TFY4235/FY8904: Computational Physics Assignment 3: Radioactivity and photon transport

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Guillermo Garrido-Hernandez

Introduction

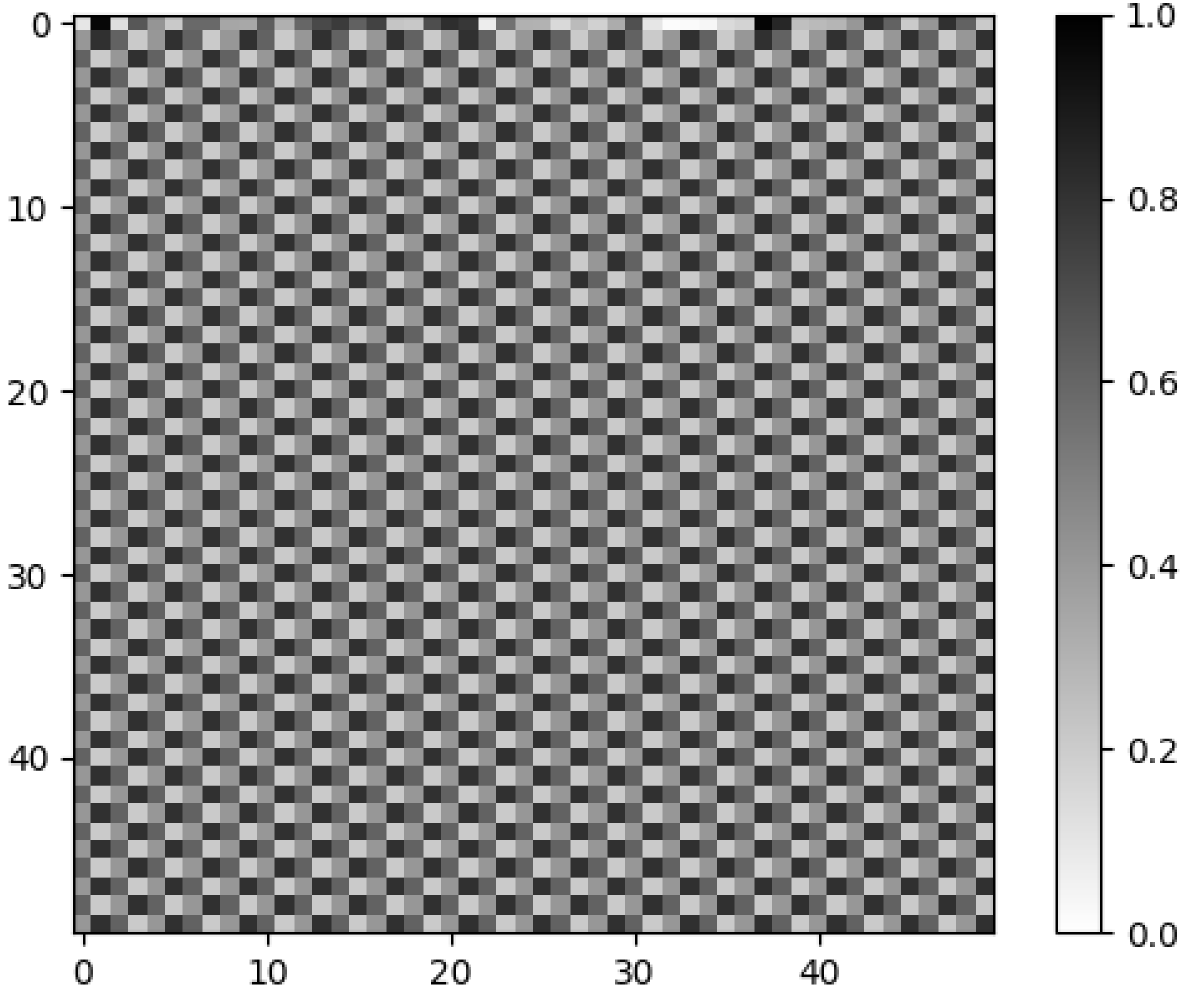
- Briefly study how random (or pseudorandom) number generators work.
- Create a MC algorithm to replicate nuclear decay. Study the uncertainty of this results.
- Model two (photoelectric effect and Comptom scattering) of the three interactions that photons undergo when moving through matter. Study photon trajectories and energy losses.
- Combine the nuclear decay with the photon interaction simulations to look at how the energy loss distributions from primary photons look like when a radioactive source is placed within a homogeneous medium.

