Optics of Surface Disordered Systems
No Disorder — No Fun!

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and several students.....
Even Pauli was Challenged by the Surface....

Wolfgang Ernst Pauli
(1900 – 1958)

Pauli is quoted for saying:

*God made the bulk; the surface was invented by the devil!*

I wonder:

What would Pauli have thought about a *randomly rough surface*....?
However, *roughness* can also be beneficial....

Roughness increases the efficiency of solar cells

[Appl. Phys. Lett. 94, 211101 (2009)]

[J. Appl. Phys. 101, 074903 (2007)]
Introduction

Theoretical Background
- Scattering geometry
- How to characterize randomly rough surfaces
- Physical observables

Physical phenomena and their origin
- The enhanced backscattering phenomenon
- The satellite peak phenomenon
- The forward scattering enhancement
- Angular intensity correlation functions

Conclusions and Outlooks

A talk about coherent effects in surface random systems and their physical origins!
Some history:
- Lord Rayleigh (1877 (?)
- Mandel’shtam (1913)
- Rice (1951)
- M. V. Berry (1979):
  - Diffractal
- McGurn, et al. (1985)
  - Multiple Scattering Phenomenon

Why should one care:
- Scientific interesting problem
  - fundamental interest
  - astrophysics
- Industrial applications
  - electronics
  - energy sector
  - seismic
  - medical sector
- Military applications
  - radar technology

The transition from specular to diffuse scattering:
Rough surface scattering is complex

Computer simulation of light scattered from a rough metal surface

The *speckle patterns* are complex and they do depend on parameters like

- surface roughness
- surface correlations
- angle of incidence
- material
For a one-dimensional scattering geometry one introduce the fundamental field quantity

\[ \Phi_{\nu}(x_1, x_3|\omega) = \begin{cases} 
    H_2(x_1, x_3|\omega), & \nu = p, \\
    E_2(x_1, x_3|\omega), & \nu = s, 
\end{cases} \]

that should satisfy the Helmholtz equation

\[(\partial^2_{x_1} + \partial^2_{x_3} + \varepsilon \frac{\omega^2}{c^2})\Phi_{\nu}(x_1, x_3|\omega) = 0.\]

Boundary Conditions:

\[\Phi_{\nu}^+(x_1, x_3|\omega)|_{x_3em}=\zeta(x_1) = \Phi_{\nu}^-(x_1, x_3|\omega)|_{x_3em}=\zeta(x_1)\]

\[\frac{1}{\kappa_{\nu}^+(\omega)} \partial_n \Phi_{\nu}^+(x_1, x_3|\omega)|_{x_3em}=\zeta(x_1) = \frac{1}{\kappa_{\nu}^-(\omega)} \partial_n \Phi_{\nu}^-(x_1, x_3|\omega)|_{x_3em}=\zeta(x_1)\]

where \(\partial_n\) is the normal derivative

\[\partial_n = n \cdot \nabla = \frac{-\zeta'(x_1) \partial_{x_1} + \partial_{x_3}}{\sqrt{1 + (\zeta'(x_1))^2}},\]

\[\kappa_{\nu}^\pm(\omega) = \begin{cases} 
    \varepsilon(\omega), & \nu = p \\
    \mu(\omega), & \nu = s 
\end{cases} \]
Theoretical Background

Scattering Geometry

\[ \theta_0 : \text{angle of incidence} \]
\[ \theta_s : \text{angle of scattering} \]
\[ \theta_t : \text{angle of transmission} \]
\[ \theta_s = \theta_0 : \text{specular direction} \]
\[ \varepsilon_{\pm}(\omega) : \text{dielectric functions} \]
\[ \zeta(x_1) : \text{surface profile function} \]

Asymptotic forms of the fields

\[
\Phi^+(x_1, x_3 | \omega) = e^{ikx_1 - i\alpha_+(k, \omega)x_3} + \int_{-\infty}^{\infty} \frac{dq}{2\pi} R_v(q | k) e^{iqx_1 + i\alpha_+(q, \omega)x_3}, \quad x_3 > \max(\zeta)
\]
\[
\Phi^-(x_1, x_3 | \omega) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} T_v(p | k) e^{ipx_1 - i\alpha_-(p, \omega)x_3}, \quad x_3 < \min(\zeta)
\]

\[
k = \sqrt{\varepsilon_+(\omega) (\omega/c) \sin \theta_0},
\]
\[
q = \sqrt{\varepsilon_+(\omega) (\omega/c) \sin \theta_s},
\]
\[
p = \sqrt{\varepsilon_-(\omega) (\omega/c) \sin \theta_t}
\]
\[
\alpha_{\pm}(k, \omega) = \sqrt{\varepsilon_{\pm}(\omega) \frac{\omega^2}{c^2} - k^2}
\]
\[
= \sqrt{\varepsilon_{\pm}(\omega) \frac{\omega}{c} \cos \theta_0}
\]
The physical observable we will be interested in is the **Mean Differential Reflection Coefficient** (mean DRC).

