TFY4240

Problemset 9 Autumn 2014

NTNU

Institutt for fysikk

Problem 1.

Let the scalar potential, $V(\boldsymbol{r}, t)$, and vector potential, $\boldsymbol{A}(\boldsymbol{r}, t)$, be given in some arbitrary gauge. Make a gauge transformation

$$V(\boldsymbol{r}, t) \rightarrow V'(\boldsymbol{r}, t) = V(\boldsymbol{r}, t) - \partial_t \Lambda(\boldsymbol{r}, t), \tag{1a}$$

$$\boldsymbol{A}'(\boldsymbol{r}, t) \rightarrow \boldsymbol{A}'(\boldsymbol{r}, t) = \boldsymbol{A}(\boldsymbol{r}, t) + \boldsymbol{\nabla} \Lambda(\boldsymbol{r}, t), \tag{1b}$$

defined via the gauge function $\Lambda(\boldsymbol{r}, t)$, so that the resulting potentials $V'(\boldsymbol{r}, t)$ and $\boldsymbol{A}'(\boldsymbol{r}, t)$ are in the Lorentz gauge. Obtain the equation satisfied by $\Lambda(\boldsymbol{r}, t)$, and identify in particular the source terms for this equation?

Problem 2.

In the lectures, it was shown that the vector potential $\boldsymbol{A}(\boldsymbol{r}, t)$ is given by the relation

$$\boldsymbol{A}(\boldsymbol{r}, t) = \frac{\mu_0}{4\pi} \int \mathrm{d}^3 r' \, \frac{\boldsymbol{J}(\boldsymbol{r}', t_r)}{R}, \tag{2a}$$

where

$$t_r = t - \frac{R}{c}, \tag{2b}$$

$$R = \left| \boldsymbol{r} - \boldsymbol{r}' \right|. \tag{2c}$$

Use this relation to obtain the expression for the magnetic induction $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ and show that it can be written as:

$$\boldsymbol{B}(\boldsymbol{r}, t) = \frac{\mu_0}{4\pi} \int \mathrm{d}^3 r' \left[\frac{\boldsymbol{J}(\boldsymbol{r}', t_r)}{R^2} + \frac{\dot{\boldsymbol{J}}(\boldsymbol{r}', t_r)}{cR} \right] \times \hat{\boldsymbol{R}}. \tag{3}$$