Random number generator

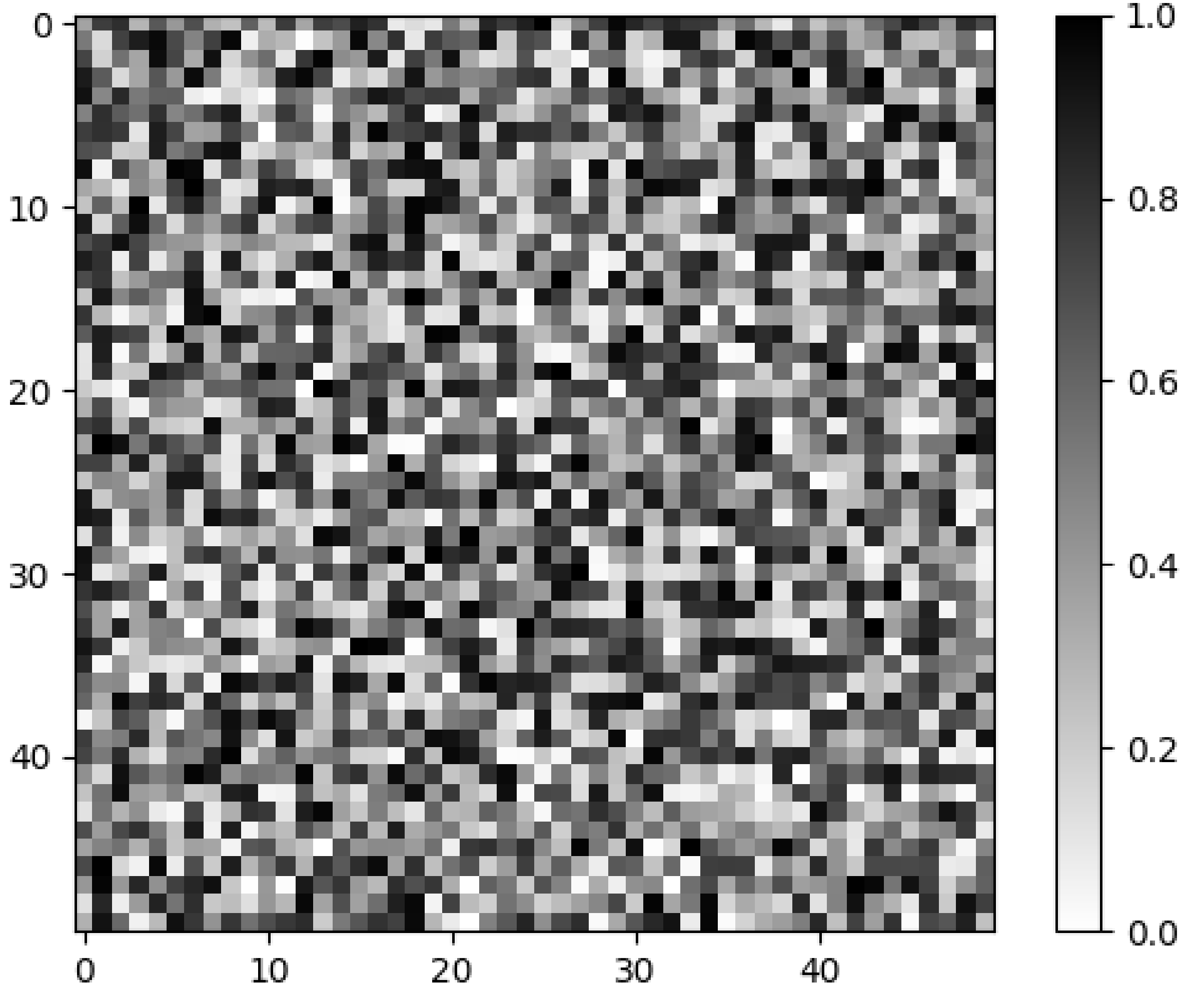
- Two random number generators (middle square method/linear congruential method).

- Study the random number distributions (does it loop at some point? When? Visualize? Try to find better/optimal parameters.