**Definition (Mean DRC)**

The mean DRC, $\langle \partial R_\nu / \partial \theta_s \rangle$, is the fraction of the power flux incident on the surface that is scattered into an angular interval of width, $d\theta_s$, about the scattering direction $\theta_s$.

The incident/scattered power flow can be obtained from the 3-component of the (complex) Poynting vector, $\mathbf{S} = \mathbf{E} \times \mathbf{H}^*$:

$$P = \int dx_1 dx_2 \, Re \langle S_3 \rangle_t$$

$$\langle \partial R_\nu / \partial \theta_s \rangle = \langle p_{sc}(\theta_s) \rangle_{P_{inc}} = \frac{1}{L} \frac{\omega^2}{2\pi c} \cos^2 \theta_s \cos \theta_0 \left| R_\nu(q|k) \right|^2$$

$k = \sqrt{\varepsilon + \omega c \sin \theta_0}$

$q = \sqrt{\varepsilon + \omega c \sin \theta_s}$

$\varepsilon$
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\[
P = \int dx_1 dx_2 \, \text{Re} \langle S_3 \rangle_t
\]

\[
\langle \partial R_v / \partial \theta_s \rangle = \langle \frac{p_{sc}(\theta_s)}{P_{inc}} \rangle = \frac{1}{L} \frac{\omega}{2\pi c} \cos^2 \theta_s \cos \theta_0 \langle |R_v(q|k)|^2 \rangle,
\]

\[
k = \sqrt{\varepsilon + \frac{\omega}{c}} \sin \theta_0
\]

\[
q = \sqrt{\varepsilon + \frac{\omega}{c}} \sin \theta_s
\]
Theoretical Background

Physical Observables

The Coherent and Incoherent contribution to the mean DRC

- The mean DRC, $\left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle$ is an experimental accessible quantity
- $R_\nu(q|k)$ is the *scattering* (or reflection) *amplitude* for polarization $\nu$
- The main goal is to obtain $R_\nu(q|k)$ (the difficult part)
Physical Observables

The Coherent and Incoherent contribution to the mean DRC

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- \( R_\nu(q|k) \) is the scattering (or reflection) amplitude for polarization \( \nu \)
- The main goal is to obtain \( R_\nu(q|k) \) (the difficult part)

A simple rewriting of the expression for the mean DRC:

\[
\langle |R_\nu(q|k)|^2 \rangle = \langle |R_\nu(q|k)|^2 \rangle - |\langle R_\nu(q|k) \rangle|^2 + |\langle R_\nu(q|k) \rangle|^2
\]

incoherent

coherent

gives that it has two components — the \textit{coherent} (or specular) and the \textit{incoherent} (or diffuse); (not easily done experimentally)

The \textit{incoherent} and \textit{coherent} contribution of the mean DRC

\[
\langle \frac{\partial R_\nu}{\partial \theta_s} \rangle = \langle \frac{\partial R_\nu}{\partial \theta_s} \rangle_{incoh} + \langle \frac{\partial R_\nu}{\partial \theta_s} \rangle_{coh}
\]
Statistical properties of the surface roughness

Let \( \zeta(x_1) \) denote the *surface profile function*

- height distribution
- height-height correlation function

Normally one assumes that \( \zeta(x_1) \) is a single-valued, differentiable function of \( x_1 \) that constitutes a stationary zero-mean Gaussian random process so that

\[
\langle \zeta(x_1) \rangle = 0 \quad \text{and} \quad \langle \zeta(x_1) \zeta(x'_1) \rangle = \sigma^2 \mathcal{W}(|x_1 - x'_1|), \quad \mathcal{W}(0) = 1
\]

where \( \langle \cdot \rangle \) is the ensemble average and \( \mathcal{W}(x_1) \) is the transverse correlation function.

Also useful is the *power spectrum* of the roughness defined by

\[
g(|k|) = \int_{-\infty}^{\infty} dx_1 \, \mathcal{W}(|x_1|) e^{-ikx_1}
\]
How to Characterize Randomly Rough Surfaces

Surface topographies from real life!

