Optics of Surface Disordered Systems

No Disorder — No Fun!

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- Elin S[ø]ndergård (Saint-Gobain, France)
- Damien Vandembroucq (ESPCI,France)

and several students.....

Even Pauli was Challenged by the Surface....

Wolfgang Ernst Pauli
(1900 – 1958)

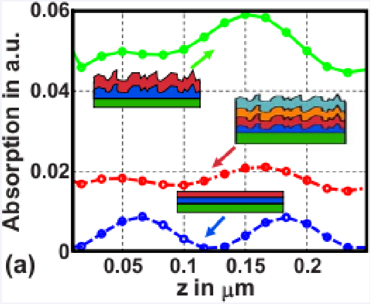
Pauli is quoted for saying:

God made the bulk;
the surface was invented by the devil!

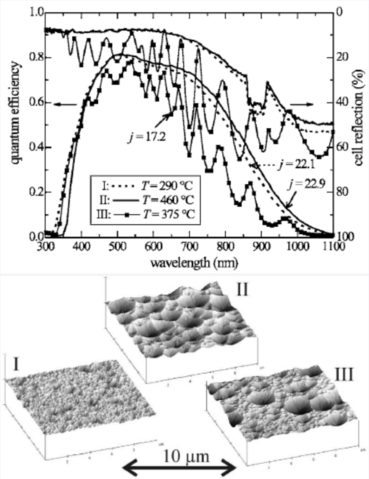
I wonder:

What would Pauli have thought about a *randomly rough surface*....?

However, *roughness* can also be beneficial....



[Appl. Phys. Lett.**94**, 211101 (2009)]



[J. Appl. Phys. **101**, 074903 (2007)]

Roughness increases the efficiency of solar cells

Outline

1. Introduction
2. Theoretical Background
   - Scattering geometry
   - How to characterize randomly rough surfaces
   - Physical observables
3. Physical phenomena and their origin
   - The enhanced backscattering phenomenon
   - The satellite peak phenomenon
   - The forward scattering enhancement
   - Angular intensity correlation functions
4. Conclusions and Outlooks

A talk about coherent effects in surface random systems and their physical origins!

Motivation

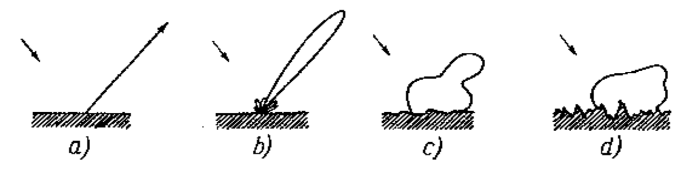
Some history:

- Lord Rayleigh (1877 (?))
- Mandel'shtam (1913)
- Rice (1951)
- M. V. Berry (1979):
  - Diffractal
- McGurn, *et al.* (1985)
  - Multiple Scattering Phenomenon

Why should one care:

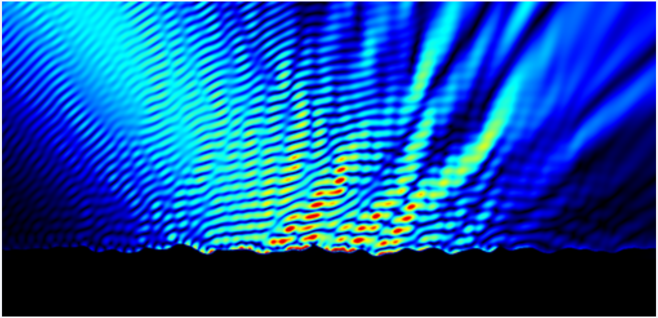
- Scientific interesting problem
  - fundamental interest
  - astrophysics
- Industrial applications
  - electronics
  - energy sector
  - seismic
  - medical sector
- Military applications
  - radar technology

The transition from specular to diffuse scattering:



Rough surface scattering is complex

Computer simulation of light scattered from a rough metal surface



[After Thomas Berg]

The *speckle patterns* are complex and they do depend on parameters like

- surface roughness
- surface correlations
- angle of incidence
- material

Basic electromagnetic theory

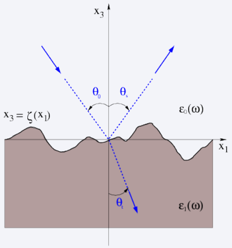
Relevant equations for a one-dimensional geometry

For a one-dimensional scattering geometry one introduce the fundamental field quantity

$$\Phi_\nu(x_1,x_3|\omega) = \begin{cases} H_2(x_1,x_3|\omega), & \nu = p, \\ E_2(x_1,x_3|,\omega), & \nu = s, \end{cases}$$

that should satisfy the Helmholtz equation

$$(\partial^2_{x_1} + \partial^2_{x_3} + \varepsilon\frac{\omega^2}{c^2})\Phi_\nu(x_1,x_3|\omega) = 0.$$



Boundary Conditions:

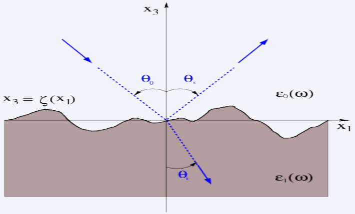
$$\Phi^+_\nu(x_1,x_3|\omega)\big|_{x_3=\zeta(x_1)} = \Phi^-_\nu(x_1,x_3|\omega)\big|_{x_3=\zeta(x_1)}$$

$$\frac{1}{\kappa^+_\nu(\omega)}\,\partial_n\Phi^+_\nu(x_1,x_3|\omega)\big|_{x_3=\zeta(x_1)} = \frac{1}{\kappa^-_\nu(\omega)}\,\partial_n\Phi^-_\nu(x_1,x_3|\omega)\big|_{x_3=\zeta(x_1)}$$

where ∂_n is the normal derivative

$$\partial_n = \mathbf{n}\cdot\nabla = \frac{-\zeta'(x_1)\partial_{x_1} + \partial_{x_3}}{\sqrt{1+(\zeta'(x_1))^2}}, \qquad \kappa^\pm_\nu(\omega) = \begin{cases} \varepsilon_\pm(\omega), & \nu = p \\ \mu_\pm(\omega), & \nu = s \end{cases}$$

Scattering Geometry



θ_0	: angle of incidence
θ_s	: angle of scattering
θ_t	: angle of transmission
$\theta_s = \theta_0$	: specular direction
$\varepsilon_\pm(\omega)$	: dielectric functions
$\zeta(x_1)$	: surface profile function

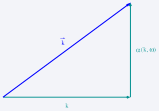
Asymptotic forms of the fields

$$\Phi^+_\nu(x_1, x_3|\omega) = e^{ikx_1 - i\alpha_+(k,\omega)x_3} + \int_{-\infty}^{\infty} \frac{dq}{2\pi} R_\nu(q|k) e^{iqx_1 + i\alpha_+(q,\omega)x_3}, \qquad x_3 > \max(\zeta)$$

$$\Phi^-_\nu(x_1, x_3|\omega) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} T_\nu(p|k) e^{ipx_1 - i\alpha_-(p,\omega)x_3}, \qquad x_3 < \min(\zeta)$$

$$k = \sqrt{\varepsilon_+(\omega)}\,(\omega/c) \sin\theta_0,$$
$$q = \sqrt{\varepsilon_+(\omega)}\,(\omega/c) \sin\theta_s,$$
$$p = \sqrt{\varepsilon_-(\omega)}\,(\omega/c) \sin\theta_t$$

$$\alpha_\pm(k,\omega) = \sqrt{\varepsilon_\pm(\omega)\frac{\omega^2}{c^2} - k^2} = \sqrt{\varepsilon_\pm(\omega)}\,\frac{\omega}{c}\cos\theta_0$$



Physical Observable
The Mean Differential Reflection Coefficient

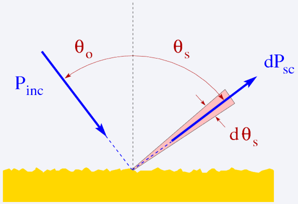
The physical observable we will be interested in is the Mean Differential Reflection Coefficient (mean DRC)

Definition (Mean DRC)

The mean DRC, $\langle \partial R_\nu / \partial \theta_s \rangle$, is the fraction of the power flux incident on the surface that is scattered into an angular interval of width, $d\theta_s$, about the scattering direction θ_s.

