TFY4240
Problemset 7 Autumn 2014

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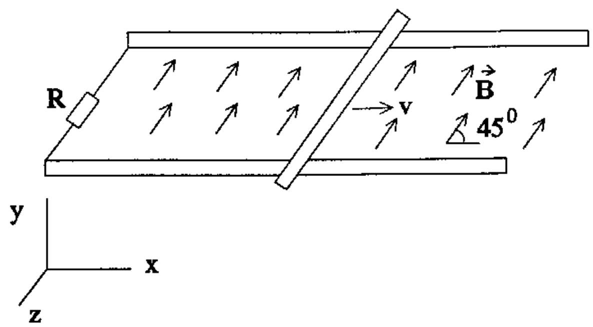
Problem 1.



A conducting rod can slide without frictions along two parallel rails as shown in the figure below. The rails are connected at one end by a resistance R, so that the system form a closed circuit as illustrated in the figure. The circuit is located in the horizontal plane, *i.e.* the xz-plane, with the rails oriented along the x-axis with their left edges at $x = 0$. The system is placed in a magnetic induction $\boldsymbol{B}$ that is in the xy-plane and forms an angle $45°$ with the plane of the circuit. Hence it follows that $B_x > 0$, $B_y > 0$, and $B_z = 0$, and we assume that the field strength is $|\boldsymbol{B}| = B$. The rod moves at constant velocity v towards the right (see figure).

a) Calculate the magnetic flux through the circuit as function of the position x of the metal rod. Obtain an expression for the current in the circuit and give its directing.

b) Due to the current in the circuit and the external magnetic induction a force will act on the rod. Give the magnitude and direction of the force acting on the rod. What is the mechanical power needed to move the rod? Compare this power to the Ohmic heat loss in the resistance R.

c) The force under point b) will also have a vertical component. For sufficiently strong magnetic induction B, the rod will leave the rail for a short period of time. Calculate the smallest value for B needed for this to happen when it is given that mass of the rod is m and the g is the acceleration of gravity.

We will now study how a magnetic induction can be used to make an object levitating (Norwegian : "sveve"). This phenomenon is known as *magnetic levitation* and it is used, for instance, in magnetic levitation train like the Maglev train in Shanghai.

Consider a conducting plane placed in the xz-plane, and where a uniform (time-independent) current density, $\boldsymbol{J}$, is "flowing". This will cause a homogeneous magnetic induction field to exist in the region above the plane. Assume that $\boldsymbol{J}$ is so that the magnitude of the magnetic induction is $B = 2.0$T. A conducting rod is now placed above the plane so that it is parallel to J.

d) What is the smallest current I that has to go through the rod in order to levitate $1,000$ kg/m of the rod? What direction must the current in the rod have?

Problem 2.

Let $\boldsymbol{v}$ be a general vector and $\overline{\overline{T}}$ and $\bar{\bar{t}}$ general tensors of order 2.

a) Write down the coordinate free form for the quantities $\boldsymbol{v}$, and $\bar{\bar{t}}$, and $\overline{\overline{T}}$ in terms of the basis vectors $\hat{\boldsymbol{x}}_i (i = 1, 2, 3)$.

b) Consider the relation

$$\overline{\overline{T}} = \boldsymbol{v} \times \bar{\bar{t}}.$$

Obtain a coordinate free expression for $\overline{\overline{T}}$ expressed in terms of the components of $\boldsymbol{v}$ and $\bar{\bar{t}}$ in addition to the basis vectors. Use this result to obtain the components of T_{ij}.

c) If $\overline{\overline{F}}(\boldsymbol{r})$ is a position dependent tensor or order 2, obtain the components of $\boldsymbol{\nabla} \cdot \overline{\overline{F}}(\boldsymbol{r})$.