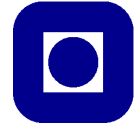


TFY4240

Problemset 9 Autumn 2014

NTNU


 Institutt for
fysikk
Problem 1.

Let the scalar potential, $V(\mathbf{r}, t)$, and vector potential, $\mathbf{A}(\mathbf{r}, t)$, be given in some arbitrary gauge. Make a gauge transformation

$$V(\mathbf{r}, t) \rightarrow V'(\mathbf{r}, t) = V(\mathbf{r}, t) - \partial_t \Lambda(\mathbf{r}, t), \quad (1a)$$

$$\mathbf{A}'(\mathbf{r}, t) \rightarrow \mathbf{A}'(\mathbf{r}, t) = \mathbf{A}(\mathbf{r}, t) + \nabla \Lambda(\mathbf{r}, t), \quad (1b)$$

defined via the gauge function $\Lambda(\mathbf{r}, t)$, so that the resulting potentials $V'(\mathbf{r}, t)$ and $\mathbf{A}'(\mathbf{r}, t)$ are in the Lorentz gauge. Obtain the equation satisfied by $\Lambda(\mathbf{r}, t)$, and identify in particular the source terms for this equation?

Problem 2.

In the lectures, it was shown that the vector potential $\mathbf{A}(\mathbf{r}, t)$ is given by the relation

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int d^3r' \frac{\mathbf{J}(\mathbf{r}', t_r)}{R}, \quad (2a)$$

where

$$t_r = t - \frac{R}{c}, \quad (2b)$$

$$R = |\mathbf{r} - \mathbf{r}'|. \quad (2c)$$

Use this relation to obtain the expression for the magnetic induction $\mathbf{B} = \nabla \times \mathbf{A}$ and show that it can be written as:

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int d^3r' \left[\frac{\mathbf{J}(\mathbf{r}', t_r)}{R^2} + \frac{\dot{\mathbf{J}}(\mathbf{r}', t_r)}{cR} \right] \times \hat{\mathbf{R}}. \quad (3)$$