The incident/scattered power flow can be obtained from the 3-component of the (complex) Poynting vector, $\mathbf{S} = \mathbf{E} \times \mathbf{H}^*$:

$$P = \int dx_1\, dx_2\, Re \langle S_3 \rangle_t$$



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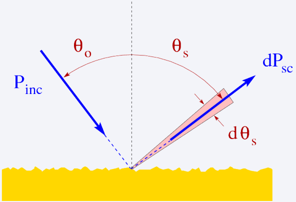

$$\left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle = \left\langle \frac{p_{sc}(\theta_s)}{P_{inc}} \right\rangle = \frac{1}{L_1} \frac{\omega}{2\pi c} \frac{\cos^2 \theta_s}{\cos \theta_0} \left\langle \left| R_\nu(q|k) \right|^2 \right\rangle, \qquad \begin{aligned} k &= \sqrt{\varepsilon_+} \frac{\omega}{c} \sin \theta_0 \\ q &= \sqrt{\varepsilon_+} \frac{\omega}{c} \sin \theta_s \end{aligned}$$

Physical Observable

The Coherent and Incoherent contribution to the mean DRC

- The mean DRC, $\left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle$ is an experimental accessible quantity
- $R_\nu(q|k)$ is the *scattering* (or reflection) *amplitude* for polarization ν
- The main goal is to obtain $R_\nu(q|k)$ (the difficult part)

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A simple rewriting of the expression for the mean DRC:

$$\left\langle |R_\nu(q|k)|^2 \right\rangle = \underbrace{\left\langle |R_\nu(q|k)|^2 \right\rangle - |\langle R_\nu(q|k)\rangle|^2}_{\text{incoherent}} + \underbrace{|\langle R_\nu(q|k)\rangle|^2}_{\text{coherent}}$$

gives that it has two components — the *coherent* (or specuar) and the *incoherent* (or diffuse); (not easily done experimentally)

The *incoherent* and *coherent* contribution of the mean DRC

$$\left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle = \left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle_{incoh} + \left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle_{coh}$$

Statistical properties of the surface roughness

Let $\zeta(x_1)$ denote the *surface profile function*

- height distribution
- height-height correlation function

Normally one assumes that $\zeta(x_1)$ is a single-valued, differentiable function of x_1 that constitutes a stationary zero-mean Gaussian random process so that

$$\begin{aligned} \langle \zeta(x_1) \rangle &= 0 \\ \langle \zeta(x_1)\zeta(x_1') \rangle &= \sigma^2 W(|x_1 - x_1'|), \qquad W(0) = 1 \end{aligned}$$

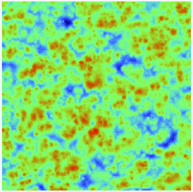
where $\langle \cdot \rangle$ is the ensemble average and $W(x_1)$ is the transverse correlation function.

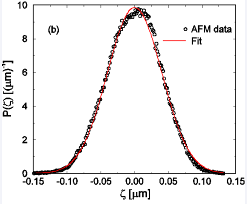
Also useful is the power spectrum of the roughness defined by

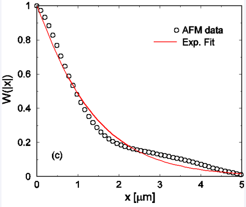
$$g(|k|) = \int_{-\infty}^{\infty} dx_1 \, W(|x_1|) e^{-ikx_1}$$

Surface topographies from real life!

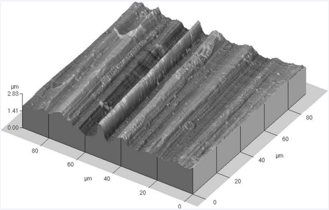
A plastic surface



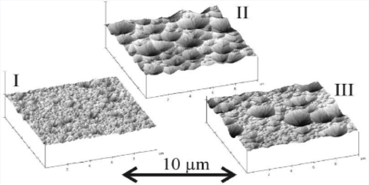




Cold rolled Al surface (self-affine)



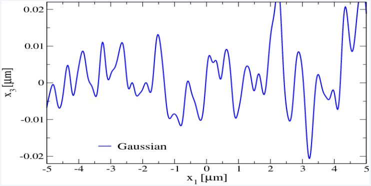
Surfaces for Photovoltaic

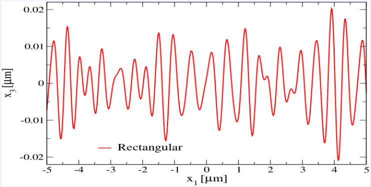


The Power Spectrum

Question : Does the form of $g(|k|)$ really matter much for the scattering?

Two different power spectra : $\sigma = 10$nm; $\lambda = 457.9$nm; p-polarization





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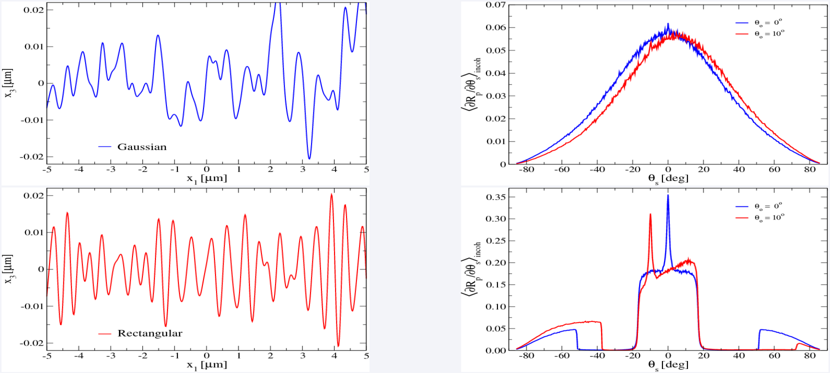

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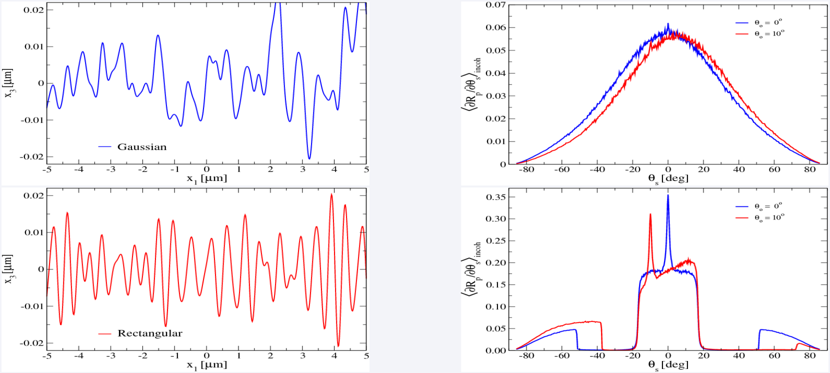
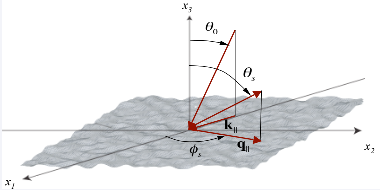


Small amplitude perturbation theory predicts that (to lowest order)

$$\left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle \propto g(|q-k|)$$

Scattering from Strongly Rough Surfaces

Scattering from a rough Gaussian correlated *perfectly conducting* surface

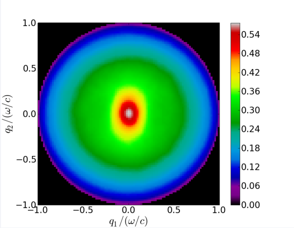


Surface Parameters

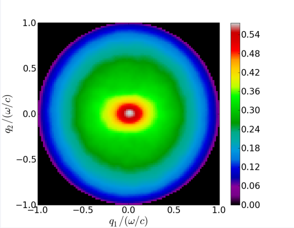
RMS-roughness	$\sigma = \lambda$
Correlation length	$a = 2\lambda$

Normal incidence : $\theta_0 = 0^\circ$

- $p \rightarrow p + s$ (both pol. recorded)



- $s \rightarrow p + s$

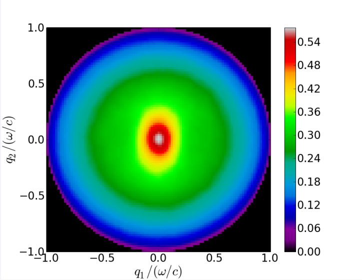


[Simonsen, Maradudin, Leskova 2009]

Scattering from Strongly Rough Surfaces

But the bright red spots are not specular peaks......