A plastic surface

Cold rolled Al surface (self-affine)

Surfaces for Photovoltaic
**Question**: Does the form of $g(|k|)$ really matter much for the scattering?

Two different power spectra: $\sigma = 10\text{nm}$; $\lambda = 457.9\text{nm}$; $p$-polarization
**The Power Spectrum**

**Question**: Does the form of $g(|k|)$ really matter much for the scattering?

Two different power spectra: $\sigma = 10\text{nm}; \lambda = 457.9\text{nm}; \rho$-polarization.

![Power Spectrum Diagram](image)

- **Gaussian**
- **Rectangular**

Small amplitude perturbation theory predicts that (to lowest order)

$$\langle \frac{\partial R_{\nu}}{\partial \theta} \rangle \propto g(|q-k|).$$
**The Power Spectrum**

**Question**: Does the form of \( g(|k|) \) really matter much for the scattering?

Two different power spectra: \( \sigma = 10\text{nm}; \lambda = 457.9\text{nm}; p\)-polarization

Small amplitude perturbation theory predicts that (to lowest order)

\[
\left\langle \frac{\partial R_\gamma}{\partial \theta_s} \right\rangle \propto g(|q - k|)
\]
Scattering from a rough Gaussian correlated *perfectly conducting* surface

Surface Parameters

- RMS-roughness: $\sigma = \lambda$
- Correlation length: $a = 2\lambda$

Normal incidence: $\theta_0 = 0^\circ$

$p \rightarrow p + s$ (both pol. recorded)

$s \rightarrow p + s$

[Simonsen, Maradudin, Leskova 2009]
Scattering from Strongly Rough Surfaces
But the bright red spots are not specular peaks......

$\theta_0 = 0^\circ$; $p$-polarization inc.

- The red “hot-spot” is not specular reflection
- The diffuse scattering dominates completely ($10^4$ stronger)

$$\left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle_{incoh} \gg \left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle_{coh}$$

The intense spot to the left is located in the back-scattering direction. The back scattering enhancement exists for both $p$- and $s$-polarization of the incident light.
Scattering from Strongly Rough Surfaces
But the bright red spots are not specular peaks......

\[ \theta_0 = 20^\circ; \quad p\text{-polarization inc.} \]

- The red “hot-spot” is not specular reflection
- The diffuse scattering dominates completely \((10^4\) stronger\)

\[
\left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle_{incoh} \gg \left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle_{coh}
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Scattering from Strongly Rough Surfaces
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- The intense spot to the left is located in the back-scattering direction
- The back scattering enhancement exists for both $p$- and $s$-polarization of the incident light

$\theta_0 = 20^\circ$; \hspace{0.5cm} s-polarization inc.
Scattering from Strongly Rough Surfaces

But the bright red spots are not specular peaks......

\[ \theta_0 = 20^\circ; \quad s\text{-polarization inc.} \]

- The red “hot-spot” is not specular reflection
- The diffuse scattering dominates completely \(10^4\) stronger

\[ \left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle_{\text{incoh}} \gg \left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle_{\text{coh}} \]

- The intense spot to the left is located in the back-scattering direction
- The back scattering enhancement exists for both \(p\)- and \(s\)-polarization of the incident light

Enhanced Back-Scattering Peaks Exist for both \(p\) and \(s\)-polarization, but what causes the phenomena?

This phenomena was first predicted based on perturbation theory in 1985.
Scattering from Strongly Rough Surfaces
A Fuller Picture: A Comparison

\[ \theta_0 = 0^\circ \]

\[ \theta_0 = 20^\circ \text{ (from the left)} \]

\[ \theta_0 = 40^\circ \]
Enhanced backscattering is due to constructive interference between paths being scattered multiple times by the grooves in the roughness.

- In the presence of coherence (no phase difference) the intensity becomes
  \[ I = |A + B|^2 = |A|^2 + A^* B + AB^* + |B|^2 = 4|A|^2 \quad (A = B) \]
- When coherence is lost
  \[ I = |A|^2 + |B|^2 = 2|A|^2 \quad (A \simeq B) \]

In absence of single scattering the Enhanced Back-Scattering Peaks should be twice of its background (but single scattering will normally also contribute).
Rough Surface Scattering Quiz
Where is the $p$- and $s$-polarized light scattered?

$\theta_0 = 0^\circ$

$p \rightarrow p + s$

$\theta_0 = 20^\circ$

$p \rightarrow p + s$
Rough Surface Scattering Quiz
Where is the $p$- and $s$-polarized light scattered?

