TFY4240

Problemset 8 Autumn 2014

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Problem 1.

The *time-average* of a function $f(t)$ is defined as

$$\langle f(t)\rangle_t = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} dt\, f(t).$$

a) Derive that $\langle\cos(\omega t)\rangle_t = \langle\sin(\omega t)\rangle_t = 0$,

b) and that $\langle\cos^2(\omega t)\rangle_t = \langle\sin^2(\omega t)\rangle_t = 1/2$.

Problem 2.

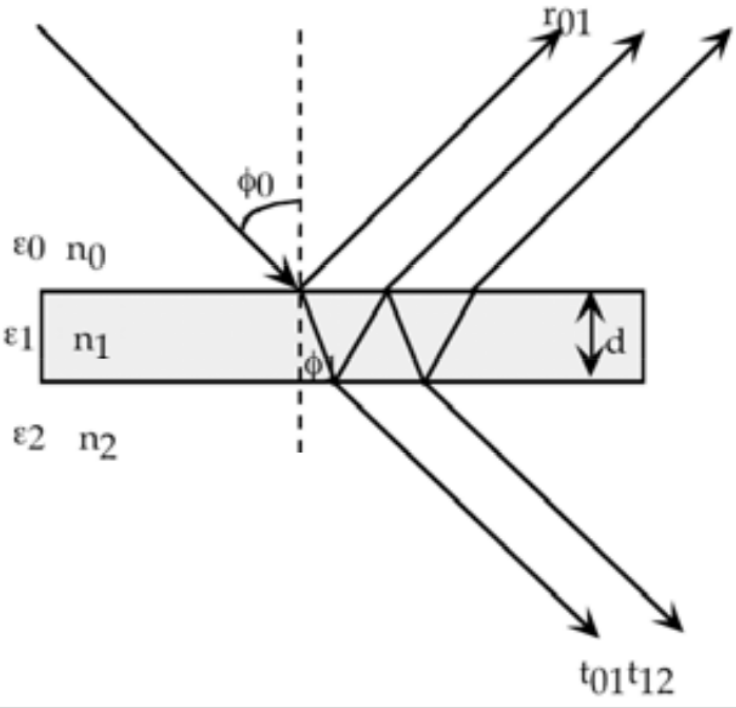

Figure 1: Film geometry

In this problem we will consider the scattering and transmission of light through a film of thickness d as shown in the figure below. The medium of incidence, film and substrate are numbered 0, 1 and 2, respectively. Medium i has dielectric constant ε_i (and refractive index $n_i = \sqrt{\varepsilon_i}$. Let r_{01} denote the Fresnel amplitude for reflection at the interface separating medium 0 and 1, and t_{01} denotes the corresponding transmission amplitude (for a given angle of incidence and linear polarization). The amplitudes r_{12} and t_{12} are defined in a similar fashion, but now for the interface separating media 1 and 2.

a) Give the four boundary conditions that the electromagnetic field has to satisfy at any interface. Describe in words how these conditions can be used to solve the scattering problem.

b) The above approach based on the boundary conditions is straight forward, but somewhat lengthy. However, an alternative approach is to use what we already know for the refection/transmission at a single interface and to use a geometric interpretation for the scattering process.

Show that the reflection and transmission amplitudes, r and t, for the film system can be expressed as

$$r = \frac{r_{01} + r_{12}\exp\left(2\mathrm{i}\delta_1\right)}{1 + r_{12}r_{01}\exp\left(2\mathrm{i}\delta_1\right)}, \tag{1}$$

and

$$t = \frac{t_{01}t_{12}\exp\left(\mathrm{i}\delta_1\right)}{1 + r_{12}r_{01}\exp\left(2\mathrm{i}\delta_1\right)}, \tag{2}$$

where δ_1 is a phase constant. *Hint*: take advantage of the sum of a geometric series and use that $r_{01}^2 + t_{01}t_{10} = 1$.

c) Determine the phase constant in terms of relevant quantities.

d) Show that the following relation exist

$$r_{02} = \frac{r_{01} + r_{12}}{1 + r_{12}r_{01}}. \tag{3}$$

For simplicity, you may assume normal incidence when showing this. Moreover, explain in words why this relation has to be satisfied.

e) A microwave antenna producing waves at a frequency of $10GHz$ is built into a plastic box. What is the thinnest wall-thickness that the box must have in order to be"invisible" for the microwaves? Assume that the microwaves are incident normally onto the walls of the box. The refractive index of the plastic material is 2.5