$\theta_0 = 0^\circ$; p-polarization inc.



- The red “hot-spot” is not specular reflection
- The diffuse scattering dominates completely (10^4 stronger)

$$\left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle_{incoh} \gg \left\langle \frac{\partial R}{\partial \Omega_s} \right\rangle_{coh}$$

Scattering from Strongly Rough Surfaces

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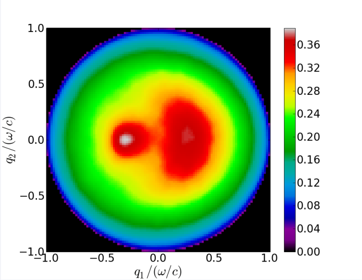

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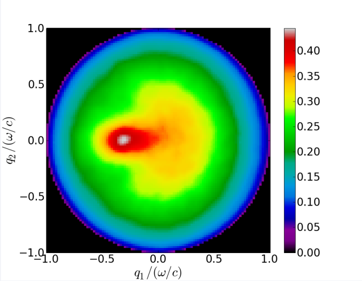

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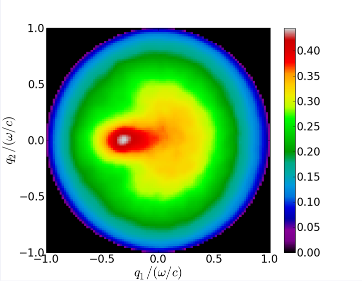

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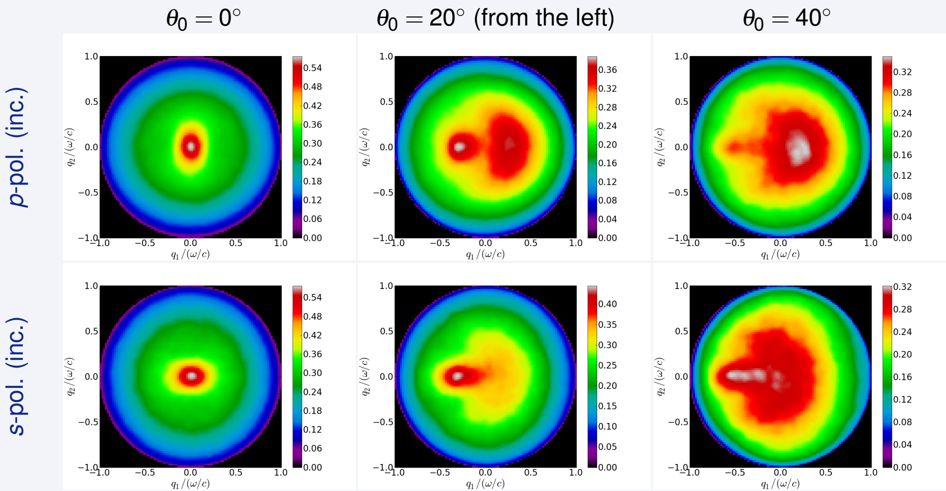
Enhanced Back-Scattering Peaks Exist for both *p* and *s*-polarization, but what causes the phenomena?

This phenomena was first predicted based on perturbation theory in 1985.

Scattering from Strongly Rough Surfaces

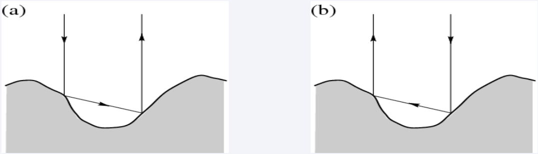
A Fuller Picture: A Comparison

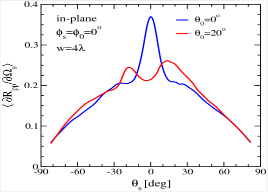


Scattering from Strongly Rough Surfaces

What is giving rise to these peaks in the back-scattering direction?

Enhanced backscattering is due to constructive interference between paths being scattered multiple times by the grooves in the roughness





- In the presence of coherence (no phase difference) the intensity becomes

$$I = |A+B|^2 = |A|^2 + A^*B + AB^* + |B|^2 = 4|A|^2 \quad (A = B)$$

- When coherence is lost

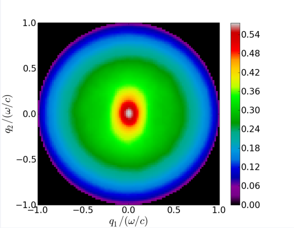
$$I = |A|^2 + |B|^2 = 2|A|^2 \quad (A \simeq B)$$

In absence of single scattering the Enhanced Back-Scattering Peaks should be twice of its background (but single scattering will normally also contribute)

Rough Surface Scattering Quiz

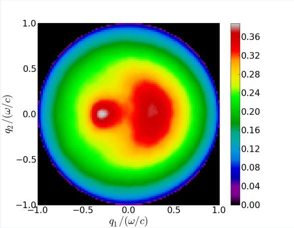
Where is the *p*- and *s*-polarized light scattered?

$\theta_0 = 0^\circ$



$p \to p + s$

$\theta_0 = 20^\circ$



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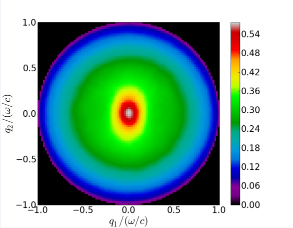
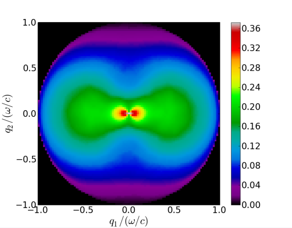
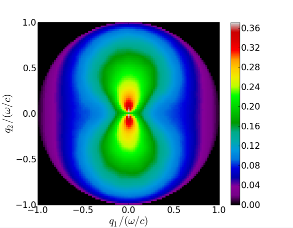


$p \to p + s$

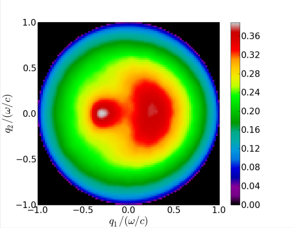


$p \to p$



$p \to s$

$\theta_0 = 20^\circ$



$p \to p + s$

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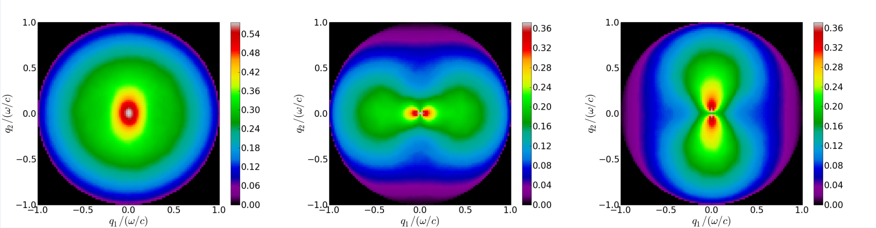
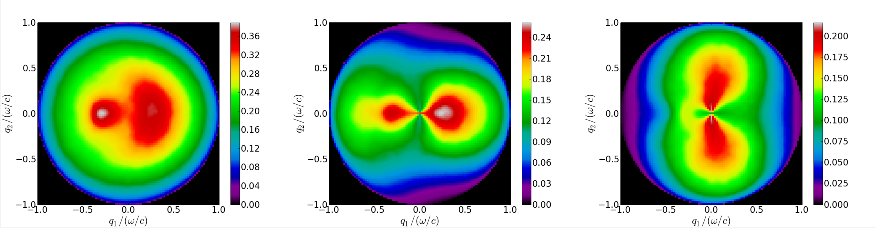