$\theta_0 = 0^\circ$

$p \rightarrow p + s$

$p \rightarrow p$

$p \rightarrow s$

$\theta_0 = 20^\circ$

$p \rightarrow p + s$
Rough Surface Scattering Quiz
Where is the $p$- and $s$-polarized light scattered?

$\theta_0 = 0^\circ$

$\theta_0 = 20^\circ$
Rough Surface Scattering Quiz
Comparison between $p$- and $s$-polarized incident light

$\theta_0 = 20^\circ$

$s \rightarrow p + s$

$s \rightarrow p$

$s \rightarrow s$

$p \rightarrow p + s$

$p \rightarrow p$

$p \rightarrow s$
Scattering from Weakly Rough Surfaces
Can backscattering peaks be observed?

Question
Do we also have enhanced backscattering for weakly rough surfaces?

Challenges:

- **single scattering dominates** for weakly rough surfaces
  - backscattering peaks will rise little over the single scattering background
  - experimental noise will make them (too?) hard to observe

- what is scattered multiple times in order to produce the backscattering peak for weakly rough surfaces?
Scattering from Weakly Rough Surfaces
Can backscattering peaks be observed?

Question
Do we also have enhanced backscattering for weakly rough surfaces?

Numerical example: Gaussian correlated rough silver surface
\( \sigma = 10\text{nm}; \ a = 200\text{nm}; \ \text{and} \ \lambda = 457.9\text{nm} \)

![Graphs showing backscattering for p-polarization and s-polarization with \( \theta_o = 0^\circ \) and \( \theta_o = 10^\circ \).](image)

Are there any backscattering peaks here? (Take a closer look at the curves for p-polarization)
West and O’Donnell realized that single scattering more-or-less completely masked potential backscattering peaks.

Their (creative) solution was
- to experimentally implemented a power spectrum where single scattering was forbidden over the angular interval of interest.
- the power spectrum they suggested is called a rectangular (West-O’Donnell) power spectrum.
West and O’Donnell realized that single scattering more-or-less completely masked potential backscattering peaks.

Their (creative) solution was
- to experimentally implemented a power spectrum where single scattering was forbidden over the angular interval of interest
- the power spectrum they suggested is called a rectangular (West-O’Donnell) power spectrum.

They reasoned as follows:

- Single scattering contribution
  
  \( (k = \frac{\omega}{c} \sin \theta_0; \ q = \frac{\omega}{c} \sin \theta_s) \)

  \[ \langle \frac{\partial R_v}{\partial \theta_s} \rangle \propto g(|q - k|) \]

- Single scattering forbidden for
  
  \( (k, q) \) where

  \[ g(|q - k|) = 0 \]

When e.g. \( k = 0 \), then \( q = \frac{\omega}{c} \sin \theta_s \) and single scattering is only allowed for

\[ |\theta_s| \geq \theta_s^- = \sin^{-1}(k_-/(w/c)) \]
Scattering from Weakly Rough Surfaces
A designed power spectrum

Numerical example: Rectangular power spectrum (Movie: Mean DRC vs $\theta_0$)

$\sigma = 10\text{nm}; \lambda = 457.9\text{nm}; k_- = 0.782\omega/c$ and $k_+ = 1.366\omega/c$

For normal incidence ($k = 0$) one finds $\theta_s^- = \sin^{-1}(q_-/(\omega/c)) = 51.4^\circ$
Scattering from Weakly Rough Surfaces
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Summary: Backscattering enhancements for weakly rough surfaces
Backscattering peaks do exist also for weakly rough surfaces, but only in $p$-polarization!
Scattering from Weakly Rough Surfaces
Why are backscattering only observed for $p$-polarization?

- For strongly rough surfaces we had enhanced backscattering peaks for both $p$- and $s$-polarization, not only in $p$-polarization.
- Different mechanisms must therefore give rise to them for strongly and weakly rough surfaces.

Question
What is origin of the enhanced backscattering for weakly rough surfaces?
Scattering from Weakly Rough Surfaces

Why are backscattering only observed for \( p \)-polarization?

- For strongly rough surfaces we had enhanced backscattering peaks for both \( p \)- and \( s \)-polarization, not only in \( p \)-polarization.
- Different mechanisms must therefore give rise to them for strongly and weakly rough surfaces.

**Question**

What is origin of the enhanced backscattering for weakly rough surfaces?

