

Assignment 1, TFY4240 Electromagnetic Theory midterm project, Fall 2014

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In assignment 1, we will solve the Laplace equation in 2 dimensions by the method of separation of variables; this is a variation of Example 3.4 in Griffiths. The problem is to determine the potential in a long, square, hollow tube, where the four walls have different potentials. The boundary conditions are as follows:

$$\begin{aligned}V(x = 0, y) &= 0, \\V(x = L, y) &= 0, \\V(x, y = 0) &= 0, \\ \text{and } V(x, y = L) &= V_0(x).\end{aligned}\tag{1}$$

Your assignment is as follows:

1. Set up the equation you need to solve, and make it dimensionless (e.g. use $\xi = x/L$ as a variable instead of x).
2. Formulate a solution in terms of Fourier coefficients, in a way suitable for numerical calculation.
3. Write a computer program that takes any function $V_0(x)$, and calculates $V(x, y)$ inside the square tube. It should be possible to choose to what order your Fourier series should be summed up to, in order to investigate convergence issues.
4. Plot $V_0(x)$, $V(x, y)$, and $V(x, y)$ on the boundaries (e.g. $V(x, y = L)$) to verify fulfillment of the boundary conditions.
5. Check how fast the calculation converges towards the correct result. In detail, you should compare how many Fourier orders you need for different V_0 functions.

6. Calculate and plot $\vec{E}(x, y)$ using your method of choice.

To plot the potential, $V(x, y)$, there are several possibilities. Possibilities include using a contour plot (`pyplot.contour`) or a filled contour plot (`pyplot.contourf`). To plot the electric field, there are two possible approaches. One is to simply plot the vector arrows (`pyplot.quiver`) with their length being proportional to the electric field. Another possibility is to plot the vector length $|\vec{E}(x, y)|$ as a contour plot, and separately (or on top) plot the field direction by using vector arrows of length 1 (unit vectors). Check out the Matplotlib gallery for inspiration: <http://matplotlib.sourceforge.net/gallery.html>

Some example functions you can play with are:

$$V_0(x) = \begin{cases} V_c \sin\left(\frac{m\pi x}{L}\right), & m = 1, 2, 3, \dots \\ V_c \left[1 - \left(\frac{x}{L} - \frac{1}{2}\right)^4\right] \\ V_c \theta(x - L/2)\theta(3L/4 - x) \end{cases} \quad (2)$$

Here, V_c is a constant that you can choose freely¹, and θ is the Heaviside step function. Play around with some of your own functions as well, and have fun!

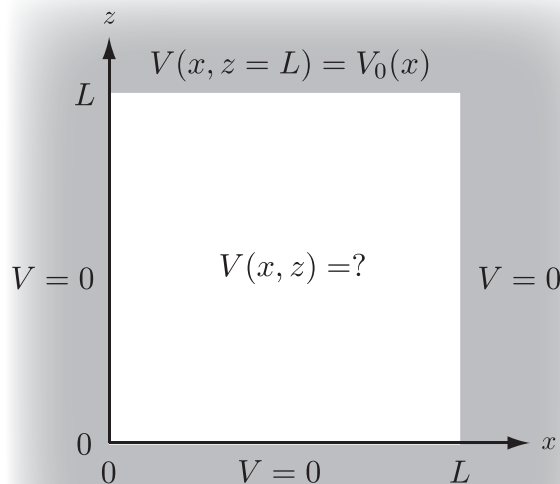


Figure 1: Sketch of the problem geometry.

¹You should “eliminate” V_c in problem 1.