$p \rightarrow p+s$ | $p \rightarrow p$ | $p \rightarrow s$

$\theta_0 = 20^\circ$

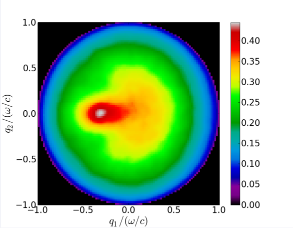


$p \rightarrow p+s$ | $p \rightarrow p$ | $p \rightarrow s$

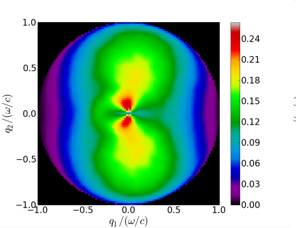
Rough Surface Scattering Quiz

Comparison between *p*- and *s*-polarized incident light

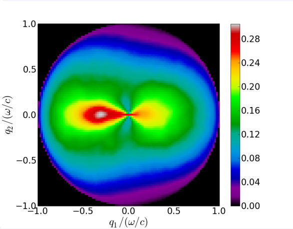
$\theta_0 = 20^\circ$ s-pol



$s \rightarrow p + s$

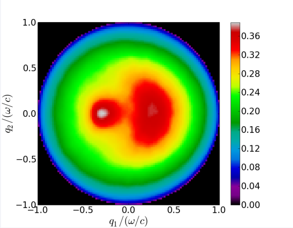


$s \rightarrow p$

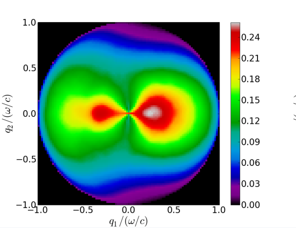


$s \rightarrow s$

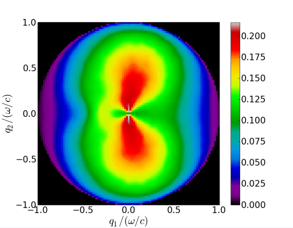
$\theta_0 = 20^\circ$ p-pol



$p \rightarrow p + s$



$p \rightarrow p$



$p \rightarrow s$

Scattering from Weakly Rough Surfaces

Can backscattering peaks be observed?

Question

Do we also have enhanced backscattering for weakly rough surfaces?

Challenges:

- single scattering dominates for weakly rough surfaces
  - backscattering peaks will rise little over the single scattering background
  - experimental noise will make them (too?) hard to observe
- what is scattered multiple times in order to produce the backscattering peak for weakly rough surfaces?

Scattering from Weakly Rough Surfaces

Can backscattering peaks be observed?

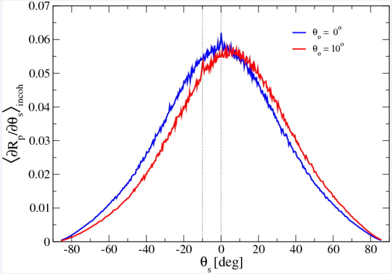
Question

Do we also have enhanced backscattering for weakly rough surfaces?

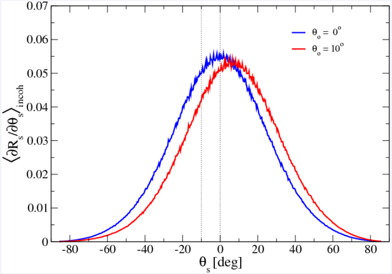
Numerical example : Gaussian correlated rough silver surface

$\sigma = 10\text{nm};\ a = 200\text{nm};\ \text{and}\ \lambda = 457.9\text{nm}$

p-polarization



s-polarization



Are there any backscattering peaks here? (Take a closer look at the curves for *p*-polarization)

Scattering from Weakly Rough Surfaces

A designed power spectrum

- West and O'Donnell realized that single scattering more-or-less completely masked potential backscattering peaks
- Their (creative) solution was
  - to experimentally implemented a power spectrum where single scattering was forbidden over the angular interval of interest
  - the power spectrum they suggested is called a rectangular (West-O'Donnell) power spectrum

Scattering from Weakly Rough Surfaces

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They reasoned as follows:

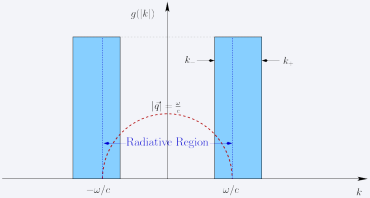
- single scattering contribution $(k = \frac{\omega}{c}\sin\theta_0;\ q = \frac{\omega}{c}\sin\theta_s)$

$$\left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle \propto g(|q-k|)$$

- Single scattering forbidden for (k, q) where

$$g(|q-k|) = 0$$

Suggested power spectrum:



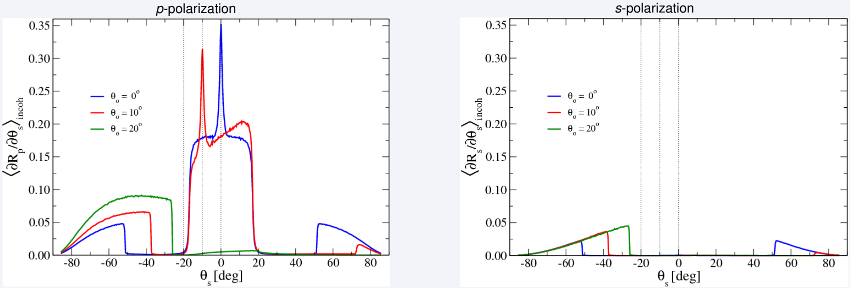
When *e.g.* $k = 0$, then $q = \frac{\omega}{c}\sin\theta_s$ and single scattering is only allowed for $|\theta_s| \geq \theta_s^- = \sin^{-1}(k_-/(w/c))$

Scattering from Weakly Rough Surfaces

A designed power spectrum

Numerical example : Rectangular power spectrum (**Movie**: Mean DRC vs θ_0)

$\sigma = 10\text{nm}$; $\lambda = 457.9\text{nm}$; $k_- = 0.782\omega/c$ and $k_+ = 1.366\omega/c$



For normal incidence ($k = 0$) one finds $\theta_s^- = sin^{-1}(q_-/(\omega/c)) = 51.4^\circ$

Scattering from Weakly Rough Surfaces

A designed power spectrum

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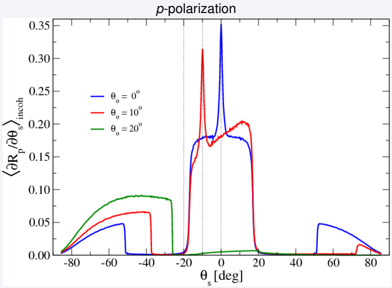
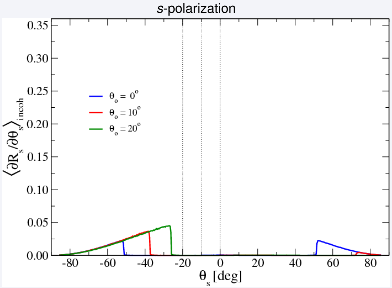
$$\sigma = 10\text{nm};\ \lambda = 457.9\text{nm};\ k_- = 0.782\omega/c \text{ and } k_+ = 1.366\omega/c$$





For normal incidence ($k = 0$) one finds $\theta_s^- = sin^{-1}(q_-/(\omega/c)) = 51.4^\circ$

Summary : Backscattering enhancements for weakly rough surfaces

Backscattering peaks do exist also for weakly rough surfaces, but only in *p*-polarization!