Middle square method



Linear congruential method



Radioactive decay

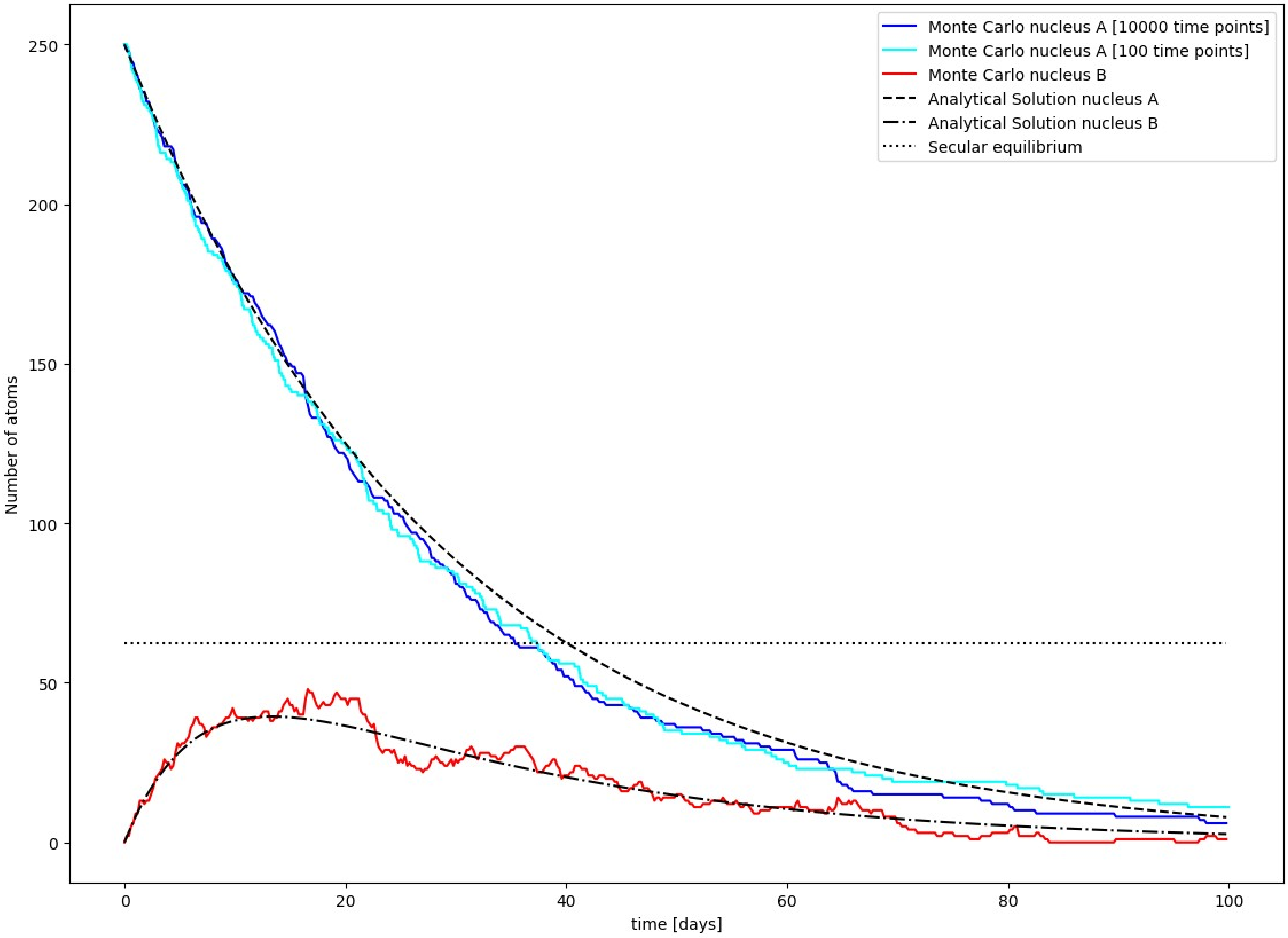
where

- Monte Carlo solution for the problem:

1. Select a total time and amount of time steps.
2. Based on the radioactive decay equation define a probability for decay/no-decay.
3. Each time step daw a random number. Put together with decay/no-decay probability and count events.
4. Run multiple times and aggregate the results.

- Radioactive decay chain:

- What if for we got or/and ?