**Answer**

Weakly rough surfaces showing the enhanced backscattering phenomenon, support some kinds of *surface waves*. For instance for a metal surface such waves are *Surface Plasmon Polaritons*.
Constructive interference between (time-revered) surface wave paths

Surface waves are characterized by decaying fields perpendicular to the interface (both directions)

Power spectrum

Angular distribution
Scattering from Weakly Rough Surfaces
Surface Plasmon Polaritons (SPP)

**Polariton**: An elementary electromagnetic wave that can couple to one of the elementary excitations of a condensed medium (plasmons, phonons, magnons)

**Surface Plasmon Polariton (SPP)**
A plasmon polariton where the associated electromagnetic field is confined to the surface separating the two dielectric media

**Ex**: planar 1d metal surface to vacuum \((Im\varepsilon(\omega) = 0)\)

\[
\Phi^\pm(x_1, x_3|\omega) = A^\pm e^{ikx_1} e^{i\beta^\pm(\omega)x_3}
\]

\[
\beta^\pm(\omega) = \sqrt{k^2 - \varepsilon^\pm(\omega)} \geq 0
\]

The boundary conditions at \(x_3 = 0\) give

\[
A^+ = A^- \equiv A
\]

\[
\left[ \frac{\beta^+(\omega)}{k^+_v(\omega)} + \frac{\beta^-(\omega)}{k^-_v(\omega)} \right] A = 0
\]

- Along \(x_1\) : wave-like
- Along \(x_3\) : decaying

- **S-pol.** : No SPPs exist
- **P-pol.** : SPPs can exist only if \(\varepsilon^+\varepsilon^- < 0\) (different signs)
Scattering from Weakly Rough Surfaces
The dispersion relation for Surface Plasmon Polaritons

The dispersion relation is

\[ k_{spp}(\omega) = \sqrt{\frac{\varepsilon_+ (\omega) \varepsilon_- (\omega)}{\varepsilon_+ (\omega) + \varepsilon_- (\omega)}} \frac{\omega}{c} \]

Free electron metal \((\varepsilon_{\infty} = 1)\)

- Light incident on a flat surface cannot excite SPPs
- Light-line in vacuum: \(\omega = kc\)
- Surface plasmons: \(\omega_{sp} = \omega_p/\sqrt{2}\).

Vacuum-silver @ \(\lambda = 457.9\)nm:

\[ \operatorname{Re} k_{spp}(\omega) = 1.074 \frac{\omega}{c} \]

\[ \omega_{spp}(k) = \begin{cases} ck, & k \to 0, \quad \text{(photon-like)} \\ \frac{\omega_p}{\sqrt{2}}, & k \to \infty, \quad \text{(plasmon-like)} \end{cases} \]
Experimental verification for strongly rough Al surfaces; \( \sigma = 1 - 2\mu m; \)

\( a = 1.8\mu m, \) s-polarization, \( \theta_0 = 0^\circ \) (left) and \( \theta_0 = -20^\circ \) (right)

Fig. 1. Polarized (circles) and depolarized (crosses) far-field mean intensity as a function of angle from a gaussian diffuser in normal incidence (\( a \approx 1.8\mu m, \sigma_h \approx 1-2\mu m, \lambda = 0.633\mu m \)). The plane of the scan is perpendicular to the incident electric field vector.

Fig. 2. Polarized (circles) and depolarized (crosses) mean intensity from the same diffuser and conditions as fig. 1, but the diffuser has an in-plane tilt of 20° with respect to the source. Angles plotted are with respect to the normal to the mean surface.
Experimental status of enhanced backscattering
Weakly rough surfaces

Satellite Peaks
What are they?

Dielectric film geometry

- $d$: mean thickness of the film
- the film supports guided-modes
- $d/\lambda$ is important for this phenomenon

$\theta_0 = 0^\circ (\theta_\pm = \pm 17.7^\circ)$

$\theta_0 = 5^\circ (\theta_\pm = 12.1^\circ, -23.1^\circ)$

(s-pol.; $\lambda = 0.6328\,\mu m$, $\delta = 30\,nm$, $k_- = 0.82\,\omega/c$, $k_+ = 1.97\,\omega/c$; $d = 500\,nm$)

The structure supports $N$ guided waves of wave-numbers: $q_1(\omega), \ldots, q_N(\omega)$.

Consider the paths $(ABCD)_m$ and the time-reversed partner $(\bar{A}C\bar{B}\bar{D})_n$.