Scattering from Weakly Rough Surfaces

Why are backscattering only observed for *p*-polarization?

- For strongly rough surfaces we had enhanced backscattering peaks for both *p*- and *s*-polarization, not only in *p*-polarization.
- Different mechanisms must therefore give rise to them for strongly and weakly rough surfaces

Question

What is origin of the enhanced backscattering for weakly rough surfaces

Scattering from Weakly Rough Surfaces

Why are backscattering only observed for *p*-polarization?

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Question

What is origin of the enhanced backscattering for weakly rough surfaces

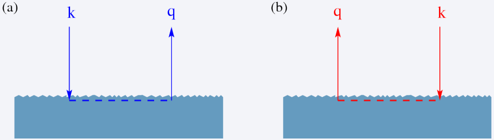
Answer

Weakly rough surfaces showing the enhanced backscattering phenomenon, support some kinds of *surface waves*. For instance for a metal surface such waves are Surface Plasmon Polaritons

Scattering from Weakly Rough Surfaces

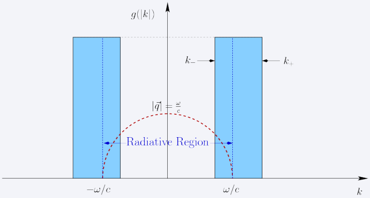
The scattering mechanism for a weakly rough metallic surface

Constructive interference between (time-revered) surface wave paths

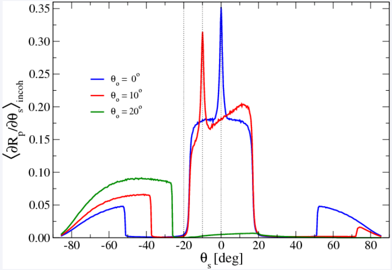


Surface waves are charactered by decaying fields perpendicular to the interface (both directions)

Power spectrum



Angular distribution



Scattering from Weakly Rough Surfaces
Surface Plasmon Polaritons (SPP)

Polariton : An elementary electromagnetic wave that can couple to one of the elementary excitations of a condensed medium (plasmons, phonons, magnons)

Surface Plasmon Polariton (SPP)

A plasmon polariton where the associated electromagnetic field is confined to the surface separating the two dielectric media

Ex : planar 1d metal surface to vacuum ($Im\,\varepsilon(\omega) = 0$)

$$\Phi_\nu^\pm(x_1, x_3|\omega) = A_\nu^\pm\, e^{ikx_1}\, e^{\mp\beta_\pm(\omega)x_3}$$

$$\beta_\pm(\omega) = \sqrt{k^2 - \varepsilon_\pm \frac{\omega^2}{c^2}} \geq 0$$

The boundary conditions at $x_3 = 0$ give

$$A_\nu^+ = A_\nu^- \equiv A_\nu$$

$$\left[\frac{\beta_+(\omega)}{\kappa_\nu^+(\omega)} + \frac{\beta_-(\omega)}{\kappa_\nu^-(\omega)}\right] A_\nu = 0$$

- Along x_1 : wave-like
- Along x_3 : decaying

- S-pol. :No SPPs exist
- P-pol. : SPPs can exist **only** if $\varepsilon_+\varepsilon_- < 0$ (different signs)

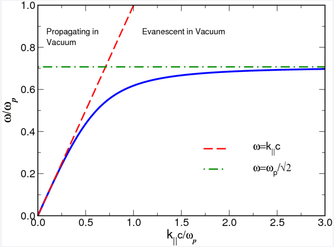
Scattering from Weakly Rough Surfaces

The dispersion relation for Surface Plasmon Polaritons

The dispersion relation is

$$k_{spp}(\omega) = \sqrt{\frac{\varepsilon_+(\omega)\varepsilon_-(\omega)}{\varepsilon_+(\omega)+\varepsilon_-(\omega)}}\,\frac{\omega}{c}$$

Free electron metal ($\varepsilon_\infty = 1$)



- Light incident on a *flat* surface cannot excite SPPs
- Light-line in vacuum : $\omega = kc$
- Surface plasmons : $\omega_{sp} = \omega_p/\sqrt{2}$.
- Vacuum-silver @ $\lambda = 457.9$nm:

$$\mathrm{Re}\, k_{spp}(\omega) = 1.074\,\frac{\omega}{c}$$

$$\omega_{spp}(k) = \begin{cases} ck, & k \to 0, \quad \text{(photon-like)} \\ \dfrac{\omega_p}{\sqrt{2}}, & k \to \infty, \quad \text{(plasmon-like)} \end{cases}$$

Experimental status of enhanced backscattering
Strongly rough surfaces

Méndez and O'Donnell, Opt. Commun. **61**, 91 (1987)

Experimental verification for strongly rough Al surfaces; $\sigma = 1-2\mu$m; $a = 1.8\mu$m, s-polarization, $\theta_0 = 0^\circ$ (left)and $\theta_0 = -20^\circ$ (right)

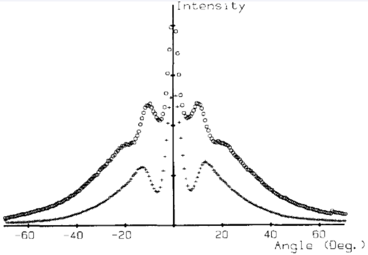


Fig. 1. Polarized (circles) and depolarized (crosses) far-field mean intensity as a function of angle from a gaussian diffuser in normal incidence ($a \approx 1.8$ μm, $\sigma_h \approx 1$–2 μm, $\lambda = 0.633$ μm). The plane of the scan is perpendicular to the incident electric field vector.

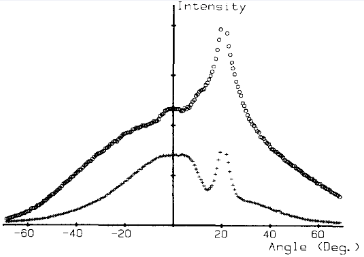


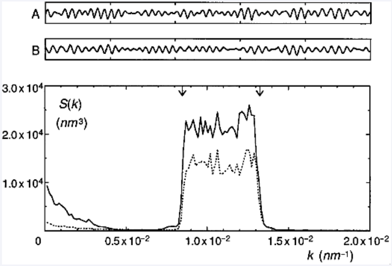
Fig. 2. Polarized (circles) and depolarized (crosses) mean intensity from the same diffuser and conditions as fig. 1, but the diffuser has an in-plane tilt of 20° with respect to the source. Angles plotted are with respect to the normal to the mean surface.

Experimental status of enhanced backscattering

Weakly rough surfaces

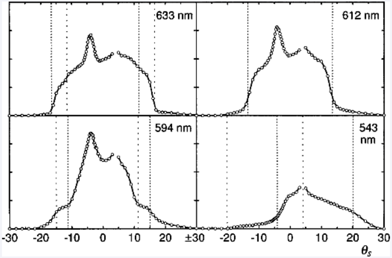
West and O'Donnell, J. Opt. Soc. Am. A. **12**, 390 (1995)

Experimental power spectrum



Scattering distribution for $\theta_0 = 4^\circ$

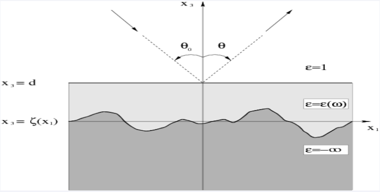
Vertical dashed lines are the limit of the out-coupling of SPP's



Satellite Peaks

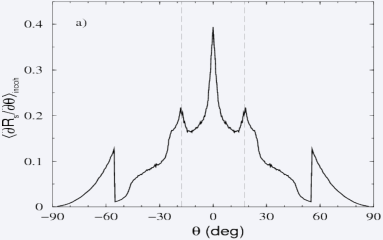
What are they?