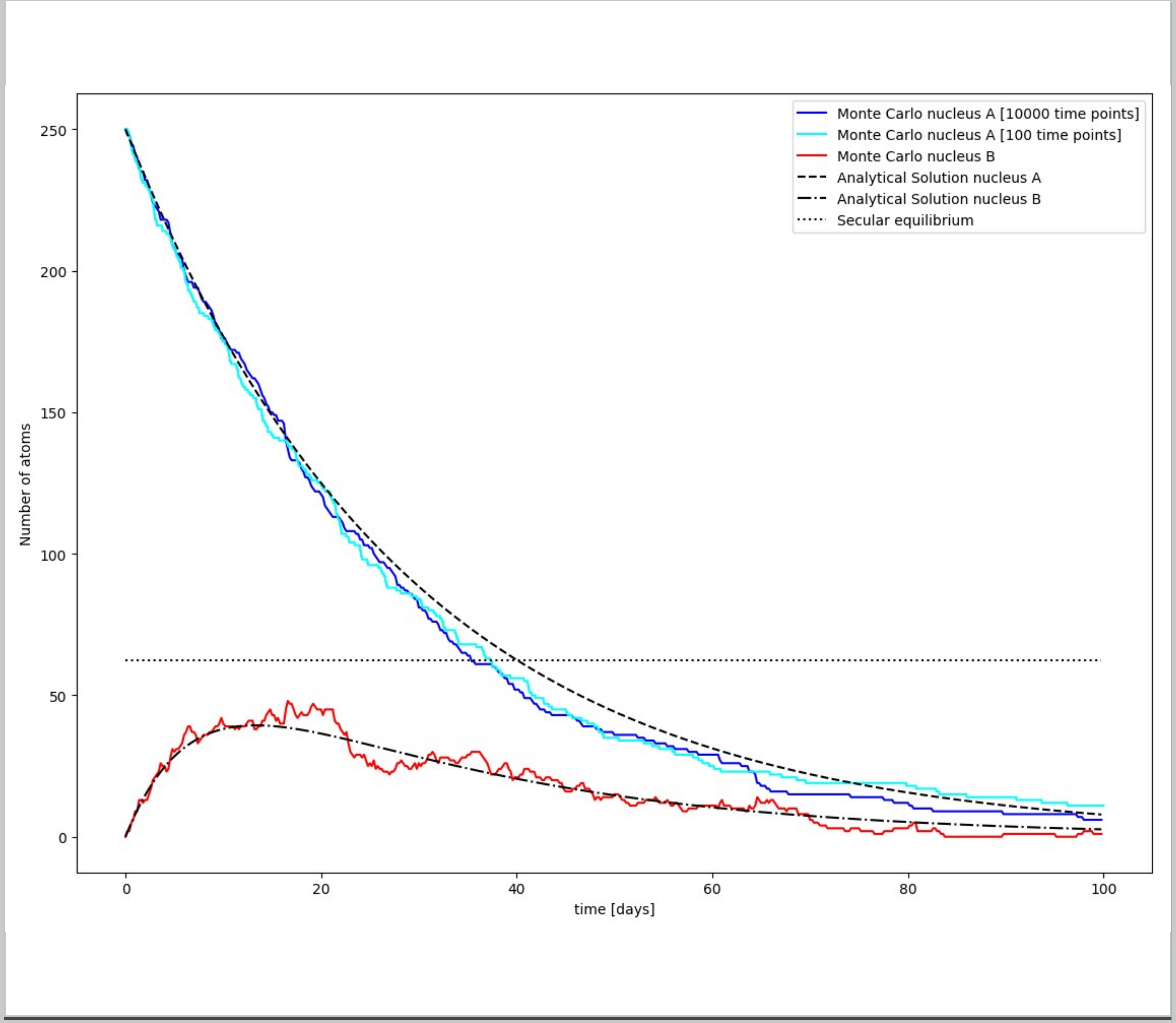
Monte Carlo nucleus A [10000 time points]
Monte Carlo nucleus A [100 time points]
Monte Carlo nucleus B
Analytical Solution nucleus A
Analytical Solution nucleus B
Secular equilibrium
Number of atoms
time [days]
0
50
100
150
200
250
0
20
40
60
80
100

- Results uncertainties?

- Error of ? Time dependent?

- How do the results for a specific t distribute around the analytical ?

- λ_1/λ_2 ratio for secular equilibrium?



Photon interactions with matter

- Photo electric effect (dominates for low energy):

Absoption/Interaction probability dependent on incident photon’s energy and target’s atomic number.

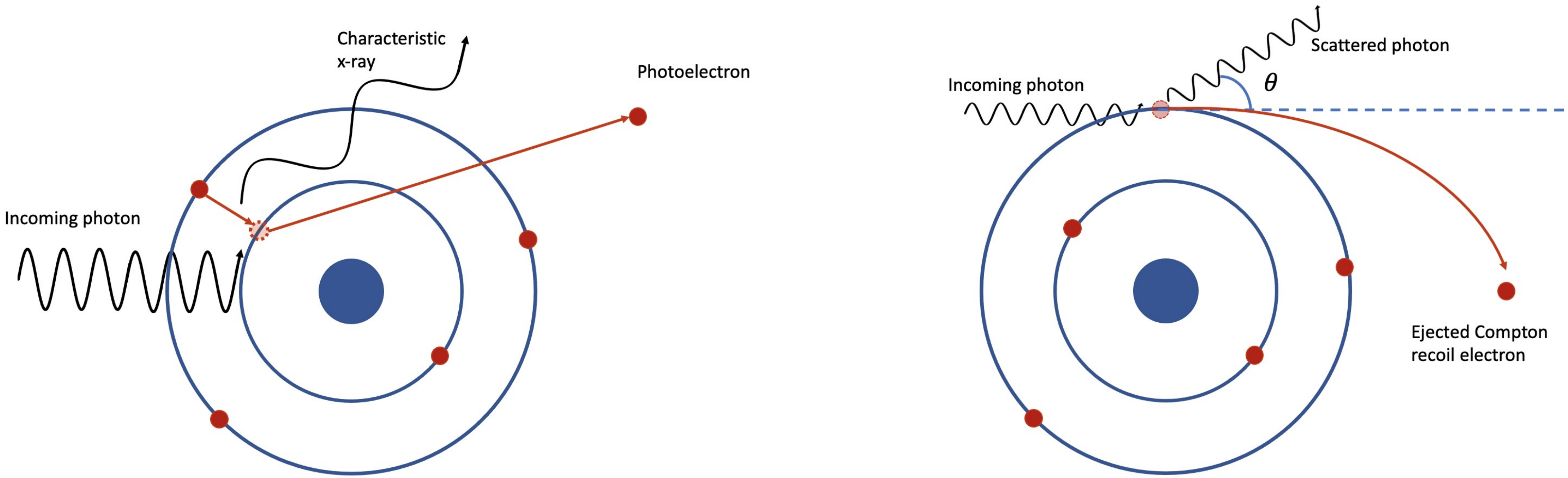
- Compton scattering (dominates for intermediate energy):

Interaction probability dependent on incident photon’s energy and target’s atomic number.