They have phase difference:

$$\Delta \phi_{nm} = r_{BC} \cdot (k_0 + k_s) + |r_{BC}| [q_n(\omega) - q_m(\omega)]$$
Satellite Peaks
The origin of the phenomena

- The structure supports $N$ guided waves of wave-numbers: $q_1(\omega), \ldots, q_N(\omega)$.
- Consider the paths $(ABCD)_m$ and the time-reversed partner $(\bar{A}CBD\bar{D})_n$
- They have phase difference:

$$
\Delta \phi_{nm} = r_{BC} \cdot (k_0 + k_s) + |r_{BC}| [q_n(\omega) - q_m(\omega)]
$$

We have coherence when $\Delta \phi_{nm} = 0$, i.e. when

$$
\sin \theta_s = -\sin \theta_0 \pm \frac{1}{\sqrt{\varepsilon_0(\omega)}} \frac{c}{\omega} [q_n(\omega) - q_m(\omega)].
$$

- $q_n = q_m \Rightarrow k_s = -k_o$ : Enhanced Backscattering
- $q_n \neq q_m \Rightarrow k_s \neq -k_o$ : Satellite Peaks
We now return to the single rough interface problem, but now with a double rectangular power spectrum.

\[
\begin{align*}
  k_+^{(1)} &= 0.782 \omega / c \\
  k_-^{(1)} &= 1.366 \omega / c \\
  k_-^{(2)} &= 2.048 \omega / c \\
  k_+^{(2)} &= 2.248 \omega / c
\end{align*}
\]

- The heights of the two rectangles are \( \gamma_1 \) and \( \gamma_2 \).
- They are the coupling constants for the process \( q \rightarrow k \) with \( \pm k \in [k_-^{(1)}, k_+^{(1)}] \) and \( \pm k \in [k_-^{(2)}, k_+^{(2)}] \), respectively.
- Movie: Mean DRC vs. \( \theta_0 \) for \( \gamma_1 = \gamma_2 \)

Numerical results: $\theta_0 = 10^\circ$, $p$-polarization, $\sigma = 10\text{nm}$ (Movie: Mean DRC vs $\gamma_2$)

For some values of $\gamma_2/\gamma_1$ we have also a peak at the forward direction $\theta_s = \theta_0$ in addition to that at $\theta_s = -\theta_0$ but what is causing this behavior....?
Forward-Scattering Peaks
Another Coherent Effect

Numerical results: $\theta_0 = 10^\circ$, $p$-polarization, $\sigma = 10$nm (Movie: Mean DRC vs $\gamma_2$)

For some values of $\gamma_2/\gamma_1$ we have also a peak at the forward direction $\theta_s = \theta_0$ in addition to that at $\theta_s = -\theta_0$

But what is causing this behavior....?
Forward-Scattering Peaks
What are their origin?

Relevant paths:

### Phases for path A and B

$$\phi_A = kx_1 - qx_4 + \beta_{1-4}$$

$$\phi_B = kx_1' - qx_4' + \beta_{1'-4'}$$

$$\Delta \phi_{BA} = \phi_B - \phi_A$$

$$= k(x_1' - x_1) - q(x_4' - x_4) + \beta_{1'-4'} - \beta_{1-4}$$

### Path A and B:

$k \rightarrow -k_{spp} \rightarrow k_{spp} \rightarrow -k_{spp} \rightarrow q$

$$k = \frac{\omega}{c} \sin \theta_0$$

$$q = \frac{\omega}{c} \sin \theta_s$$
Forward-Scattering Peaks
What are their origin?

Phases for path A and B

\[
\phi_A = kx_1 - qx_4 + \beta_{1-4}
\]
\[
\phi_B = kx_1' - qx_4' + \beta_{1'-4'}
\]
\[
\Delta \phi_{BA} = \phi_B - \phi_A
\]
\[
= k(x_1' - x_1) - q(x_4' - x_4) + \beta_{1'-4'} - \beta_{1-4}
\]

By assuming \(x_4' - x_1' = x_4 - x_1\) it follows that

\[
\Delta \phi_{BA} = (k - q)(x_1' - x_1),
\]

and requiring phase-coherence gives

\[
\Delta \phi_{BA} = 0 \quad \Rightarrow \quad q = k
\]

or

\[
\theta_s = \theta_0
\]
Forward-Scattering Peaks

What are their origin?

Relevant paths:

Phases for path A and C (if allowed)

\[ \Delta \phi_{CA} = \phi_C - \phi_A \]

\[ = (q - k)(x_1 - x_1') + (q + k)\Delta x \]

where \( \Delta x = x_4 - x_1 = x_4' - x_1' \).