Dielectric film geometry

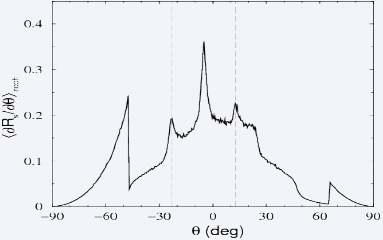


- d: mean thickness of the film
- the film supports guided-modes
- d/λ is important for this phenomenon

$\theta_0 = 0^o$ ($\theta_\pm = \pm 17.7^o$)



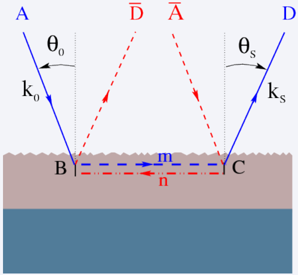
$\theta_0 = 5^o$ ($\theta_\pm = 12.1^o, -23.1^o$)



(s-pol.; $\lambda = 0.6328\mu m$, $\delta = 30nm$, $k_- = 0.82\,\omega/c$, $k_+ = 1.97\,\omega/c$; $d = 500nm$)

Reference : Freilikher *et al.*, Phys. Lett. A **193**, 467 (1994); Simonsen and Maradudin, Opt. Commun. **162**, 99 (1999).

Satellite Peaks
The origin of the phenomena

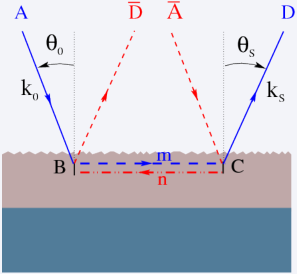


- The structure supports N guided waves of wave-numbers: $q_1(\omega), \ldots, q_N(\omega)$.
- Consider the paths $(ABCD)_m$ and the time-reversed partner $(\bar{A}CB\bar{D})_n$
- They have phase difference:

$$\Delta\phi_{nm} = \mathbf{r}_{BC} \cdot (\mathbf{k}_0 + \mathbf{k}_s) + |\mathbf{r}_{BC}| \left[q_n(\omega) - q_m(\omega) \right]$$

Satellite Peaks

The origin of the phenomena



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- They have phase difference:

$$\Delta\phi_{nm} = \mathbf{r}_{BC}\cdot(\mathbf{k}_0 + \mathbf{k}_s) + |\mathbf{r}_{BC}|\,[q_n(\omega) - q_m(\omega)]$$

We have coherence when $\Delta\phi_{nm} = 0$, *i.e.* when

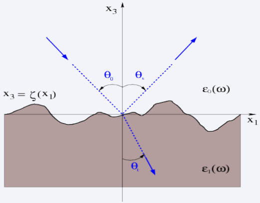
$$\sin\theta_s = -\sin\theta_0 \pm \frac{1}{\sqrt{\varepsilon_0(\omega)}}\frac{c}{\omega}\,[q_n(\omega) - q_m(\omega)]\,.$$

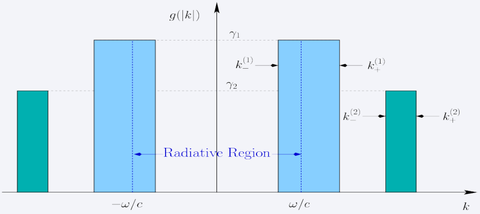
- $q_n = q_m \Rightarrow \mathbf{k}_s = -\mathbf{k}_o$: Enhanced Backscattering
- $q_n \neq q_m \Rightarrow \mathbf{k}_s \neq -\mathbf{k}_o$: Satellite Peaks

Forward-Scattering Peaks

Geometry and Power Spectrum

We now return to the single rough interface problem, but now with a double rectangular power spectrum





$$k_-^{(1)} = 0.782\omega/c \qquad k_+^{(1)} = 1.366\,\omega/c$$

$$k_-^{(2)} = 2.048\,\omega/c \qquad k_+^{(2)} = 2.248\,\omega/c$$

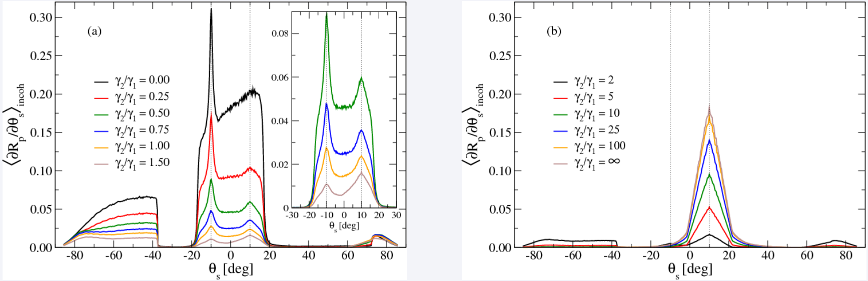
- The heights of the two rectangles are γ_1 and γ_2.
- They are the coupling constants for the process $q \to k$ with $\pm k \in [k_-^{(1)}, k_+^{(1)}]$ and $\pm k \in [k_-^{(2)}, k_+^{(2)}]$, respectively.
- Movie: Mean DRC vs. θ_0 for $\gamma_1 = \gamma_2$

Ref. : J. Opt. Soc. Am. A **18**, 1507 (2001); J. Opt. Soc. Am. A **20**, 2338 (2003).

Forward-Scattering Peaks

Another Coherent Effect

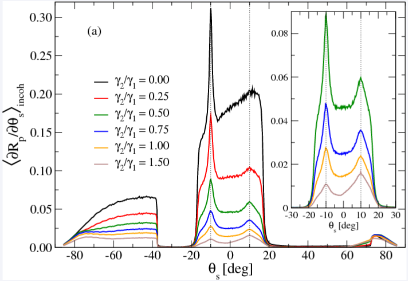
Numerical results : $\theta_0 = 10^\circ$, p-polarization, $\sigma = 10$nm (Movie: Mean DRC vs γ_2)

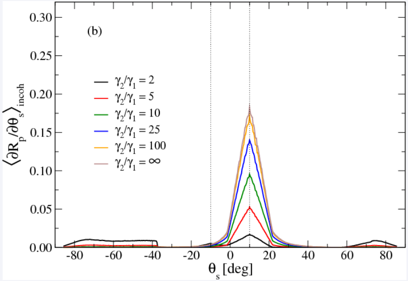


Forward-Scattering Peaks

Another Coherent Effect

Numerical results : $\theta_0 = 10^\circ$, p-polarization, $\sigma = 10$nm (Movie: Mean DRC vs γ_2)





Forward-Scattering Peak

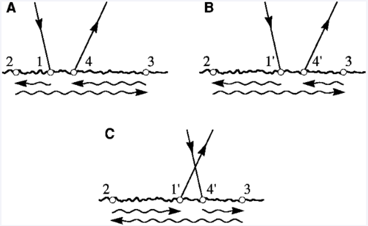
For some values of γ_2/γ_1 we have also a peak at the forward direction $\theta_s = \theta_0$ in addition to that at $\theta_s = -\theta_0$

But what is causing this behavior....?

Forward-Scattering Peaks

What are their origin?

