TFY4235/FYS8904
Spring 2022

NTNU

Institutt for fysikk

Problem 1.

This is a pre-assignment[1] that is dedicated to the study of a *random walk process* (or Brownian motion). The aim is to study the probability distribution for the return to the starting point, the "origin", for the random walker. Let the position of the walker at time t be denoted

$$x(t) = \sum_{n=0}^{t} \xi_n, \tag{1}$$

where time is understood to be measured in units of the time interval Δt. Here ξ_n denotes some arbitrary *uncorrelated* random deviates of finite variance and *zero* mean. For reasons of specificity, you may assume ξ_n to be uniformly distributed random numbers on the interval from -1 to 1 [$U(-1,1)$], or standard Gaussian random deviates [$N(0,1)$].

a) Calculate the probability distribution function, $p(t)$ that the walker returns to the starting point for the *first time* after t steps.

Comment: if the first step satisfies $\xi_1 > 0$, then t is determined by the *smallest* time so that the condition $x(t) < 0$ is satisfied for the first time. The desired $p(t)$ is the probability distribution function of such waiting times.

b) This probability distribution function will have the form

$$p(t) \sim t^{-\alpha}. \tag{2}$$

Determine the numerical value of the exponent α.

c) For a random walker starting at $x = 0$ at $t = 0$, calculate the probability distribution function for the waiting time for the *first passage* of the level $x = a > 0$, *i.e.* the smallest waiting time for $x(t) \geq a$. This problem is called the *level crossing problem*. What is the form of the pdf in this case?

d) Finally obtain the pdf of waiting times for reaching either level $x = a$ or level $x = -a$ for the first time (still starting from $x = 0$ at $t = 0$). What is the form if the pdf in this case?.

[1] The pre-assignment is *not* part of the set of assignments that can be selected on the exam for full solution submission.