Only for \( \Delta x = 0 \) does one get the same phase condition as for \( \Delta \phi_{BA} \).

\[ k = \frac{\omega}{c} \sin \theta_0 \]

\[ q = \frac{\omega}{c} \sin \theta_s \]
Forward-Scattering Peaks
What are their origin?

Relevant paths:

Phases for path A and C (if allowed)

\[ \Delta \phi_{CA} = \phi_C - \phi_A \]
\[ = (q - k)(x_1 - x_1') + (q + k)\Delta x \]

where \( \Delta x = x_4 - x_1 = x_{4'} - x_{1'} \).

Only for \( \Delta x = 0 \) does one get the same phase condition as for \( \Delta \phi_{BA} \).

\[ k = \frac{\omega}{c} \sin \theta_0 \]
\[ q = \frac{\omega}{c} \sin \theta_s \]

Take home message

The counter propagation \( \pm k_{spp} \rightarrow \mp k_{spp} \) is essential for the forward scattering peak phenomenon.
Forward-Scattering Peaks

Amount of coherent/incoherent light vs. $\gamma_2 / \gamma_1$

Fraction of incident power being reflected

$$\mathcal{U}_\nu = \int_{-\pi/2}^{\pi/2} d\theta_s \left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle$$
Forward-Scattering Peaks
Amount of coherent/incoherent light vs. $\gamma_2/\gamma_1$

Fraction of incident power being reflected

$$U = \int_{-\pi/2}^{\pi/2} d\theta_s \left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle$$

Take home message

By changing the correlations along the surface the scattering properties can be changed dramatically.
Angular Intensity Correlation Functions

The definition

\[ C(q, k|q', k') = \langle I(q|k) I(q'|k') \rangle - \langle I(q|k) \rangle \langle I(q'|k') \rangle \]

where the intensity \( I(q|k) \) is defined as (\( k = \omega c \sin \theta_0 \) etc)

\[ I(q|k) = \frac{1}{L} \left| S(q|k) \right|^2 = \frac{1}{L} \alpha_0(q, \omega) \alpha_0(k, \omega) \left| R(q|k) \right|^2 \]
Angular Intensity Correlation Functions

The definition

Definition:

\[ C(q, k | q', k') = \langle I(q | k) I(q' | k') \rangle - \langle I(q | k) \rangle \langle I(q' | k') \rangle \]

where the intensity \( I(q | k) \) is defined as \( k = \frac{\omega}{c} \sin \theta_0 \) etc

\[ I(q | k) = \frac{1}{L_1} \left( \frac{\omega}{c} \right) |S(q | k)|^2 = \frac{1}{L_1} \left( \frac{\omega}{c} \right) \frac{\alpha_0(q, \omega)}{\alpha_0(k, \omega)} |R(q | k)|^2 \]
Angular Intensity Correlation Functions

Different types

One can separate the angular intensity correlation functions into several terms:

\[ C(q, k|q', k') = C^{(1)} + C^{(1.5)} + C^{(2)} + C^{(3)} \]

where

- \( C^{(1)} \) – Short range correlation functions
- \( C^{(1.5)} \) – Intermediate range correlation functions
- \( C^{(2)} \) – Long range correlation functions
- \( C^{(3)} \) – Infinite range correlation functions

The \( C^{(1.5)} \) is unique to rough surface scattering (and has e.g. no analogy in random bulk systems)!
Angular Intensity Correlation Functions
Short Range Correlations

\[ C^{(1)}(q, k|q', k') = \frac{\varepsilon_0 \omega^2}{L_1^2 c^2} \left| \langle \delta S(q|k)\delta S^*(q'|k') \rangle \right|^2, \quad \delta S = S - \langle S \rangle \]

\[ = \frac{2\pi \delta(q - k - q' + k')}{L_1} C^{(1)}_0(q, k|q', q' - q + k) \]

\[ C^{(10)}(q, k|q', k') = \frac{2\pi \delta(q - k + q' - k')}{L_1} C^{(10)}_0(q, k|q', q' + q + k). \]

where \( C^{(1)}_0 \) and \( C^{(10)}_0 \) are envelopes independent of \( L_1 \)

They are non-zero only when the momentum transfer \( (\Delta_{qk} = q - k) \) satisfies

- \( C^{(1)} \): \( \Delta_{qk} = \Delta_{q'k'} \)
  - Memory Effect: \( k = k', q = q' \)
  - Reciprocal Memory Effect \( (S(q|k) = S(-k|-q)) : k = -q', q = -k' \)