Relevant paths:



Path A and B :

$$k \to -k_{spp} \to k_{spp} \to -k_{spp} \to q$$

$$k = \frac{\omega}{c} \sin\theta_0$$
$$q = \frac{\omega}{c} \sin\theta_s$$

Phases for path A and B

$$\phi_A = kx_1 - qx_4 + \beta_{1-4}$$
$$\phi_B = kx_{1'} - qx_{4'} + \beta_{1'-4'}$$

$$\Delta\phi_{BA} = \phi_B - \phi_A$$
$$= k(x_{1'} - x_1) - q(x_{4'} - x_4) + \beta_{1'-4'} - \beta_{1-4}$$

Forward-Scattering Peaks

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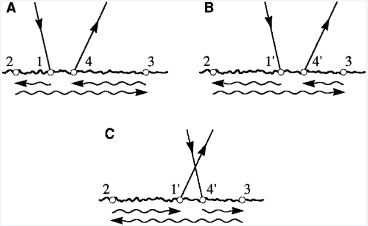

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$$\Delta\phi_{BA} = \phi_B - \phi_A$$

$$= k(x_{1'} - x_1) - q(x_{4'} - x_4) + \beta_{1'-4'} - \beta_{1-4}$$

By assuming $x_{4'} - x_{1'} = x_4 - x_1$ it follows that

$$\Delta\phi_{BA} = (k - q)(x_{1'} - x_1),$$

and requiring *phase-coherence* gives

$$\Delta\phi_{BA} = 0 \qquad \Rightarrow \qquad q = k$$

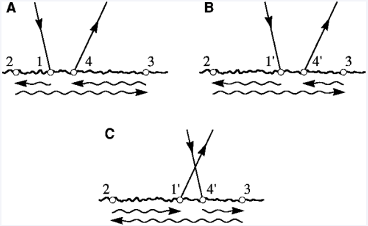
or

$$\theta_s = \theta_0$$

Forward-Scattering Peaks

What are their origin?

Relevant paths:



$$k = \frac{\omega}{c} \sin\theta_0$$

$$q = \frac{\omega}{c} \sin\theta_s$$

Phases for path A and C (if allowed)

$$\Delta\phi_{CA} = \phi_C - \phi_A$$
$$= (q-k)(x_1 - x_{1'}) + (q+k)\Delta x$$

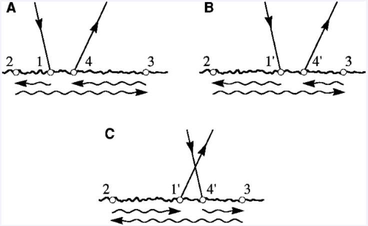
where $\Delta x = x_4 - x_1 = x_{4'} - x_{1'}$.

Only for $\Delta x = 0$ does one get the same phase condition as for $\Delta\phi_{BA}$.

Forward-Scattering Peaks

What are their origin?

Relevant paths:



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where $\Delta x = x_4 - x_1 = x_{4'} - x_{1'}$.

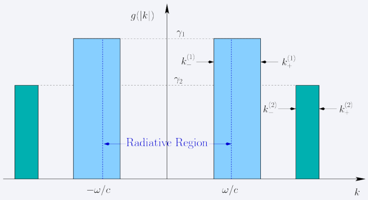
Only for $\Delta x = 0$ does one get the same phase condition as for $\Delta\phi_{BA}$.

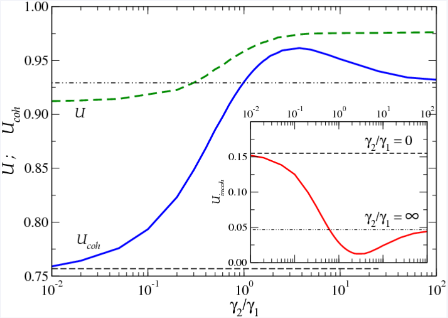
Take home message

The counter propagation $\pm k_{spp} \to \mp k_{spp}$ is essential for the forward scattering peak phenomenon

Forward-Scattering Peaks

Amount of coherent/incoherent light *vs.* $gamma_2 / gamma_1$



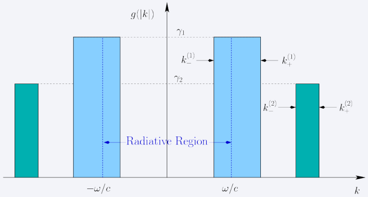


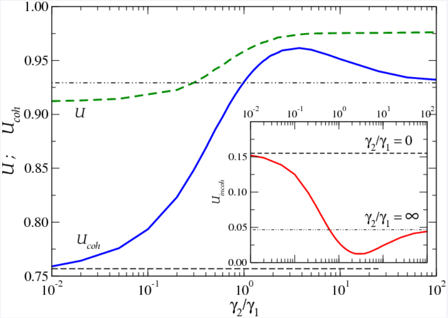
Fraction of incident power being reflected

$$\mathscr{U}_\nu = \int_{-\pi/2}^{\pi/2} d\theta_s \left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle$$

Forward-Scattering Peaks

Amount of coherent/incoherent light *vs.* $gamma_2/gamma_1$





Fraction of incident power being reflected

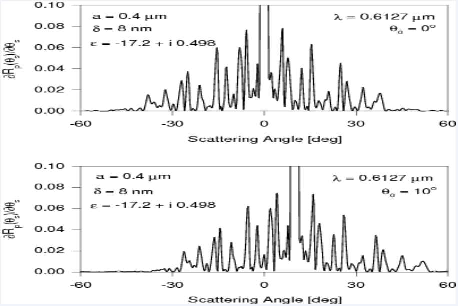
$$\mathscr{U}_\nu = \int_{-\pi/2}^{\pi/2} d\theta_s \left\langle \frac{\partial R_\nu}{\partial \theta_s} \right\rangle$$

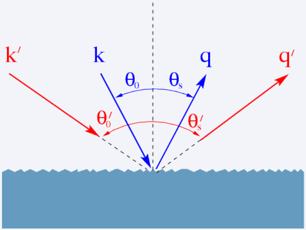
Take home message

By changing the correlations along the surface the scattering properties can be changed dramatically

Angular Intensity Correlation Functions

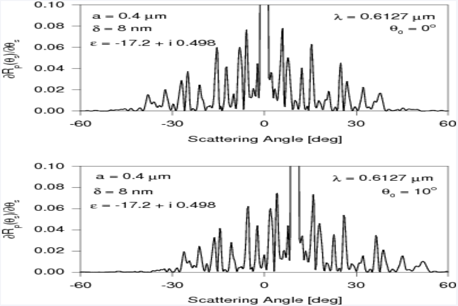
The definition

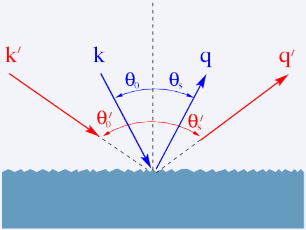




Angular Intensity Correlation Functions

The definition





Definition:

$$C(q,k|q',k') = \langle I(q|k)I(q'|k')\rangle - \langle I(q|k)\rangle \langle I(q'|k')\rangle$$

where the intensity $I(q|k)$ is defined as ($k = \frac{\omega}{c}\sin\theta_0$ *etc*)

$$I(q|k) = \frac{1}{L_1}\left(\frac{\omega}{c}\right)|S(q|k)|^2 = \frac{1}{L_1}\left(\frac{\omega}{c}\right)\frac{\alpha_0(q,\omega)}{\alpha_0(k,\omega)}|R(q|k)|^2$$

Angular Intensity Correlation Functions

Different types

One can separate the angular intensity correlation functions into several terms:

$$C(q,k|q',k') = C^{(1)} + C^{(1.5)} + C^{(2)} + C^{(3)}$$

where

- $C^{(1)}$ – Short range correlation functions
- $C^{(1.5)}$ – Intermediate range correlation functions
- $C^{(2)}$ – Long range correlation functions
- $C^{(3)}$ – Infinite range correlation functions

The $C^{(1.5)}$ is unique to rough surface scattering (and has *e.g.* no analogy in random bulk systems)!