Klein-Nishina equation to estimate deflection after interaction.

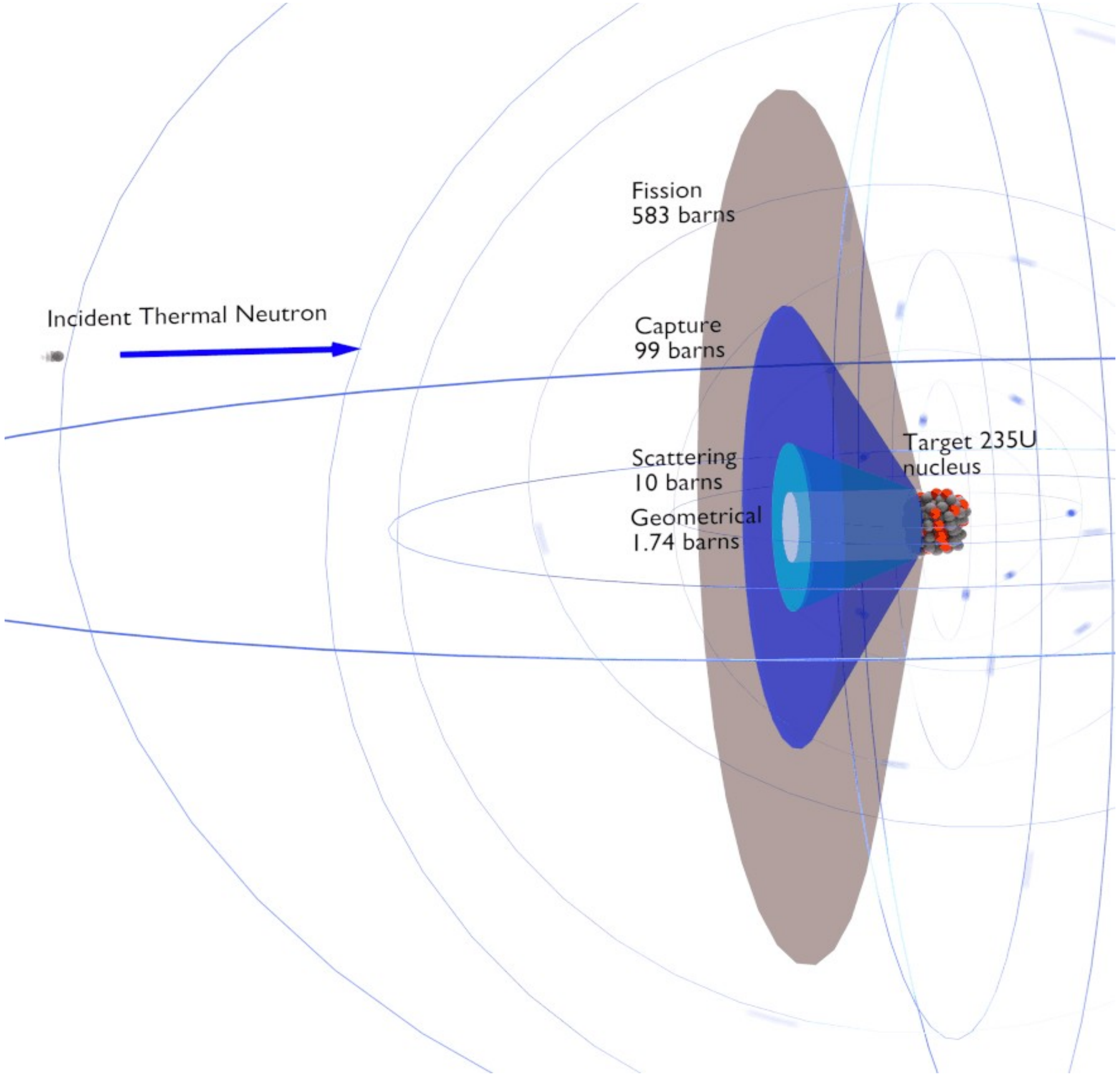
- Pair production (dominates for high energy):

Not relevant for our energy range.


Characteristic
x-ray
Photoelectron
Incoming photon
Scattered photon
Incoming photon
θ
Ejected Compton
recoil electron

Cross-section

- If the projectile particle hits this effective surface the projectile and the target would interact (if it doesn't there will be no interaction).
- This effective surface is referred to as a **cross-section**.
- Different interaction mechanisms (e.g., PE and Compton scattering) will have different cross-sections and therefore different interaction probabilities.