- \( C^{(10)} \): \( \Delta_{qk} = -\Delta_{q'k'} \)

**Note**: \( C^{(10)} \) is unique to surface scattering!
Angular Intensity Correlation Functions

Simulations and Experiments

small Simulations (p-pol.):

- Gaussian silver surface: $\delta = 5nm$, $a = 100nm$, $\theta_0 = 20^\circ$, $\theta_s = -10^\circ$

- Memory effect
  - $\theta'_s = \theta_s = -10^\circ$ ($q' = q$)

- Reciprocal Memory Effect:
  - $\theta'_s = -\theta_0 = -20^\circ$ ($q' = -k$)

Experiments (p-pol):

- West-O'Donnell gold surface: $k_- = 0.83\omega/c$, $k_+ = 1.30\omega/c$,
  $\Delta_- = 0.04\omega/c$, $\sigma = 15.5nm$, $\theta_0 = 20^\circ$, $\theta_s = -10^\circ$

- (This corresponds to: $-C^{(1)}; \cdots -C^{(10)}$)

Ref.: West and O'Donnell, PRB 59, 2393 (1999)
Angular Intensity Correlation Functions

Physical meaning of short range correlation functions

\[ C^{(1)} : \Delta q_k = \Delta q'_k' \]

If \( k \) is changed to \( k' = k + \Delta k \) the entire speckle pattern changes such that a feature originally at \( q \) moves to \( q' = q + \Delta k \)

\[ C^{(10)} : \Delta q_k = -\Delta q'_k' \]

If \( k \) is changed to \( k' = k + \Delta k \) the entire speckle pattern changes such that a feature originally at \( q = k - \Delta q \) moves to \( q' = k + \Delta q \), (symmetry with respect to the specular direction)

These effects can be seen directly in the specular patterns!
Angular Intensity Correlation Functions
Long and infinite range correlations $C^{(N)}$

$$C^{(N)}(q, k|q', k') = \frac{\varepsilon_0 \omega^2}{L_1^2 c^2} \{ \delta S(q|k) \delta S^*(q|k) \delta S(q'|k') \delta S^*(q'|k') \} \propto \frac{1}{L_1}$$

It will be hard to observe experimentally!

$$C^{(N)}(q, k|q', k') = C^{(1.5)}(q, k|q', k') + C^{(2)}(q, k|q', k') + C^{(3)}(q, k|q', k')$$

- $C^{(N)}$ has a complex peak structure
- $C^{(1.5)}$ is unique to surface scattering

[p-pol : $\theta_o = 20^o$, $\theta_s = -10^o$, $\delta = 5nm$ and $a = 100nm$]
For weakly rough surfaces we saw:

- No peaks in the $C^{(1)}$ correlation function for s-polarization (No SPPs)
- $C^{(1)}$ and $C^{(10)}$ are of the same magnitude

No surprise that multiple scattering of volume waves is causing peaks in $C^{(1)}$ for strongly rough surfaces also in s-polarization.

Note: $C^{(10)}$ vanishes for strongly rough surfaces.
Angular Intensity Correlation Functions

Statistics of the Field Amplitudes

Measurements of the $C(q, k|q', k')$ can provide information about the amplitude of the scattered field (Leskova et al. 2000)

- Only $C^{(1)}$ is observed
  - $R(q|k)$ is a circular complex Gaussian random process defined as $(A = A_1 + iA_2, B = B_1 + iB_2)$
    $$\langle A_1 B_1 \rangle = \langle A_2 B_2 \rangle \quad \langle A_1 B_2 \rangle = -\langle A_2 B_1 \rangle$$
  - this implies that
    $$\langle AB \rangle = 0 \quad \Rightarrow \quad C^{(10)} \propto |\langle \delta S(q|k) \delta S(q'|k') \rangle|^2 = 0$$

- Only $C^{(1)}$ and $C^{(10)}$ are observed
  - $R(q|k)$ is a complex Gaussian random process

- $C^{(1)}$, $C^{(10)}$ and $C^{(N)}$ are observed
  - $R(q|k)$ is a non-Gaussian random process

Summary

Angular correlation functions can tell us about the statistics properties of the scattered field.
Conclusions

- Rough surface scattering is rich
- There might be “order in the chaos”
- The height-height correlations are important for the scattering
- The roughness can be used to tune the optical properties

A renewed interest in rough surface scattering has been witnessed during the last years (probably) due to its potential applications.


Thank you for your Attention!