Angular Intensity Correlation Functions
Short Range Correlations

$$C^{(1)}(q,k|q',k') = \frac{\varepsilon_0}{L_1^2}\frac{\omega^2}{c^2}\left|\langle \delta S(q|k)\delta S^*(q'|k')\rangle\right|^2, \quad \delta S = S - \langle S\rangle$$

$$= \frac{2\pi\delta(q-k-q'+k')}{L_1} C_0^{(1)}(q,k|q',q'-q+k)$$

$$C^{(10)}(q,k|q',k') = \frac{2\pi\delta(q-k+q'-k')}{L_1} C_0^{(10)}(q,k|q',q'+q+k).$$

where $C_0^{(1)}$ and $C_0^{(10)}$ are *envelopes* independent of L_1

They are non-zero only when the momentum transfer ($\Delta_{qk} = q-k$) satisfies

- $C^{(1)}$: $\Delta_{qk} = \Delta_{q'k'}$
  - Memory Effect : $k = k'$, $q = q'$
  - Reciprocal Memory Effect ($S(q|k) = S(-k|-q)$) : $k = -q'$, $q = -k'$
- $C^{(10)}$: $\Delta_{qk} = -\Delta_{q'k'}$

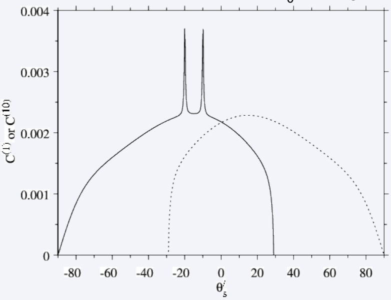
Note : $C^{(10)}$ is unique to surface scattering!

Angular Intensity Correlation Functions

Simulations and Experiments

small Simulations (p-pol.):

Gaussian silver surface : $\delta = 5nm$, $a = 100nm$, $\theta_0 = 20^o$, $\theta_s = -10^o$

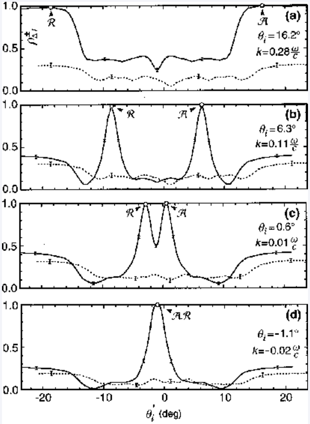


- Memory effect
  - $\theta_s' = \theta_s = -10^o$ $(q' = q)$
- Reciprocal Memory Effect :
  - $\theta_s' = -\theta_0 = -20^o$ $(q' = -k)$

Experiments (p-pol):

West-O'Donnell gold surface : $k_- = 0.83\omega/c$, $k_+ = 1.30\omega/c$,

$\Delta - = 0.04\omega/c$, $\sigma = 15.5nm$, $\theta_0 = 20^o$, $\theta_s = -10^o$

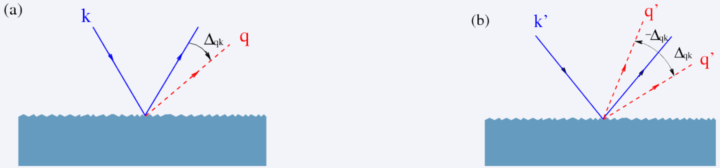


(This corresponds to : — $C^{(1)}$; - - - $C^{(10)}$)

Ref. : West and O'Donnell, PRB **59**, 2393 (1999)

Angular Intensity Correlation Functions

Physical meaning of short range correlation functions



- $C^{(1)}$: $\Delta_{qk} = \Delta_{q'k'}$
  If k is changed to $k' = k + \Delta k$ the entire speckle pattern changes such that a feature originally at q moves to $q' = q + \Delta k$
- $C^{(10)}$: $\Delta_{qk} = -\Delta_{q'k'}$
  If k is changed to $k' = k + \Delta k$ the entire speckle pattern changes such that a feature originally at $q = k - \Delta q$ moves to $q' = k + \Delta q$, (symmetry with respect to the specular direction)

These effects can be seen directly in the specular patterns!

Angular Intensity Correlation Functions

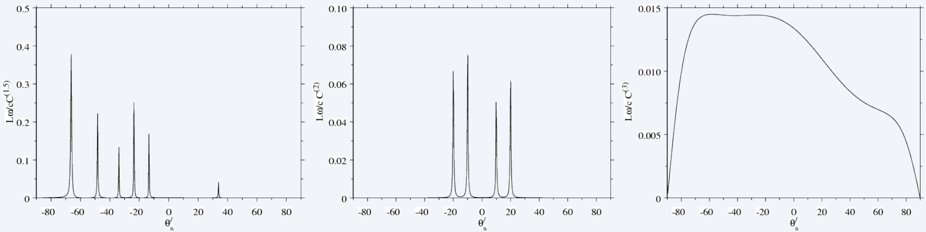
Long and infinite range correlations $C^{(N)}$

$$C^{(N)}(q,k|q',k') = \frac{\varepsilon_0}{L_1^2}\frac{\omega^2}{c^2}\left\{\delta S(q|k)\delta S^*(q|k)\delta S(q'|k')\delta S^*(q'|k')\right\} \propto \frac{1}{L_1}$$

It will be hard to observe experimentally!

$$C^{(N)}(q,k|q',k') = C^{(1.5)}(q,k|q',k') + C^{(2)}(q,k|q',k') + C^{(3)}(q,k|q',k')$$

- $C^{(N)}$ has a complex peak structure
- $C^{(1.5)}$ is *unique* to surface scattering



[p-pol : $\theta_o = 20^o$, $\theta_s = -10^o$, $\delta = 5nm$ and $a = 100nm$]

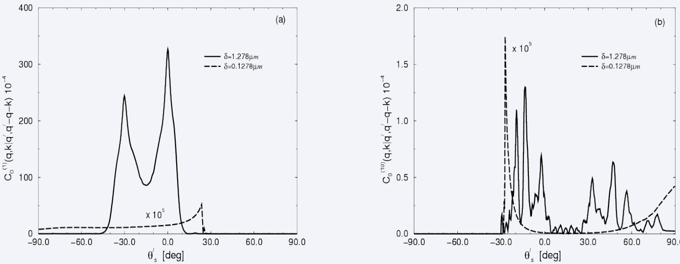
Angular Intensity Correlation Functions

Strongly rough surfaces

For weakly rough surfaces we saw :

- No peaks in the $C^{(1)}$ correlation function for s-polarization (No SPPs)
- $C^{(1)}$ and $C^{(10)}$ are of the same magnitude

No surprise that multiple scattering of volume waves is causing peaks in $C^{(1)}$ for strongly rough surfaces also in s-polarization



(S-pol : $\theta_0 = 30^\circ$ and $\theta_s = 0^\circ$; Gaussian silver surface $a = 3.85\mu\mathrm{m}$)

Note : $C^{(10)}$ vanishes for strongly rough surfaces

Angular Intensity Correlation Functions

Statistics of the Field Amplitudes

Measurements of the $C(q,k|q',k')$ can provide information about the amplitude of the scattered field (Leskova *et al.* 2000)

- Only $C^{(1)}$ is observed
  - $R(q|k)$ is a circular complex Gaussian random process defined as ($A = A_1 + iA_2$, $B = B_1 + iB_2$)

$$\langle A_1 B_1 \rangle = \langle A_2 B_2 \rangle \qquad \langle A_1 B_2 \rangle = -\langle A_2 B_1 \rangle$$

    this implies that

$$\langle AB \rangle = 0 \quad \Rightarrow \quad C^{(10)} \propto |\langle \delta S(q|k) \delta S(q'|k') \rangle|^2 = 0$$

- Only $C^{(1)}$ and $C^{(10)}$ are observed
  - $R(q|k)$ is a complex Gaussian random process
- $C^{(1)}$, $C^{(10)}$ and, $C^{(N)}$ are observed
  - $R(q|k)$ is a non-Gaussian random process

Summary

Angular correlation functions can tell us about the statistics properties of the scattered field.

Conclusions and outlook

Conclusions

- Rough surface scattering is rich
- There might be “order in the chaos”
- The height-height correlations are important for the scattering
- The roughness can be used to tune the optical properties

A renewed interest in rough surface scattering has been witnessed during the last years (probably) due to it potential applications.

Reference : I. Simonsen, ArXiv:cond-mat/040817

Thank you for your Attention!