Photoelectric interaction

$$\sigma_{PE} \approx 3 \cdot 10^{12} \frac{Z^4}{E_\gamma^{3.5}}$$

- Where Z is the atomic number of the target element and E_γ is the incident photon energy.

During PE interactions the incident photon is absorbed by an inner shell atomic electron.

Comptom scattering

$$\sigma_C = \begin{cases} Z 2\pi r_e^2 \left(\frac{1+k}{k^2} \left[\frac{2(1-k)}{1+2k} - \frac{\ln(1+2k)}{k} \right] + \frac{\ln(1+2k)}{k} - \frac{1+3k}{(1+2k)^2} \right) \text{if } k < 0.2 \\ Z \frac{8}{3} \pi r_e^2 \frac{1}{(1+2k)^2} \left(1+2k+\frac{6}{5}k^2 - \frac{1}{2}k^3 + \frac{2}{7}k^4 - \frac{6}{35}k^5 + \frac{8}{105}k^6 + \frac{4}{105}k^7 \right) \text{if } k > 0.2 \end{cases}$$

- Where k is the ratio of the photon energy and the electron's rest mass energy ($k = E_\gamma / E_e$) and r_e is the classical electron radius ($r_e = e^2 / E_e$).

In Compton scattering, a photon transfers a portion of its energy to a loosely bound outer shell electron of an atom.

- The effects of Compton scattering:

$$E'_{\gamma} = \frac{E_{\gamma}}{1 + k(1 - cos(\theta))}$$

$$\frac{d\sigma}{d\Omega} = \frac{\hbar^2 \alpha^2}{2E_e^2} \left(\frac{E'_{\gamma}}{E_{\gamma}} \right)^2 \left[\frac{E'_{\gamma}}{E_{\gamma}} - \frac{E_{\gamma}}{E'_{\gamma}} - sin^2(\theta) \right]$$

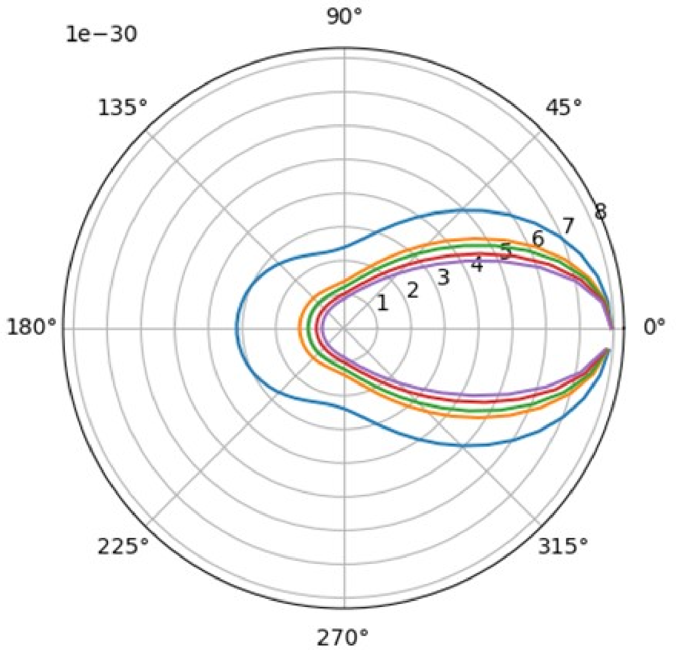
$$\frac{d\sigma}{d\Omega} = \frac{\hbar^2 \alpha^2}{2E_e^2} \left[\frac{1}{1 + k(1 - cos(\theta))} \right]^2 \left[\frac{1}{1 + k(1 - cos(\theta))} - 1 + k(1 - cos(\theta)) - sin^2(\theta) \right]$$

