TFY4240

Solution problemset 2 Autumn 2014

NTNU
Institutt for fysikk

Problem 1.
See Griffiths and lecture notes

Problem 2.

a) The method of images consists of replacing the electrostatic problem we are trying to solve with another so-called equivalent problem. The equivalent problem's solution should satisfy the same boundary conditions as the original electrostatic problem. Due to the theorem of uniqueness and its corollary, this solution is thus the same solution as the solution to our original problem.

As the metal plane is grounded, the potential has to be zero everywhere on the surface of the metal plane:

$$V|_{z=-a} = 0.$$

In addition, the normal derivative of the potential is discontinuous at the interface, due to the discontinuity of $\mathbf{D}$:

$$\begin{aligned} \mathbf{D}^{\perp}|_{z=-a} &= \sigma_f, \text{ or} \\ -\varepsilon_0 \partial_z V|_{z=-a} &= \sigma_f. \end{aligned}$$

Here, σ_f is the free surface charge density.

b) This problem is solved in Griffiths, Chapter 3.2.1. After translating the coordinate system, the solution reads:

$$V(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0}\left\{\left[x^2+y^2+z^2\right]^{-1/2} - \left[x^2+y^2+(z+2a)^2\right]^{-1/2}\right\}.$$

Inserting $z=-a$, we easily see that the two terms cancel, and that $V(z=-a)=0$.

c) This problem can seem tricky, but it is not so hard if you are systematic. We simply need to add up the contribution from an infinite number of point charges. The 'zeroth" term stems from the original point charge at the origin. Then, the next term stems from two 'mirror charges;" one for each plane. But, each of these mirror charges need a mirror charge for the opposite plane... and so on. Continuing this summation gives us positive and negative charges at:

$$\begin{aligned} q \text{ at } z &= 4na, & n &= 0, \pm 1, \pm 2, \ldots \\ -q \text{ at } z &= (4n+2)a, & n &= 0, \pm 1, \pm 2, \ldots \end{aligned}$$

The analytical expression for V is somewhat involved and it was not a requirement to state this on the exam. It is simply found by summing up the contributions from all the point charges.

d) This is done by taking the normal derivative of the potential at the interface and using the discontinuity of the normal derivative of the field, $\partial_z V|_{z=-a}$. Griffiths does this for the 'simple" image problem in Chapter 3.2.1.