift of the Brewster angle in this case is in the direction of larger values.

The corresponding transmissivities $T_s(\theta_0)$ at the frequencies $\omega = 4.10$ GHz and 4.96 GHz are plotted as functions of θ_0 in Figs. 6(a) and 6(b), respectively. Both curves qualitatively follow the curves of $1 - R_s(\theta_0)$.

We have seen from the results presented in Figs. 3 and 5 that the roughness-induced shift of the Brewster angle can be either toward smaller values (a negative shift) or toward larger values (a positive shift) depending on the frequency and polarization of the incident electromagnetic field. The rule specifying whether the shift is positive or negative at a given frequency can be stated as follows. In the absence of surface modes or waveguide modes at the frequency of the incident field, the shift of the Brewster angle is negative when the group velocity of the Brewster mode at that frequency is positive, and is positive if the group velocity is negative. The presence of guided modes at the frequency of the incident field, i.e. the presence of true poles of the averaged Green's function, makes the shift negative.

The mean thickness of the slabs we have studied until now ($D = 10$cm), is too small for the reflectivities to display the pseudo–total internal reflection effect discussed in Section 3. However, if we increase it significantly, the effect appears. In Fig. 7 we plot the reflectivity $R_p(\theta_0)$ when an electromagnetic field of frequency $\omega = 4.9$ GHz is incident on a slab of a left-handed material whose mean thickness $D = 40$cm and whose illuminated surface is a one-dimensional randomly rough surface (solid curve). We also plot the corresponding result obtained in the absence of the roughness (dashed curve). In addition to the narrow dip in $R_p(\theta_0)$ centered at $\theta_0 \cong 22.25°$, that is shifted to smaller angle by the surface roughness, it is seen that the reflectivity of the slab with a planar illuminated surface approaches unity at an angle of incidence close to the value $\theta_0 = 37.78°$ given by Eq. (3.7), and remains close to unity as θ_0 is increased toward 90°. The presence of the surface roughness makes this effect less sharply delineated, but it is nevertheless still present.

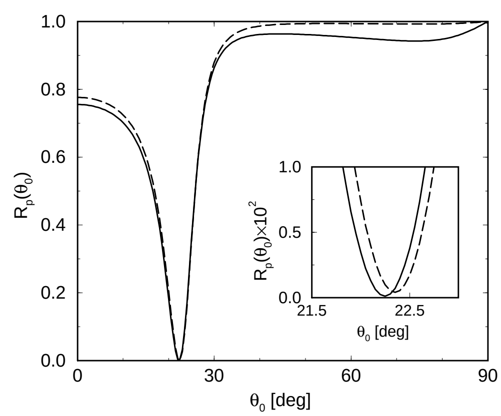

Figure 7. The reflectivity $R_p(\theta_0)$ as a function of the angle of incidence θ_0 in the case of a randomly rough illuminated surface. The frequency of the incident field is $\omega = 4.9$ GHz. The mean thickness of the slab is $D = 40$ cm.

5. CONCLUSIONS

A slab of a left-handed medium in contact with vacuum supports surface polaritons, guided waves, and Brewster modes of both p and s polarizations in suitable frequency ranges.

The reflectivities for incident electromagnetic waves of both p and s polarization as functions of the angle of incidence display the Brewster effect and a pseudo–total internal reflection effect in suitable ranges of the frequency of the incident electromagnetic wave. Surface roughness produces a shift of the Brewster angle, whose sign depends on the existence or nonexistence of surface or guided waves at the frequency of the incident field, while it makes the pseudo-total internal reflection effect less pronounced and shifts the pseudo–critical angle in the direction of larger values

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