PET110

An extended version of Problem #3 (Kearey, ch. 4, p. 97) with solutions

A zero-offset (primary) reflection event at 1.000 s (meaning t_0 or t_{01} = 1.000 s) has a normal moveout (NMO) of 5 ms at 200 m offset. (Assume that the NMO is exactly 5 ms, even though this is unrealistic.) Determine the stacking velocity using both:

(a) the hyperbolic equation and

(b) the parabolic approximation.

(c) A second primary reflection arrives with t_{02} = 2.000 s and Δt_{NMO}(200 m) = 2 ms. What numerical value of V is associated hyperbolically with this offset, this t_0 value and this Δt_{NMO} value?

(d) What is the name we give to this kind of velocity value, and what computable velocity is it approximately equal to?

(e) If we no longer assume that the Δt_{NMO} value is exact, but (more realistically) was determined to the nearest ms, (though we will still assume that the offset and t_0 values in (c) are exact) what is the uncertainty in the velocity value determined in (c)? (Use the hyperbolic expression.)

(f) Suppose we then looked at this second reflection on a trace recorded at an offset of 600 m and found a normal moveout of 18 ms, again correct to the nearest ms. Determine from this greater offset what the estimated value of V would be, and its uncertainty, using the hyperbolic expression.

(g) Determine for the case in (f) the corresponding value for V using the parabolic approximation.

(h) Assume that we're looking here at just two layers, i.e., two reflectors. Then, of course, the stacking velocity found in (a) is actually the interval velocity of the first layer (or the top layer). Estimate the interval velocity in the second layer and determine its uncertainty.

(i) Consider a multiple reflection due to a second trip down and up through the top layer (also called a 1st-order multiple). What will be its zero-offset traveltime?

(j) What will be its normal moveout at x = 200 m?

Solutions

(References are to Kearey et al., 2002.)

(a) Hyperbolic: starting with eq. (4.4):

$$t^2(x) = \left[t_0 + \Delta t_{\text{NMO}}(x)\right]^2 = t_0^2 + \frac{x^2}{V^2}.$$

Then solving for *V* gives:

$$V = \frac{x}{\sqrt{\left[t_0 + \Delta t_{\text{NMO}}(x)\right]^2 - t_0^2}} = \frac{200}{\sqrt{1.005^2 - 1.000^2}} = 1997.51 \text{ m/s}.$$

(b) Parabolic: using eq. (4.8):

$$V \approx \frac{x}{\sqrt{\left[2t_0 \Delta t_{\text{NMO}}(x)\right]}} = \frac{200}{\sqrt{\left[2 \cdot 1 \cdot (0.005)\right]}} = 2000 \text{ m/s}.$$

(c) $$V = \frac{200}{\sqrt{2.002^2 - 2^2}} = 2235.51 \text{ or } 2236 \text{ m/s}.$$

(d) This kind of velocity value is called a stacking velocity, and it's approximately equal to the RMS velocity (or root-mean-square velocity).

(e) Since the Δt_{NMO} value, 2 ms, is given to the nearest ms, its true value could be anywhere between 1.5 and 2.5 ms (which we could call 2.0 ± 0.5 ms). Then *V* could be anywhere between the values found using 1.5 ms and using 2.5 ms for Δt_{NMO}, i.e., between:

$$V_{\text{max}} = \frac{200}{\sqrt{2.0015^2 - 2^2}} = 2581.50 \text{ m/s}$$

and

$$V_{\text{min}} = \frac{200}{\sqrt{2.0025^2 - 2^2}} = 1999.38 \text{ m/s}.$$

So we could say $V = 2290 \pm 291$ m/s [compared with 2236 m/s found in (c)].

[We could also say that the uncertainty in the velocity value determined in (e) is about 300 m/s. We could have said about 290 m/s, but the last two digits are not really significant when the uncertainty is so great.]

(f) In the case of x = 600 m and Δt_{NMO} = 18.0±0.5 ms:

$$V_{\max} = \frac{600}{\sqrt{2.0175^2 - 2^2}} = 2262.84 \text{ m/s.}$$

and

$$V_{\min} = \frac{600}{\sqrt{2.0185^2 - 2^2}} = 2200.56 \text{ m/s.}$$

so now can say $V = 2232 \pm 31$ m/s and the uncertainty is about 30 m/s, much less than for x = 200 m. [Assuming the 18 ms to be exact would yield V = 2231.05 m/s.]

(g) Parabolic: again using eq. (4.8):

$$V \approx \frac{x}{\sqrt{\left[2t_0 \Delta t_{\mathrm{NMO}}(x)\right]}} = \frac{600}{\sqrt{\left[2 \cdot 2 \cdot (0.018)\right]}} = 2336 \text{ m/s.}$$

[The parabolic approximation gets worse at greater offset. If you worked it out for part (c) you'd find the difference between hyperbolic and parabolic to be less than 1 m/s at 200 m offset; but now it's a bit over 100 m/s. On the other hand, the uncertainty in V due to the uncertainty in NMO decreases at greater offset: it was about 300 m/s at 200 m offset, but is about 30 m/s at 600 m.]

(h) Use the Dix equation (p. 46, lower left) for relating RMS velocity to interval velocities; that is:

$$v_n^2 = \frac{V_{RMS,n}^2 t_n - V_{RMS,n-1}^2 t_{n-1}}{t_n - t_{n-1}}$$

or

$$v_2^2 = \frac{2232^2 \cdot 2 - 2005^2 \cdot 1}{2 - 1} \quad \text{or} \quad v_2 = 2438 \text{ m/s.}$$

Here I used 2005 m/s for $V_{RMS,1} = v_2$ after checking max- and min-values.

[If I had assumed exact NMO values and used the resulting V values (1997.51 and 2231.05) I'd have got 2442.36 m/s for v_2.]

(i) For the multiple, t_0 is twice that of the first primary, i.e., 2.000 s.

(j) If the primary reflection in the top layer is described by

$$t^2(x) = 1^2 + \frac{x^2}{2005}$$

then the 1st-order multiple in the same layer will be described by

$$t^2(x) = 2^2 + \frac{x^2}{2005}.$$

Then its moveout at x = 200 m will be given by $t(200) - t_0 = \sqrt{4 + \frac{200^2}{2005^2}} - 2 \;=\; 2.49$ ms, or about 2½ ms, about half that of the primary (with t_{01} = 1.000 s) at 200 m offset.