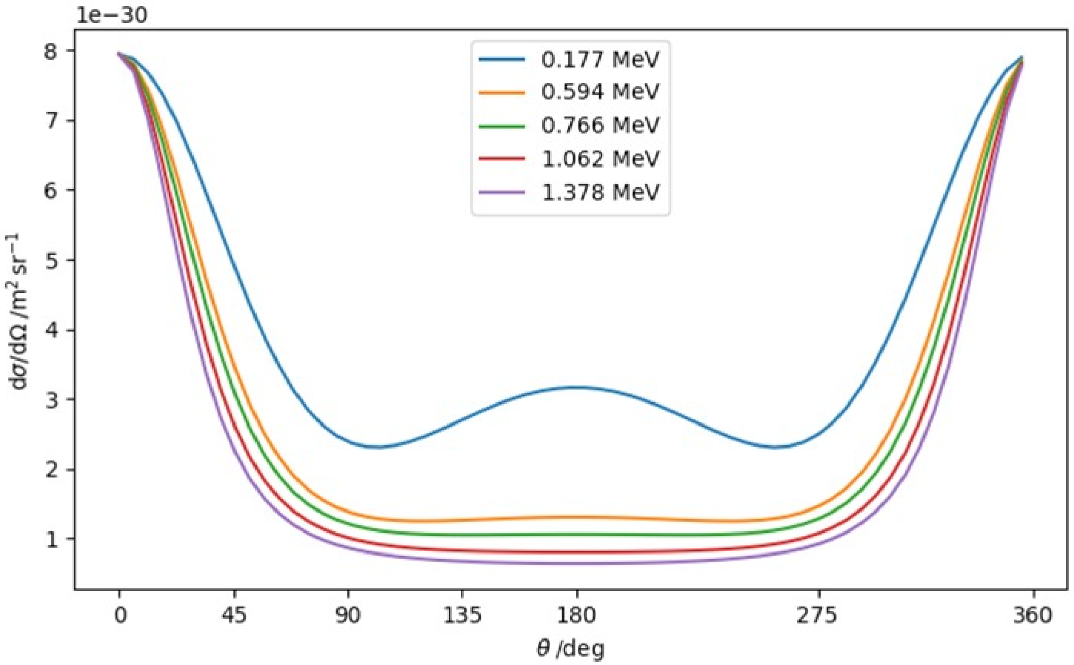
- The effects of Compton scattering:

$$E'_\gamma = \frac{E_\gamma}{1 + k(1 - \cos(\theta))}$$

$$\frac{d\sigma}{d\Omega} = \frac{\hbar^2\alpha^2}{2E_e^2}\left(\frac{E'_\gamma}{E_\gamma}\right)^2\left[\frac{E'_\gamma}{E_\gamma} - \frac{E_\gamma}{E'_\gamma} - sin^2(\theta)\right]$$

$$\frac{d\sigma}{d\Omega} = \frac{\hbar^2\alpha^2}{2E_e^2}\left[\frac{1}{1 + k(1 - \cos(\theta))}\right]^2\left[\frac{1}{1 + k(1 - \cos(\theta))} - 1 + k(1 - \cos(\theta)) - sin^2(\theta)\right]$$





Assumptions

- 2D geometry: movement on the XY-plane.
- No pair production (energy range not high enough): only Comptom scattering and PE.
- Distance between photon interactions constant.
- Absorption probability of for a photon

Simple particle transport simulation

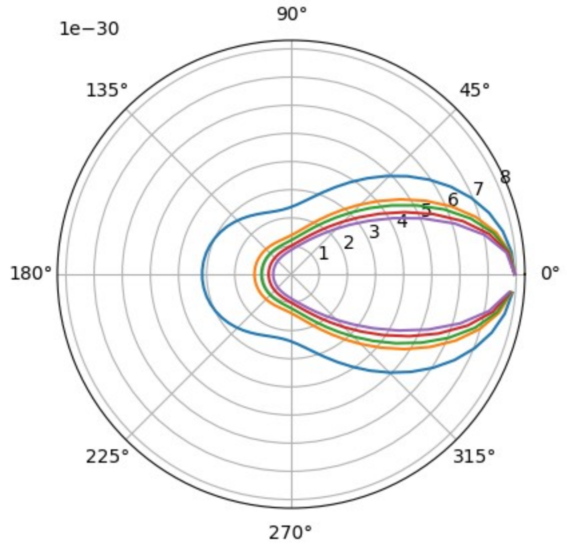
1. Calculate PE probabability. If photon absorbed stop simulation. If photon not absorbed do 2.
2. Implement Compton scatering:

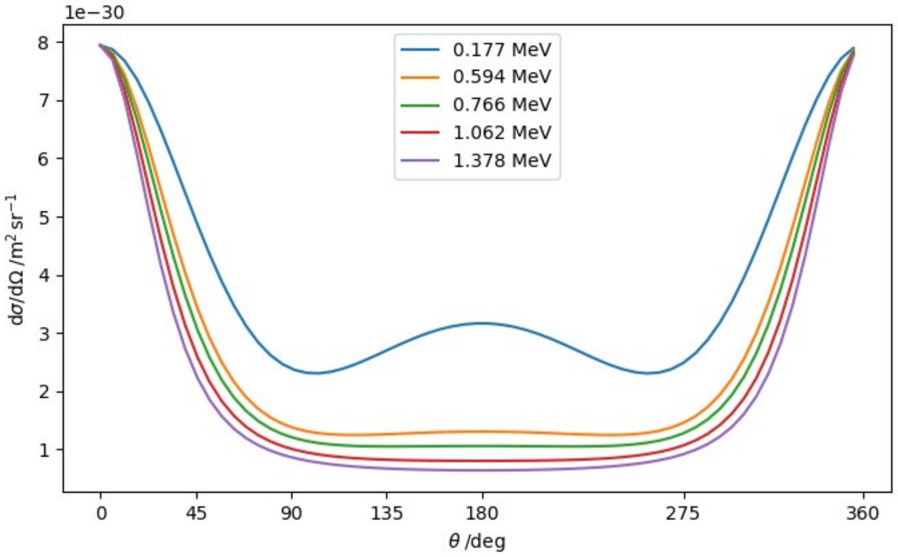
Discretize particle's path through medium.

Use Klein-Nishina to create angle deflection probability distributions.

Calculate energy loss as function of particle's deflection.

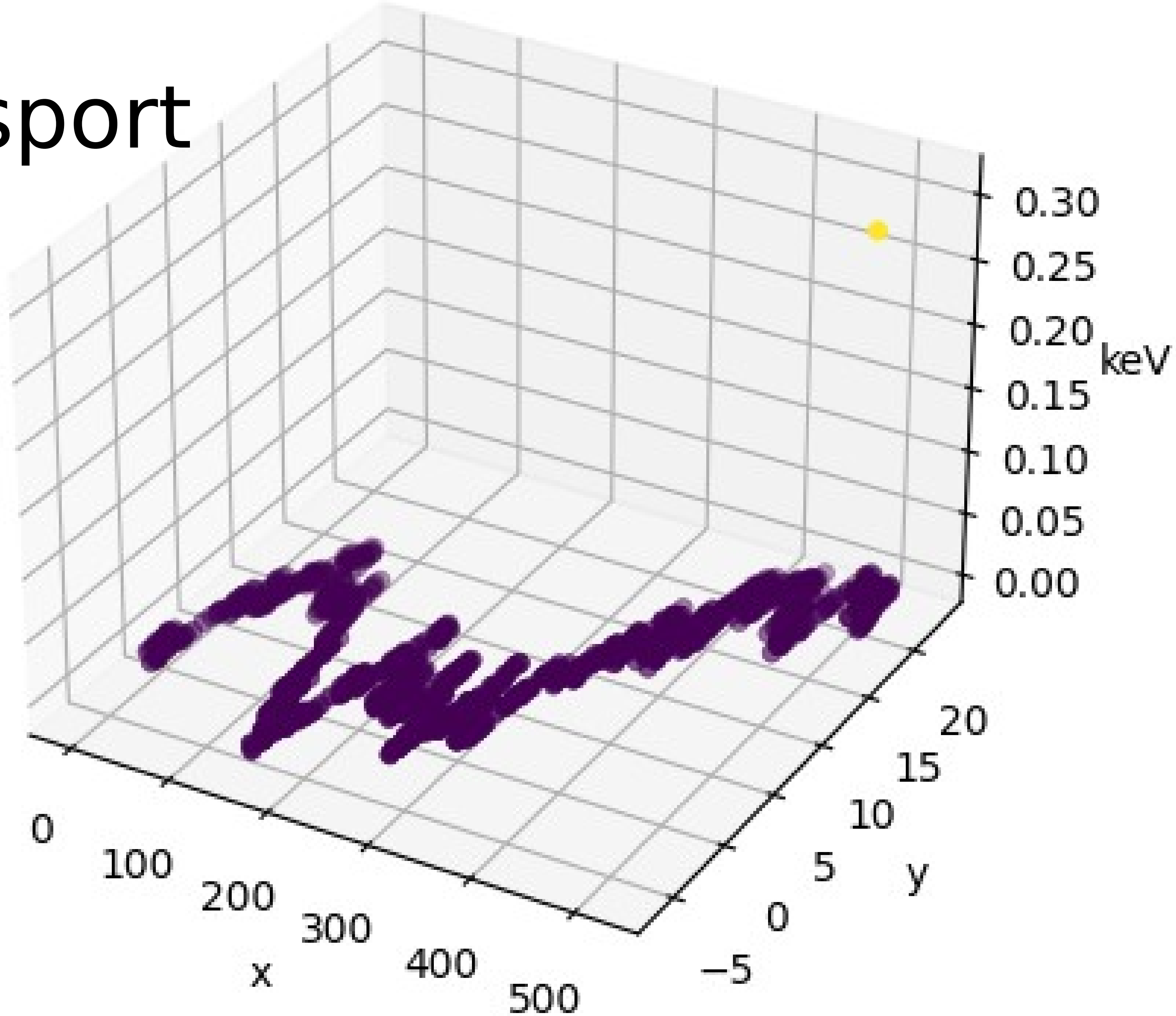
Store data and run again until photon is absorbed.





Particle transport

- Average range for different materials (different Z)?
- Plot results for trajectories and energy losses.
- Modify the step size.



Combine decay with particle transport

- Simulate decay.
- Create discrete gamma spectrum for the decay (each decay event has an P1%, P2% and P3% of decaying with a E1, E2 and E3 energy (in keV). And a uniform angle distribution probability (direction that the particle follows after decay event): Initial conditions (Energy and direction).
- Start the particle transport simulation from the initial conditions.

- GOAL: Estimate an energy loss/absoption map.